Thermal Strain

“Every day is a good day.”
Yun-Men

Real World Experience

Have you even put a soft drink or other beverage that comes in a can into the freezer, forgot about it, and then come back later to find the can ruptured and the freezer a mess?
If you have, you have experienced thermal strain first hand

Most materials when heated, tend to expand
As energy is added, the atoms in the material absorb some of the energy and actually vibrate at a higher frequency and tend to want to move apart (simple simple science explanation)

Also, when the same material is cooled, it will contract
Unfortunately for you and your beverage, water doesn’t behave like that
It expands when it cools to a point close to freezing
That is very good for life on earth
But back to mechanics of materials.
When typical engineering materials are heated, they tend to expand.
The amount that they expand with respect to their original length is known as thermal strain.

Within limits, the amount of thermal strain is linearly proportional to change in temperature of the material.
The thermal strain can then be calculated as

\[ \varepsilon_T = \alpha \Delta T \]

\( \varepsilon_T \) is the amount of thermal strain.
\( \alpha \) is the coefficient of thermal expansion.
\( \Delta T \) is the change in temperature.

Take care that your units match.
Thermal Strain

If the material is constrained when the temperature is changed, the thermal strain will not be able to be expressed and so a strain in the opposite direction will have to be developed.

Either that or the material buckles

\[ \varepsilon_T = \alpha \Delta T \]

Thermal Strain

This strain which offsets the thermal strain will have to be developed from some other stress in the material.

This responsive stress will generate a strain that is equal in magnitude and opposite in direction to the thermal strain.

\[ \varepsilon_T = \alpha \Delta T \]

Thermal Strain

If we heat a constrained material with an axial length \( L \), the amount of extension that it would have if unconstrained would be

\[ \varepsilon_T = \alpha \Delta T = \frac{\delta_T}{L} \]

\[ \delta_T = \alpha \Delta TL \]

Thermal Strain

Since the material was constrained, the same amount of strain would have to be developed in the opposite direction.

\[ \varepsilon_T = \alpha \Delta T = \frac{\delta_T}{L} \]

\[ \delta_T = \alpha \Delta TL \]
Thermal Strain

This would mean that the elongation would have to sum to 0

\[ \delta_T = \alpha \Delta TL \]

\[ \delta_T - \delta = 0 \]

Thermal Strain

The axial stress offsetting the thermal stress is generated by some force, \( P \), so we would have \( \delta_r = \alpha \Delta TL \)

\[ \delta = \frac{PL}{EA} \]

\[ \delta_r - \delta = 0 \]

\[ \alpha \Delta TL - \frac{PL}{EA} = 0 \]

Thermal Strain

Rearranging and dividing by \( L \) we have

\[ \alpha \Delta T - \frac{P}{EA} = 0 \]

Therefore the amount of force that the thermal stress will generate within a constrained object can be calculated as

\[ P = EA \alpha \Delta T \]
Thermal Strain

This is what breaks sidewalks apart when water enters pore spaces and expands and contracts

\[ P = E A \alpha \Delta T \]

Problem 3-5.12

3-5.12. The prismatic bar is made of material with modulus of elasticity \( E = 28 \times 10^6 \text{psi} \) and coefficient of thermal expansion \( \alpha = 8 \times 10^{-6} \text{°F}^{-1} \). It is fixed to a rigid wall at the left. There is a gap \( h = 0.002 \text{ in.} \) between the bar’s right end and the rigid wall. If the temperature is increased by 50°F above the bar’s initial temperature \( T \), what is the normal stress on a plane perpendicular to the bar’s axis?

Homework

- 3-5.4
- 3-5.7
- 3-5.12