7 November, 1940 – Four months after the bridge’s completion, the middle section of the Tacoma Narrows Bridge across the Tacoma Narrows in state of Washington collapsed in a windstorm.
\[ \sigma_x = \frac{My}{I_z} \]

\[ \sigma_x = \frac{(M + \Delta M)y}{I_z} \]
\[ \sum F_x = \int_{A'} \frac{M}{I_z} y dA' - \int_{A'} \frac{M + \Delta M}{I_z} y dA' + F_H = 0 \]

\[ - \int_{A'} \frac{\Delta M}{I_z} y dA' + F_H = 0 \]

\[ F_H = \int_{A'} \frac{\Delta M}{I_z} y dA' \]

\[ Q = \int_{A'} y dA' \]

\[ F_H = \frac{\Delta MQ}{I_z} \]

\[ \tau = \frac{VQ}{I_z t} \]

\[ t = \text{width of the section at some depth in the beam} \]

\[ Q = \sum \overline{y_i} A_i \]
Shear Stress in Beams II
9.9 A 1.6-m long cantilever beam supports a concentrated load of 7.2 kN, as shown below. The beam is made of a rectangular timber having a width of 120 mm and a depth of 280 mm. Calculate the maximum horizontal shear stresses at points located 35 mm, 70 mm, 105 mm, and 140 mm below the top surface of the beam. From these results, plot a graph showing the distribution of shear stresses from top to bottom of the beam.

Fig. P9.9a Cantilever beam

Fig. P9.9b Cross-sectional dimensions

Solution

Shear for cantilever beam:

\[ V = 7.2 \text{ kN} = 7,200 \text{ N} \]

Shear stress formula:

\[ \tau = \frac{VQ}{It} \]

Section properties:

\[ I = \frac{1}{12} (120 \text{ mm})(280 \text{ mm})^3 = 219.52 \times 10^6 \text{ mm}^4 \]

Distance below top surface of beam:

- 35 mm: \[ \tau = \frac{7200 \times 35}{219.52 \times 10^6} = 105.6 \text{ kPa} \]
- 70 mm: \[ \tau = \frac{7200 \times 70}{219.52 \times 10^6} = 211 \text{ kPa} \]
- 105 mm: \[ \tau = \frac{7200 \times 105}{219.52 \times 10^6} = 301 \text{ kPa} \]
- 140 mm: \[ \tau = \frac{7200 \times 140}{219.52 \times 10^6} = 321 \text{ kPa} \]

Homework

- Problem P9.10
- Problem P9.11
- Problem P9.13