Civil Engineering Hydraulics
Mechanics of Fluids

Simple Piping Problems

March 21th, 1685 – Happy Birthday
Johann Sebastian Bach

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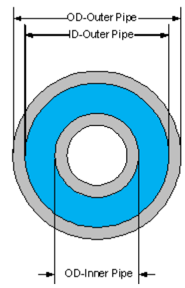
Pipe Geometries

- In the text, problems are worked out for pipes of noncircular cross sections.
- While the derivations are fine and work well, you can also use an equivalent hydraulic diameter to work on these problems.

Pipe Geometries

An annulus is developed by putting one pipe inside of another so that the center of the pipes are aligned. This is a very common setup in heat exchangers.

The blue area is the flow area we are concerned with.

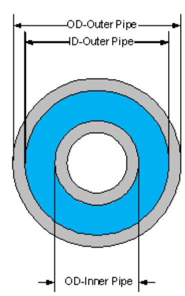


Pipe Geometries

Outer pipe is a 4-nominal, schedule 40 pipe. Inner pipe is a 2-nominal, schedule 40 pipe. we can get the inner and outer diameters for the pipes from Table C.1.

$ID_{Outer} := 0.3355ft$

$OD_{Inner} := 2.375in = 0.198\,ft$



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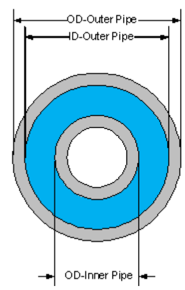
Pipe Geometries

To calculate a hydraulic diameter, we need the area and the wetted perimeter.

$$\text{Area} := \pi \cdot \frac{ID_{Outer}^{\ 2}}{4} - \pi \cdot \frac{OD_{Inner}^{\ 2}}{4} = 0.058\,\text{ft}^2$$

$$WP := \pi \cdot ID_{Outer} + \pi \cdot OD_{Inner} = 1.676\,\text{ft}$$



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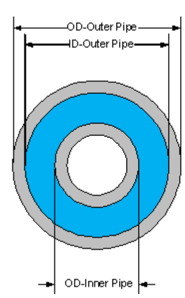
Pipe Geometries

So the equivalent D_h is

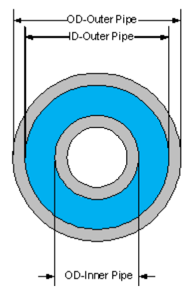
$$D_h := 4 \cdot \frac{\text{Area}}{WP} = 0.138\,\text{ft}$$



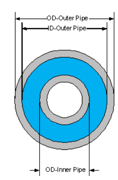
Pipe Geometries

Calculate the average flow velocity using our normal method

$$Q := 0.3\,\frac{ft^3}{s} \qquad v := \frac{Q}{Area} = 5.205\,\frac{ft}{s}$$



Pipe Geometries



Now we can calculate the Re for the flow

From Table A.5

$$\rho := 1.265 \cdot 1.94\,\frac{slug}{ft^3} = 2.454\,\frac{slug}{ft^3} \qquad \rho = 78.958\,\frac{lbm}{ft^3}$$

$$\mu := 0.752 \cdot 10^{-5}\,\frac{lbf \cdot s}{ft^2}$$

$$Re := \frac{\rho \cdot v \cdot D_h}{\mu} = 2.337 \times 10^5$$

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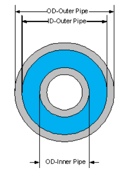
Pipe Geometries



For a cast iron pipe the ε is 0.00085 from Table 5.2 so we can calculate the ε/D_h

$$\varepsilon := 0.00085\text{ft} \qquad \text{RoughnessRatio} := \frac{\varepsilon}{D_h} = 6.178 \times 10^{-3}$$

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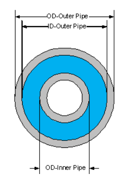
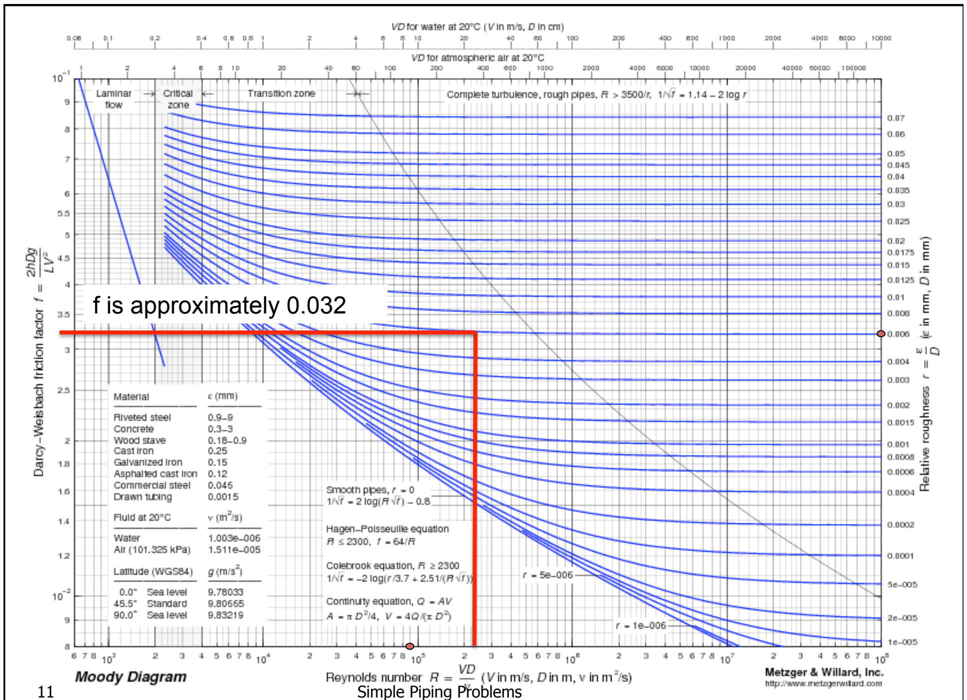
Pipe Geometries



We can use the Moody Diagram to calculate the friction factor, f



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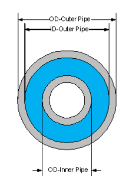
Pipe Geometries



Now we can use this friction factor to calculate the head loss in this section of pipe.

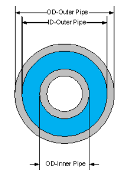
$$f := 0.032$$

$$L := 15\text{ft}$$

$$\Delta p := \frac{f \cdot L}{D_h} \cdot \frac{\rho \cdot v^2}{2} = 115.967 \frac{\text{lbf}}{\text{ft}^2}$$

In this case, all the pressure loss was due to friction.

Pipe Geometries



This is the most common way to work this type of problem.

If the pipes made of different materials you could develop an average relative roughness

$$\frac{\varepsilon}{D_{average}} = \frac{\pi D_{outer} \dfrac{\varepsilon}{D_{outer}} + \pi D_{inner} \dfrac{\varepsilon}{D_{inner}}}{\pi D_{outer} + \pi D_{inner}}$$

Pipe Geometries

Example problem 5.5 gives a different type of problem where we know the pressure drop over a length of pipe and we want to find the volumetric flow rate.

Pipe Geometries

Benzene flows through a 12-nominal schedule 80 wrought iron pipe. The pressure drop measured at points 350 m apart is 34 kPa. Determine the flow rate through the pipe.

Pipe Geometries

Using something like Solver in EXCEL, this isn't a difficult problem. However, if you need to do this problem by an iterative method, it can be solved.

Pipe Geometries

Since the pipe has a constant cross section and is horizontal, all the of the pressure loss is due to friction so we have the expression

$$p_1 - p_2 = \frac{\rho v^2}{2}\frac{fL}{D_h}$$

Pipe Geometries

The problem we have is the f is a function of v through the Re and we don't have a direct solution

$$p_1 - p_2 = \frac{\rho v^2}{2}\frac{fL}{D_h}$$

Pipe Geometries

From what is given in the problem and from what we can look up in tables we have

$\rho := 0.876 \cdot 1000 \frac{kg}{m^3}$ $\mu := 0.601 \cdot 10^{-3} \frac{N \cdot s}{m^2}$

$D := 28.89cm = 0.289\,m$ $A := 655.50cm^2 = 0.066\,m^2$

$\varepsilon := 0.0046cm = 4.6 \times 10^{-5}\,m$

$\Delta p := 34kPa$ $L := 350m$

Pipe Geometries

Rearranging the Bernoulli expression

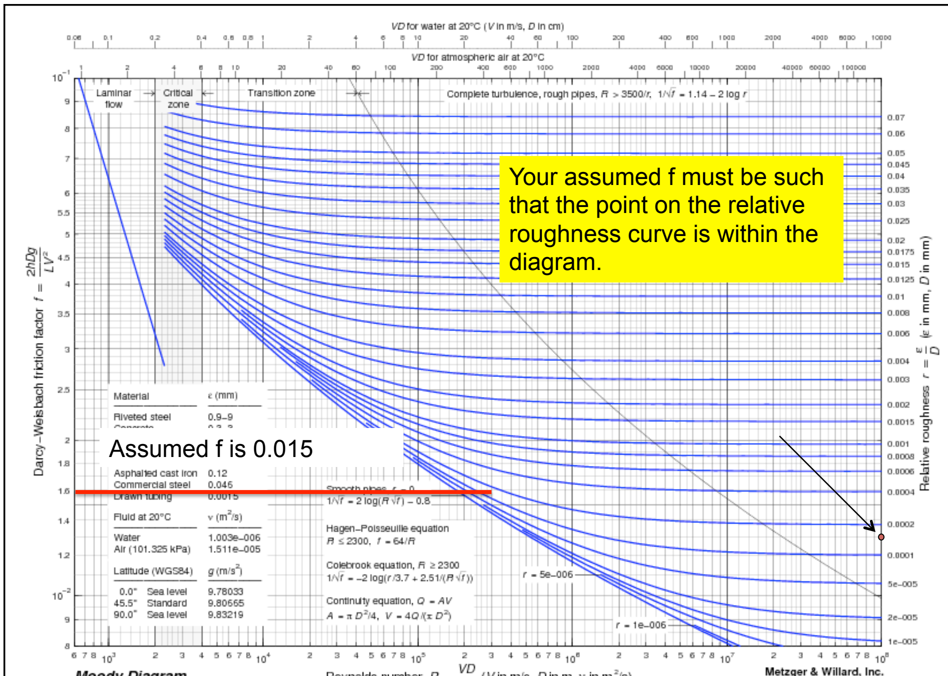
$$\Delta p = \frac{\rho v^2}{2} \frac{fL}{D_h}$$

$$\frac{2 D_h \Delta p}{\rho L} = v^2 f$$

$$v = \sqrt{\frac{2 D_h \Delta p}{f \rho L}}$$

Pipe Geometries

Now we can start by assuming a friction factor, using the roughness ratio, and getting a Re from there.



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Pipe Geometries

From the assumed f, calculate the v and the Re

$$f := 0.015 \qquad v := \frac{\sqrt{\frac{2 \cdot D_h \cdot \Delta p}{\rho \cdot L}}}{\sqrt{f}} = 2.067 \frac{m}{s}$$

$$Re := \frac{\rho \cdot v \cdot D_h}{\mu} = 8.703 \times 10^5$$

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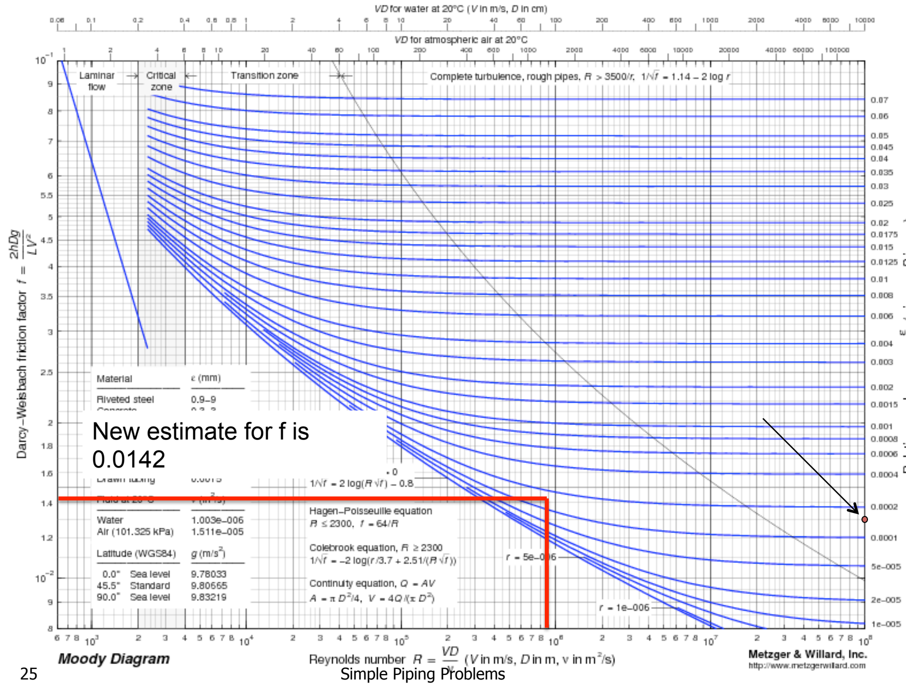
Pipe Geometries

Now, using this Re, you can go back to the Moody diagram and calculate a new estimate for f



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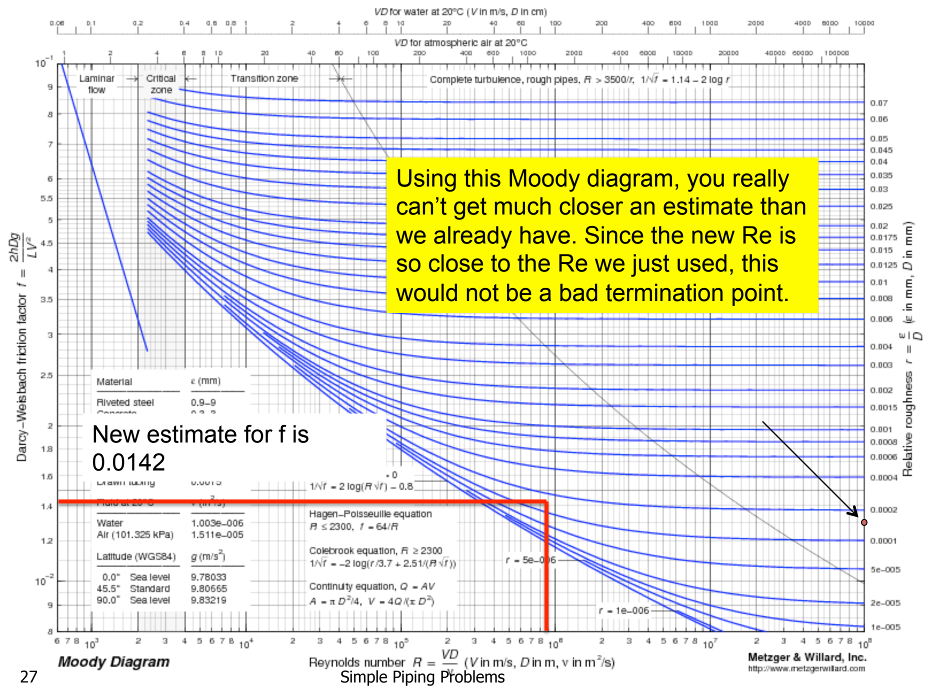
Pipe Geometries

And you can iterate the process based on the new estimate for f

$$f := 0.0142 \qquad v := \frac{\sqrt{\frac{2 \cdot D_h \cdot \Delta p}{\rho \cdot L}}}{\sqrt{f}} = 2.124\,\frac{m}{s}$$

$$Re := \frac{\rho \cdot v \cdot D_h}{\mu} = 8.945 \times 10^5$$



Pipe Geometries

So out volumetric flow rate is

$$Q := v \cdot A = 0.139 \frac{m^3}{s}$$

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Pipe Geometries

There are expressions for approximating the friction factor that do not rely on the Moody diagram.

These are all empirical equations but they do have the ability to be solved directly.

The forms are given on page 246 and 247 of the 4th edition of the text and page 267 of the 3rd edition (only two are given in the 3rd edition).

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Pipe Geometries

One of the ones not given in the 3rd edition is the Swamee-Jain equation

$$f = \frac{0.250}{\left[\log\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2}$$

Pipe Geometries

One of the ones not given in the 3rd edition is the Swamee-Jain equation

$$f = \frac{0.250}{\left[\log\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2}$$

$$\mathrm{Re} = \left(\frac{5.74}{e^{\sqrt{\frac{0.250}{f}}} - \frac{\varepsilon}{3.7D}}\right)^{\frac{1}{0.9}}$$

Pipe Geometries

Using the equation for f rather than the Moody diagram for iteration

$$f := 0.015 \qquad v := \frac{\sqrt{\frac{2 \cdot D_h \cdot \Delta p}{\rho \cdot L}}}{\sqrt{f}} = 2.067 \, \frac{m}{s}$$

$$\mathrm{Re} := \frac{\rho \cdot v \cdot D_h}{\mu} = 8.703 \times 10^5$$

$$f := \frac{0.250}{\left(\log\left(\frac{\varepsilon}{3.7 \cdot D_h} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right)^2} = 0.014$$

Pipe Geometries

Second Iteration

$$v := \frac{\sqrt{\frac{2 \cdot D_h \cdot \Delta p}{\rho \cdot L}}}{\sqrt{f}} = 2.107 \frac{m}{s}$$

$$Re := \frac{\rho \cdot v \cdot D_h}{\mu} = 8.871 \times 10^5$$

$$f := \frac{0.250}{\left(\log\left(\frac{\varepsilon}{3.7 \cdot D_h} + \frac{5.74}{Re^{0.9}}\right)\right)^2} = 0.014$$

Pipe Geometries

Third Iteration

$$v := \frac{\sqrt{\frac{2 \cdot D_h \cdot \Delta p}{\rho \cdot L}}}{\sqrt{f}} = 2.108 \frac{m}{s}$$

$$Q := v \cdot A = 0.138 \frac{m^3}{s}$$

$$Re := \frac{\rho \cdot v \cdot D_h}{\mu} = 2.091 \times 10^6$$

$$f := \frac{0.250}{\left(\log\left(\frac{\varepsilon}{3.7 \cdot D_h} + \frac{5.74}{Re^{0.9}}\right)\right)^2} = 0.014$$

Pipe Geometries

A third type of problem is where we are given the flow and the head loss and asked to choose a pipe size.

Example 5.7

Pipe Geometries

From what we are given and from the Tables in the text.

$$\rho := 0.701 \cdot 1.94 \frac{\text{slug}}{\text{ft}^3} = 43.755 \frac{\text{lbm}}{\text{ft}^3}$$

$$\mu := 1.07 \cdot 10^{-5} \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}$$

$$Q := 2 \frac{\text{ft}^3}{\text{s}} \qquad \Delta p := 40 \frac{\text{lbf}}{\text{in}^2} = 5.76 \times 10^3 \frac{\text{lbf}}{\text{ft}^2}$$

$$L := 800\text{ft}$$

Pipe Geometries

For the value of ε, the table gives a range of values. With no other information, the best estimate is to take the average of the upper and lower ranges.

$$\varepsilon := \frac{0.003\text{ft} + 0.03\text{ft}}{2} = 0.017\text{ ft}$$

Pipe Geometries

Since the pipe is horizontal and has a constant cross section, all the of the pressure loss will be due to friction. This reduces the Bernoulli equation to.

$$p_1 - p_2 = \frac{\rho v^2}{2} \frac{fL}{D_h}$$

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Pipe Geometries

$$p_1 - p_2 = \frac{\rho v^2}{2}\frac{fL}{D_h}$$

Collect all the known information on the left side

$$\frac{2\Delta p}{\rho L} = v^2 \frac{f}{D_h}$$

Since we are dealing with a pipe with a circular cross section, Dh is the pipe diameter

$$v = \frac{Q}{A} = \frac{Q}{\frac{\pi D^2}{4}}$$

$$\frac{2\Delta p}{\rho L} = \left(\frac{Q}{\frac{\pi D^2}{4}}\right)^2 \frac{f}{D_h} \Leftrightarrow \frac{2\Delta p}{\rho L} = \frac{Q^2}{\left(\frac{\pi}{4}\right)^2}\frac{f}{D_h^5} \Leftrightarrow \frac{2\Delta p}{\rho L}\frac{\left(\frac{\pi}{4}\right)^2}{Q^2} = \frac{f}{D_h^5}$$

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Pipe Geometries

This just sets up all the calculations as functions to make things a bit easier.

$$\text{Area}(\text{diameter}) := \frac{\pi \cdot \text{diameter}^2}{4}$$

$$\text{Re}(\text{density}, \text{velocity}, \text{diameter}, \text{viscosity}) := \frac{\text{density} \cdot \text{velocity} \cdot \text{diameter}}{\text{viscosity}}$$

$$\text{velocity}(\text{flowrate}, \text{area}) := \frac{\text{flowrate}}{\text{area}}$$

$$f(\text{diameter}) := \frac{0.250}{\left(\log\left(\frac{\varepsilon}{3.7 \cdot \text{diameter}} + \frac{5.74}{\text{Re}(\rho, \text{velocity}(Q, \text{Area}(\text{diameter})), \text{diameter}, \mu)^{0.9}}\right)\right)^2}$$

$$D(f) := \left[\frac{f}{\left[\frac{2 \cdot \Delta p \cdot \left(\frac{\pi}{4}\right)^2}{\rho \cdot L \cdot Q^2}\right]}\right]^{\frac{1}{5}}$$

Pipe Geometries

This just sets up all the calculations as functions to make things a bit easier.

We will start by assuming an f of 0.01 and calculating the diameter which would give us this f.

$f_1 := 0.01$ $\quad d_1 := D(f_1) = 0.361\ \text{ft}$

The subcript will just denote the iteration.

$A_1 := \text{Area}(d_1) = 0.102\ \text{ft}^2$

$v_1 := \text{velocity}(Q, A_1) = 19.549\ \frac{\text{ft}}{\text{s}}$

Pipe Geometries

This just sets up all the calculations as functions to make things a bit easier.

$f_2 := f(d_1) = 0.069$

$d_2 := D(f_2) = 0.531\ \text{ft}$

$A_2 := \text{Area}(d_2) = 0.221\ \text{ft}^2$

$v_2 := \text{velocity}(Q, A_2) = 9.042\ \frac{\text{ft}}{\text{s}}$

Pipe Geometries

This just sets up all the calculations as functions to make things a bit easier.

$$f_3 := f(d_2) = 0.058$$

$$d_3 := D(f_3) = 0.513\ \text{ft}$$

$$A_3 := \text{Area}(d_3) = 0.207\ \text{ft}^2$$

$$v_3 := \text{velocity}(Q, A_3) = 9.668\ \frac{\text{ft}}{\text{s}}$$

Pipe Geometries

This just sets up all the calculations as functions to make things a bit easier.

$$f_4 := f(d_3) = 0.059$$

$$d_4 := D(f_4) = 0.515\ \text{ft}$$

$$A_4 := \text{Area}(d_4) = 0.208\ \text{ft}^2$$

$$v_4 := \text{velocity}(Q, A_4) = 9.614\ \frac{\text{ft}}{\text{s}}$$

Pipe Geometries

- So to carry the flow what this pressure drop or lower, we would need a pipe with an inside diameter of 0.515 ft as a minimum. This is a pipe with an ID of 6.176 in.
- To get an internal diameter at least this large, you would specify a 8 nominal pipe

Homework

5.33 A 12-nominal, schedule 80 wrought iron pipe is inclined at an angle of 10° with the horizontal and conveys chloroform downhill. If the allowable pressure drop in the pipe is 2 psi and the pipe length is 100 ft, determine the volume carrying capacity in the pipe.

5.34 Ether flows through a horizontally laid, 4-nominal schedule 40 wrought iron pipe. The pressure drop measured at points 280 m apart is 100 kPa. Determine the volume flow rate through the pipe.

5.35 Syrup flows through a schedule 80, 4-nominal stainless steel pipe to a bottling machine in a production plant. The pipe is 250 ft long, and the pressure drop is 12 psia. Determine the volume flow rate. Take the properties of syrup to be the same as those of glycerine and the pipe wall to be smooth. The syrup is at room temperature.

5.36 A refinery plant separates crude oil into various components. One constituent produced is heptane, which is conveyed through a schedule 30, 14-nominal cast-iron pipe that is 150 ft long. The pressure drop in the pipe is 0.75 psi. Determine the volume flow rate of heptane in the pipe.

5.37 A rectangular conduit of dimension 5 × 7 in. conveys hydrogen. The conduit wall is asphalt coated and is 25 ft long. The hydrogen compressor provides enough power to overcome a pressure drop of 0.01 in. of water. Determine the mass flow of hydrogen.

5.38 Solve Problem 5.37 using effective diameter instead of hydraulic diameter and compare the results of the two methods.

5.39 An annular flow passage is formed by placing a 1-standard, type M copper tube within a 3-standard, type M copper tube. The annulus is 2 m long and conveys glycerine. The pressure drop over the 2-m length is 19 kPa. Determine the volume flow rate of glycerine.

Homework

5.44 A nozzle is used to provide a water spray for keeping dust from escaping from a dirt pile into the atmosphere. Just before entering the nozzle, the fluid must have a pressure of 60 psi at a flow rate of 12 ft^3/min. The water source is a city water main in which the pressure is maintained at 70 psi. The distance from the main to the proposed nozzle location is 65 ft. Select a suitable pipe size for the installation if galvanized iron pipe is all that is available.

5.45 A diesel engine is used as a power source for a generator as part of an electrical backup system for a remotely located manufacturing plant. The diesel requires 0.01 m^3/s of kerosene. A kerosene tank is located 10 m from the engine. The tank pressure is maintained at a constant 200 kPa, and the engine fuel injectors require that the kerosene be delivered at 115 kPa or less. Drawn copper tubing will be used for the fuel line; select an appropriate diameter.

5.46 Linseed oil is often used as a wood finish and can be purchased in half-gallon containers. On the assembly line, one machine can fill and cap 20 containers per minute. The linseed oil tank is located 36 ft from a machine. The oil is pumped from the tank to the machine; the pump outlet pressure is 35 psig, and the machine requires 15 psig pressure for optimum operation. Select a suitable diameter for a pipeline if drawn copper tubing is to be used.

5.47 An annulus is to be made of two type M copper tubes that are 2.5 m long. The outer tube size is limited by space and is 2-standard, whereas the size of the inner tube must be selected. The fluid is turpentine, and the available pump can overcome a pressure loss of 30 kPa while delivering 0.01 m^3/s of liquid. Select a suitable inner-tube size.