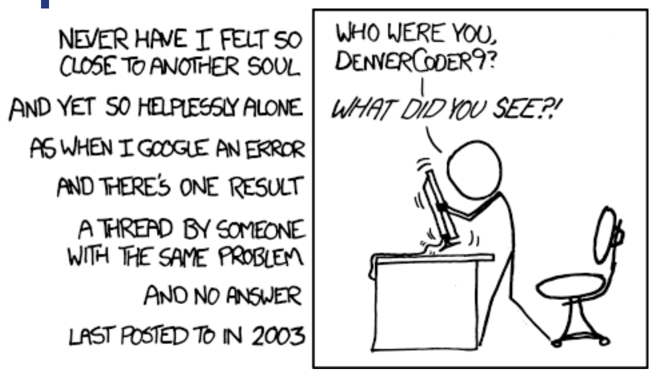
Mechanics of Materials
CIVL 3322 / MECH 3322
Deflection of Beams



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The Elastic Curve

- The deflection of a beam must often be limited in order to provide integrity and stability of a structure or machine, or
- To prevent any attached brittle materials from cracking

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The Elastic Curve

- Deflections at specific points on a beam must be determined in order to analyze a statically indeterminate system.

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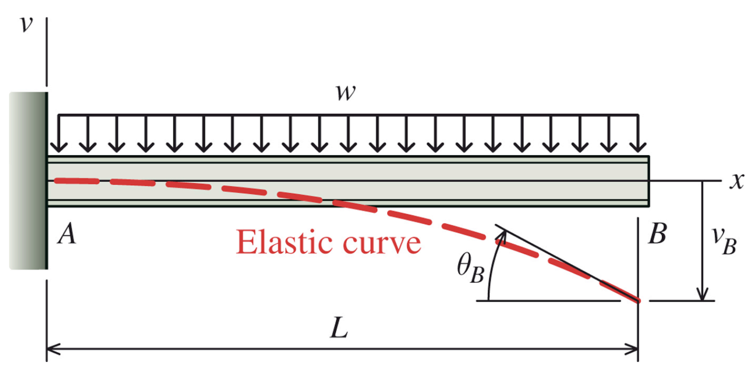
The Elastic Curve

- The curve that is formed by the plotting the position of the centroid of the beam along the longitudal axis is known as the elastic curve.

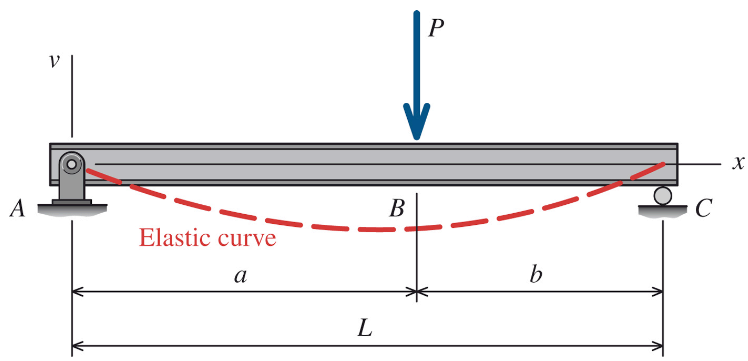
The Elastic Curve

- At different types of supports, information that is used in developing the elastic curve are provided
  - Supports which resist a force, such as a pin, restrict displacement
  - Supports which resist a moment, such as a fixed end support, resist displacement and rotation or slope

The Elastic Curve



The Elastic Curve



The Elastic Curve

- We can derive an expression for the curvature of the elastic curve at any point where ρ is the radius of curvature of the elastic curve at the point in question

$$\frac{1}{\rho} = \frac{M}{EI}$$

The Elastic Curve

- If you make the assumption to deflections are very small and that the slope of the elastic curve at any point is very small, the curvature can be approximated at any point by

$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

v is the deflection of the elastic curve

The Elastic Curve

- We can rearrange terms

$$EI\frac{d^2v}{dx^2} = M$$

The Elastic Curve

- Differentiate both sides with respect to x

$$\frac{d}{dx}\left(EI\frac{d^2v}{dx^2}\right)=\frac{dM}{dx}=V(x)$$

The Elastic Curve

- And again differentiate both sides with respect to x

$$\frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right)=\frac{dV}{dx}=w(x)$$

The Elastic Curve

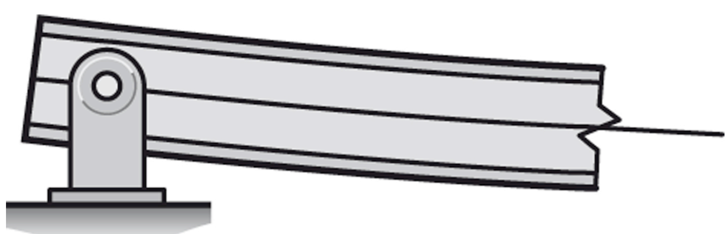
- So there are three paths to finding v

$$EI\frac{d^4v}{dx^4} = w(x)$$

$$EI\frac{d^3v}{dx^3} = V(x)$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

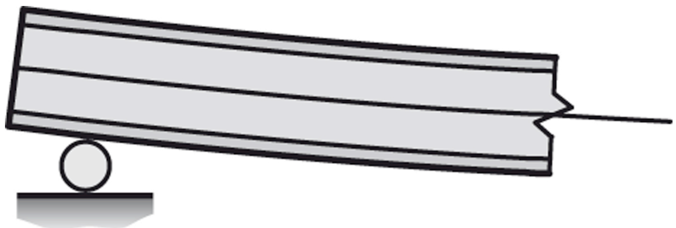
Boundary Conditions



$$v = 0$$

Pin support

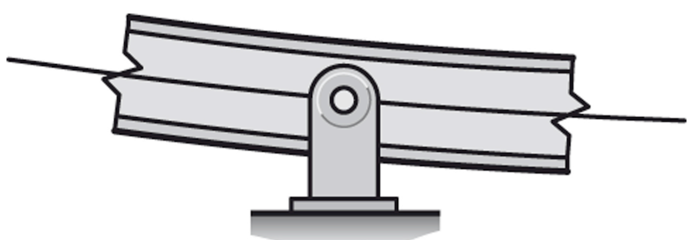
Boundary Conditions



$$v = 0$$

Roller support

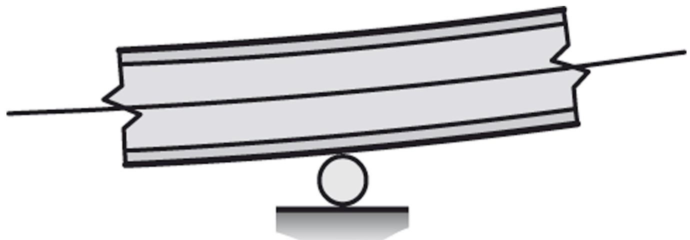
Boundary Conditions



$$v = 0$$

Pin support

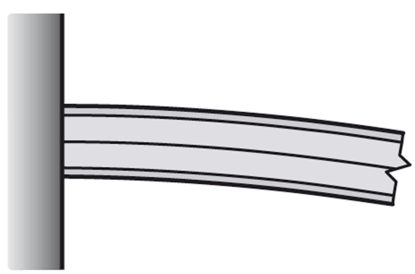
Boundary Conditions



$$v = 0$$

Roller support

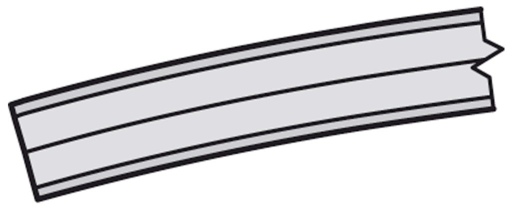
Boundary Conditions



$$dv/dx = 0$$
$$v = 0$$

Fixed support

Boundary Conditions

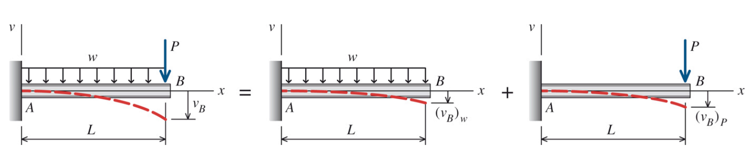


$$V = 0$$

$$M = 0$$

Free end

Combining Load Conditions



Cantilever Example

- Given a cantilevered beam with a fixed end support at the right end and a load P applied at the left end of the beam.
- The beam has a length of L.

Cantilever Example

- If we define x as the distance to the right from the applied load P, then the moment function at any distance x is given as

$$M(x) = -Px$$

Cantilever Example

- Since we have a function for M along the beam we can use the expression relating the moment and the deflection

$$M\left(x\right) = -Px$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = -Px$$

Cantilever Example

- Isolating the variables and integrating

$$EI\left(\frac{d^2v}{dx^2}\right) = -Px$$

$$EI\left(\frac{dv}{dx}\right) = -\frac{Px^2}{2} + C_1$$

Cantilever Example

- Integrating again

$$EI\left(\frac{dv}{dx}\right) = -\frac{Px^2}{2} + C_1$$

$$EIv = -\frac{Px^3}{6} + C_1 x + C_2$$

Cantilever Example

- To be able to define the function for v, we need to evaluate C_1 and C_2

$$EI\left(\frac{dv}{dx}\right) = -\frac{Px^2}{2} + C_1$$

$$EIv = -\frac{Px^3}{6} + C_1 x + C_2$$

Cantilever Example

- The right end of the beam is supported by a fixed end support therefore the slope of the deflection curve is 0 and the deflection is 0

$$EI\left(\frac{dv}{dx}\right) = -\frac{Px^2}{2} + C_1$$

$$EIv = -\frac{Px^3}{6} + C_1x + C_2$$

Cantilever Example

- In terms of boundary conditions this means

$$EI\left(\frac{dv}{dx}\right) = -\frac{Px^2}{2} + C_1 \qquad x = L : \frac{dv}{dx} = 0$$

$$EIv = -\frac{Px^3}{6} + C_1x + C_2 \qquad x = L : v = 0$$

Cantilever Example

- Evaluating the expressions at the boundary conditions

$$EI\left(\frac{dv}{dx}\right)=-\frac{Px^2}{2}+C_1 \qquad x=L:\frac{dv}{dx}=0$$

$$EI(0)=-\frac{PL^2}{2}+C_1\Rightarrow C_1=\frac{PL^2}{2} \qquad x=L:v=0$$

$$EIv=-\frac{Px^3}{6}+\frac{PL^2}{2}x+C_2$$

$$EI(0)=-\frac{PL^3}{6}+\frac{PL^2}{2}L+C_2\Rightarrow C_2=-\frac{PL^3}{3}$$

Cantilever Example

- So the expression for the slope (θ) and the deflection (v) are given by

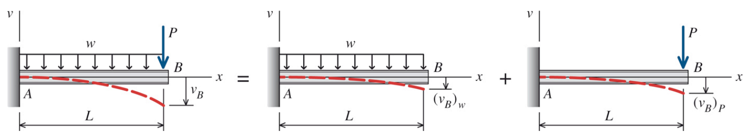
$$\theta=\frac{1}{EI}\left(-\frac{Px^2}{2}+\frac{PL^2}{2}\right) \qquad x=L:\frac{dv}{dx}=0$$

$$\theta=\frac{P}{2EI}\left(L^2-x^2\right) \qquad x=L:v=0$$

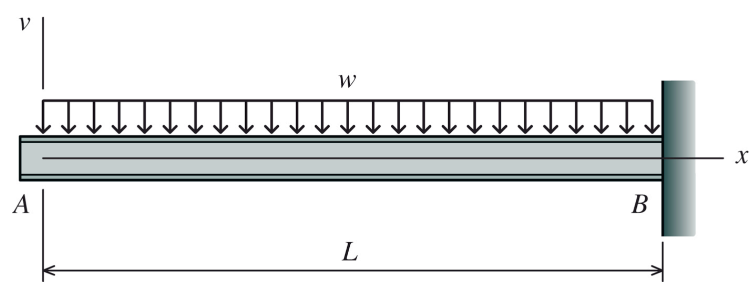
$$v=\frac{1}{EI}\left(-\frac{Px^3}{6}+\frac{PL^2}{2}x-\frac{PL^3}{3}\right)$$

$$v=\frac{P}{6EI}\left(-x^3+3xL^2-2L^3\right)$$

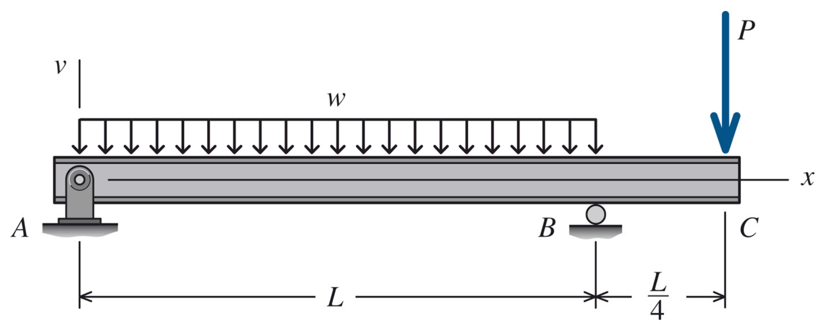
Combining Load Conditions



Example 1



Example 1



Homework

- Show a plot of the shear, bending moment, slope, and deflection curves identifying the maximum, minimum, and zero points for each curve. Use separate plots for each function.
- Show the mathematical expression(s) for each function.
- Problem P10.4
- Problem P10.8
- Problem P10.11