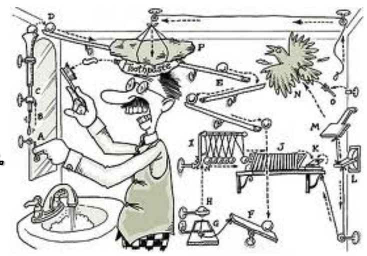
Civil Engineering Hydraulics
Mechanics of Fluids

# Flow in Pipes



***Everyone thinks is just one Bernoulli…***
***It is not so! There are many of us, truly.***
***My family is big, many scientists in this house,***
***With father Johan, nephew Jacob and brother Nicolaus.***

***But the famous principle is mine, you know,***
***It tells of the relationship of fluid flow,***
***To pressure, velocity, and density too.***
***I also invented the manometer – out of the blue!***

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# Pipe Flow

- Now we will move from the purely theoretical discussion of nondimensional parameters to a topic with a bit more that you can see and feel

# Pipe Flow

- We don't always call the transport system a pipe, that is just common usage for a conveyance with a circular cross section that usually carries water
- If the system is carrying a gas and doesn't have a circular cross section, we often call it a duct



# Pipe Flow

- It the system is very small, it may be called a tube
- It doesn't really matter what we call it, the conditions and expressions governing the system will be essentially the same

## Characteristics of Pipe Flow

- Pipe flow can be divided into three separate flow regimes or classes based on the Reynolds number of the flow
  - Laminar
  - Transitional
  - Turbulent



## Characteristics of Pipe Flow

- Laminar flow is a flow in which the fluid particles flow in step
- No pushing and shoving
- Every particle moves along the stream line and doesn't get out of line
- A drop of dye put into the water would describe a straight line in the direction of flow

# Characteristics of Pipe Flow

http://www.youtube.com/watch?v=p08_KITKP50





# Characteristics of Pipe Flow

- Transitional flow is a chaotic state
- The flow is neither turbulent or laminar but has some of the characteristics of both

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# Characteristics of Pipe Flow

- The flow could return to being laminar or could become turbulent
- There is some shoving and pushing between particles that could either be resolved and order restored or it could descend into bedlam



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# Characteristics of Pipe Flow

- Turbulent flow is where the fluid particles are moving not only along the streamline but across streamlines
- There is no set vector for any individual particle
- A drop of dye would disperse in the direction of the flow and become completely mixed into the flow very quickly

## Characteristics of Pipe Flow

- While the boundaries between each flow condition are not exact, we do make an approximation as to what the flow condition is using a non-dimensional parameter called the Reynold's Number



## Reynold's Number

- The Reynold's number (Re) for a flow is a dimensionless number that is the ratio of the Inertial forces to the Viscous forces

$$Re = \frac{Inertial\ Forces}{Viscous\ Forces} = \frac{v_{avg} D}{\nu} = \frac{\rho v_{avg} D}{\mu}$$

# Reynold's Number

- The symbol v is the kinematic viscosity and that is the ratio of the mass density to the dynamic viscosity

$$Re = \frac{Inertial\ Forces}{Viscous\ Forces} = \frac{v_{avg} D}{\nu} = \frac{\rho v_{avg} D}{\mu}$$



# Reynold's Number

- In cases where the inertial forces dominate and the Re is large, particles tend to move in and out of the streamline so the flow regime is turbulent

$$Re = \frac{Inertial\ Forces}{Viscous\ Forces} = \frac{v_{avg} D}{\nu} = \frac{\rho v_{avg} D}{\mu}$$

## Reynold's Number

- In cases where the viscous forces dominate and the Re is smaller, particles tend to stay within the streamline so the flow regime is laminar

$$Re = \frac{Inertial\ Forces}{Viscous\ Forces} = \frac{v_{avg} D}{\nu} = \frac{\rho v_{avg} D}{\mu}$$



## Reynold's Number

- The D is the characteristic length
- It has dimensions of {L}

$$Re = \frac{Inertial\ Forces}{Viscous\ Forces} = \frac{v_{avg} D}{\nu} = \frac{\rho v_{avg} D}{\mu}$$

## Reynold's Number

- The Re at which the flow transitions from transitional flow to fully turbulent flow is know as the critical Reynold's number or $Re_{Cr}$
- For a circular pipe, the number is given as 4000

$$Re = \frac{Inertial\ Forces}{Viscous\ Forces} = \frac{v_{avg} D}{\nu} = \frac{\rho v_{avg} D}{\mu}$$



## Reynold's Number

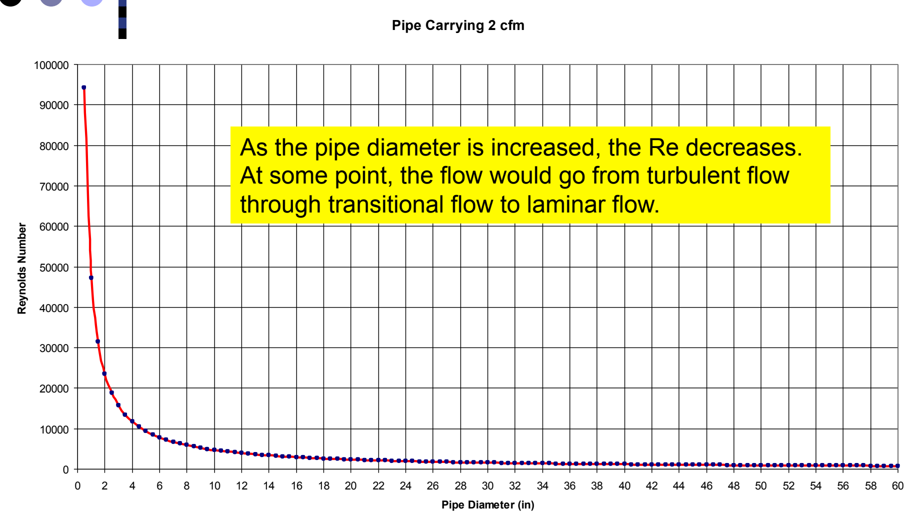
- As a general rule of thumb, we define flow regimes as
  - Re < 2100 Laminar
  - Re > 2100 and < 4000 Transitional
  - Re > 4000 Turbulent

# Reynold's Number

- If we look at a circular pipe carrying a flow of 2 $ft^3$/min (cfm)

$$Re = \frac{Inertial\ Forces}{Viscous\ Forces} = \frac{v_{avg} D}{\nu} = \frac{\rho v_{avg} D}{\mu}$$





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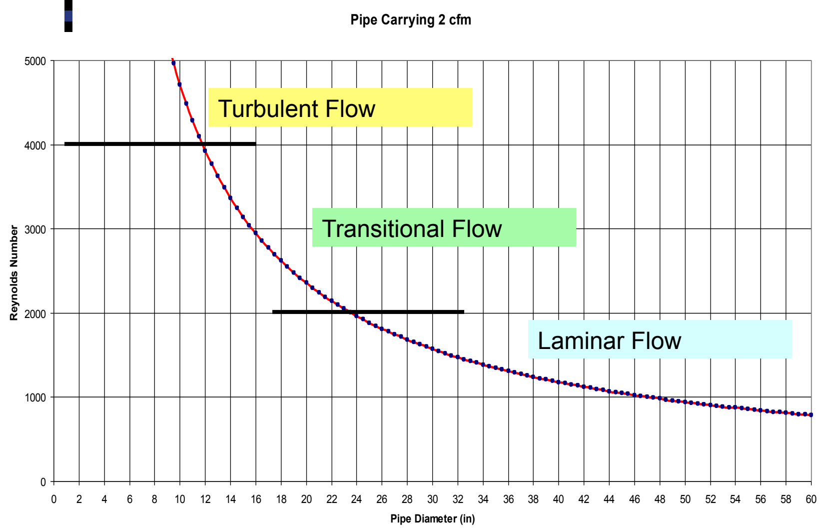


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# Effective Diameter

- As noted earlier, all the conveyance systems we encounter do not have circular cross sections.
- To better model the non-circular cross section conveyance system, we utilize a number of calculated parameters.

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# Effective Diameter

- The effective diameter, $D_{eff}$ is the diameter of a circular cross section that has the same flow area as the noncircular cross section.
- For example, a square cross section which is 2ft on a side would have an $D_{eff}$ calculated by

$$Area = (2\,ft)^2$$

$$Area = \pi \frac{(D_{eff})^2}{4}$$

$$D_{eff} = \sqrt{\frac{4(2\,ft)^2}{\pi}} = 2.26\,ft$$



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# Hydraulic Diameter

- The hydraulic diameter is another term used frequently in the analysis of conveyance systems.
- The hydraulic diameter, $D_h$, is the ratio of the area of flow divided by the wetted perimeter

$$D_h = \frac{4A}{WP}$$

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## Hydraulic Diameter

- For the square conduit considered earlier.

$$Area = (2\,ft)^2$$

$$Area = \pi\frac{(D_{eff})^2}{4}$$

$$D_{eff} = \sqrt{\frac{4(2\,ft)^2}{\pi}} = 2.26\,ft$$

$$D_h = \frac{4A}{WP} = \frac{4(2\,ft)^2}{4(2\,ft)} = 2\,ft$$



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## Hydraulic Diameter

- One case that is frequently encountered is the circular cross section.

$$D_h = \frac{4A}{WP} = \frac{4\frac{\pi D^2}{4}}{\pi D} = D$$

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# Hydraulic Radius

- And finally, there is the hydraulic radius

$$r_h = \frac{A}{WP} = \frac{\frac{\pi D^2}{4}}{\pi D} = \frac{D}{4}$$



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# Back to Reynold's Number

- The D in the Reynold's number expression is actually not the physical diameter of the pipe but an equivalent hydraulic diameter and is usually given with the symbol $D_h$

$$Re = \frac{Inertial\ Forces}{Viscous\ Forces} = \frac{v_{avg} D_h}{\nu} = \frac{\rho v_{avg} D_h}{\mu}$$

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# Example Problem

- A rectangular duct has inside dimensions of 12 by 18 in. What is the hydraulic diameter of the duct? If 3 $ft^3$/min of air at 71°F flows through the duct, determine the Reynolds number.



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# Example Problem

- A rectangular duct has inside dimensions of 12 by 18 in. What is the hydraulic diameter of the duct? If 3 $ft^3$/min of air at 71°F flows through the duct, determine the Reynolds number.

$$A := 12in \cdot \frac{1ft}{12in} \cdot 18in \cdot \frac{1ft}{12in} = 1.5\,ft^2$$

$$WP := 2 \cdot \left(12in \cdot \frac{1ft}{12in} + 18in \cdot \frac{1ft}{12in}\right) = 5\,ft$$

$$D_h := \frac{4 \cdot A}{WP} = 1.2\,ft$$

$$Q := 3\,\frac{ft^3}{min} \cdot \frac{1min}{60s} = 0.05\,\frac{ft^3}{s}$$

$$v_{avg} := \frac{Q}{A} = 0.03\,\frac{ft}{s}$$

$$\rho := 0.00232\,\frac{slug}{ft^3}$$ From Table A.3

$$\mu := 0.03801 \times 10^{-6}\,\frac{lbf \cdot s}{ft^2}$$ From Table A.3

$$Re := \frac{\rho \cdot v_{avg} \cdot D_h}{\mu \cdot \frac{\frac{slug \cdot ft}{s^2}}{lbf}} = 2441.46$$

On the original on the board in class the conversion units from lbf were incorrect. The correct conversion is 1 lbf = 1 slug * 1 ft/ $sec^2$ not 1 slug / (1 ft * $sec^2$)

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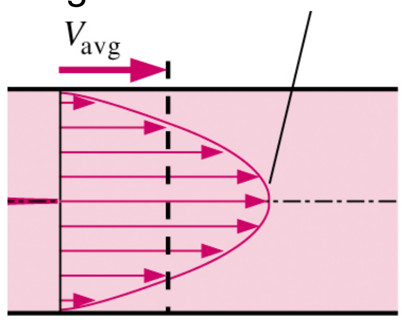
# Flow Profiles

- If you look at flow moving through a pipe and plot the velocities of the streamlines you would have a distribution of velocities approximating

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# Flow Profiles

- The velocity is not the same across the cross section of the pipe
- The flow velocity at the wall of the pipe will be equal to 0
- The velocity at any distance from the centerline of the flow can be approximated by an empirical equation.

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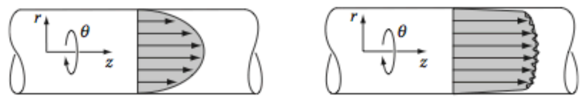
# Flow Profile and Flow Regime




| Laminar Flow | Parameter | Turbulent Flow |
| --- | --- | --- |
| $V_z$ only; $V_r = V_\theta = 0$ | Velocity | $V_r$, $V_\theta$, $V_z$ all nonzero |
| $V_z = V_z(r)$ only | Functional dependence | $V_r = V_r(r, \theta, z, t)$<br>$V_\theta = V_\theta(r, \theta, z, t)$<br>$V_z = V_z(r, \theta, z, t)$ |
| Parabolic (see above); solution from equation of motion | Velocity distribution | Determined from experimental data |
| $\frac{V_z}{V_{z\,max}} = 1 - \left(\frac{r}{R}\right)^2$<br>$\frac{V}{V_{z\,max}} = \frac{1}{2}$ | Equation | $\frac{\bar{V}_z}{V_{z\,max}} \approx \left(1 - \frac{r}{R}\right)^{1/7}$<br>$\frac{\bar{V}}{V_{z\,max}} \approx \frac{4}{5}$ |



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# Flow Characteristics

- To determine a true velocity in the pipe, we could integrate the flow profile across the pipe but rather than do that, we take the average velocity across the profile and use that in our calculation

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# Flow Characteristics

- While we typically model pipe flow considering that the velocity profile is completely developed, when there is a disruption in the flow, it takes some time (distance downstream) for the profile to become stable.
- This time (distance) is based on the flow regime in the conveyance.



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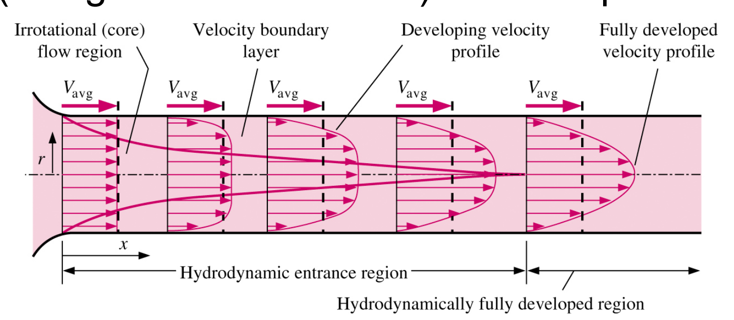
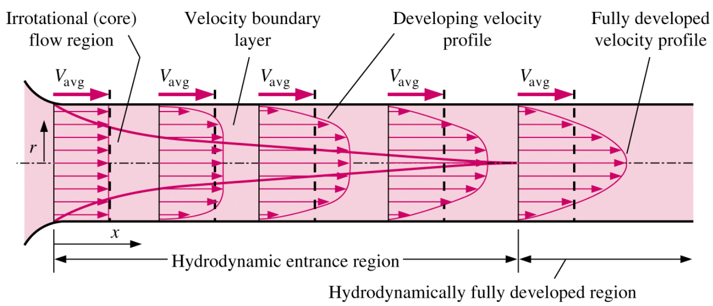
# Development of flow profiles

- When a flow moves from one cross sectional area to another, there is a transition until a consistent velocity profile (along the flow direction) is developed



# Development of flow profiles

- In this case, we have a fluid entering a pipe from a region with a much larger diameter

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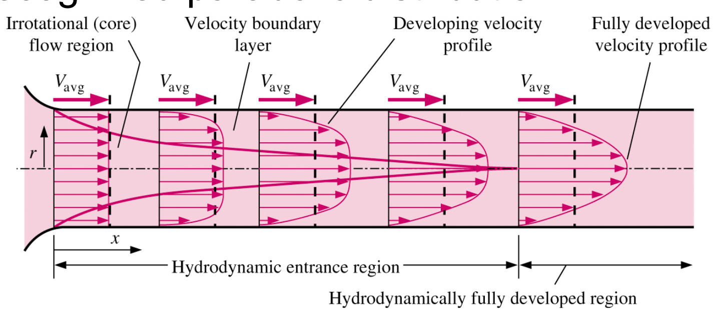
# Development of flow profiles

- The flow in the first profile has an almost constant distribution across the section but as the flow travels along the pipe, the velocity distributes until it assumes the recognized parabolic distribution

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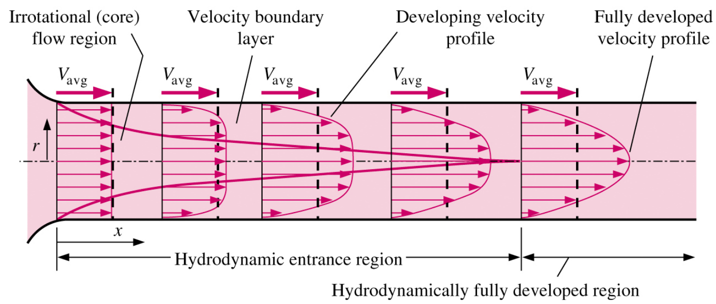
# Development of flow profiles

- The recognition that the profile does not develop instantly is important when designing measurements in stream channels and other flow systems

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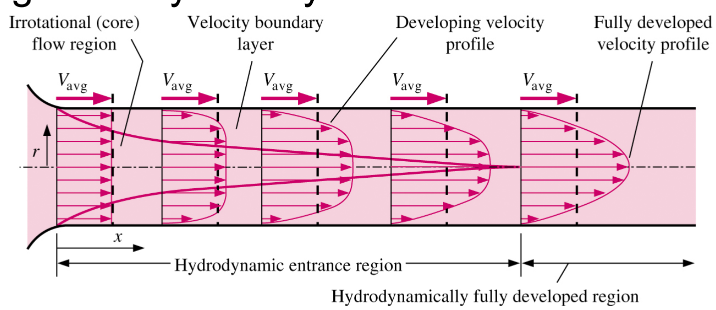
# Development of flow profiles

- If you base you analysis on an assumption of the parabolic fully developed velocity profile, taking velocity measurements in the flow in the entrance regions can significantly skew your results

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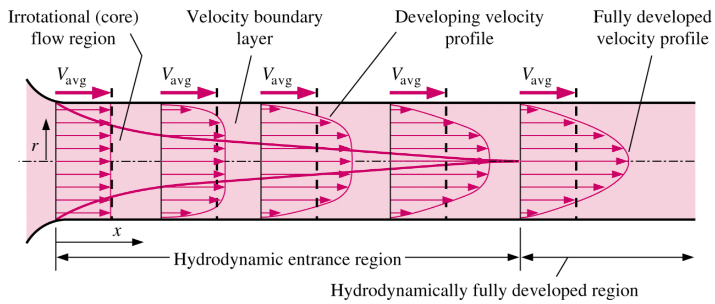
## Development of flow profiles

- There are expressions for estimating the length of the entrance region based on what the flow regime will be when the flow is fully developed

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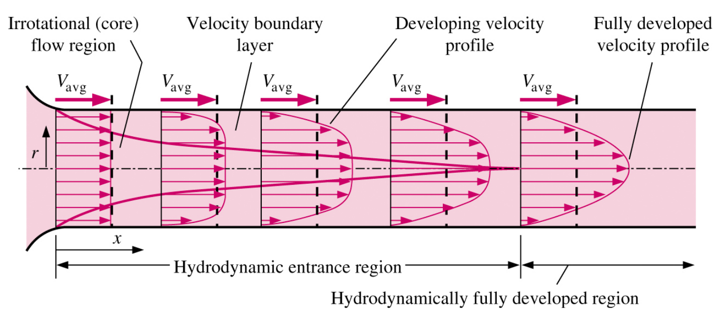
## Development of flow profiles

- In laminar flow, because of the limited mixing radially to the streamline, it takes longer for the velocity profile to fully develop

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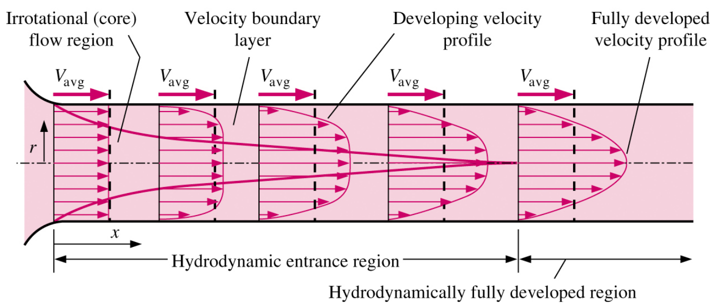
# Development of flow profiles

- An estimation of the distance is given by

$$L_{h,laminar} = 0.06\left(Re\right)\left(D_h\right)$$

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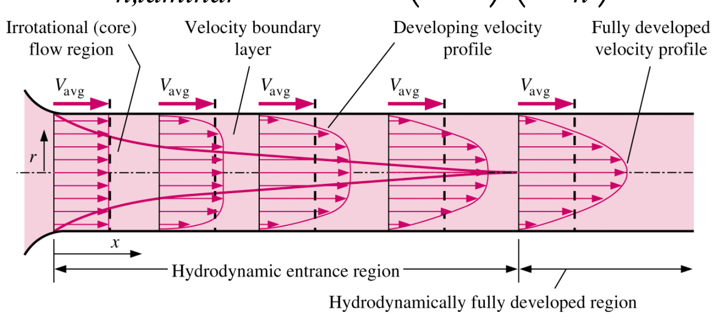
# Development of flow profiles

- Notice that this is a linear function of Re so as the transition zone is approached, the distance will become significantly longer

$$L_{h,laminar} = 0.06\left(Re\right)\left(D_h\right)$$

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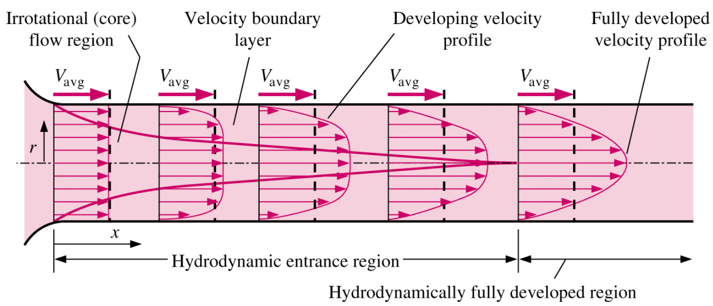
# Development of flow profiles

- At the approximate boundary of Re=2100, the length to develop would be 126 times the pipe hydraulic diameter

$$L_{h,laminar} = 0.06\left(Re\right)\left(D_h\right)$$

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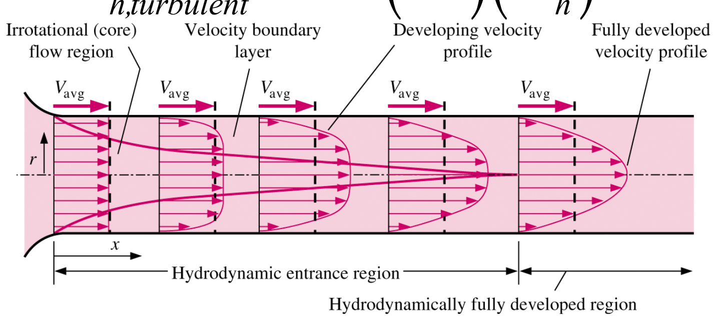
# Development of flow profiles

- Due to the mixing effects during turbulent flow, the time and therefore the distance it takes to develop the flow profile is less

$$L_{h,turbulent} = 4.4\left(Re\right)\left(D_h\right)^{\frac{1}{6}}$$

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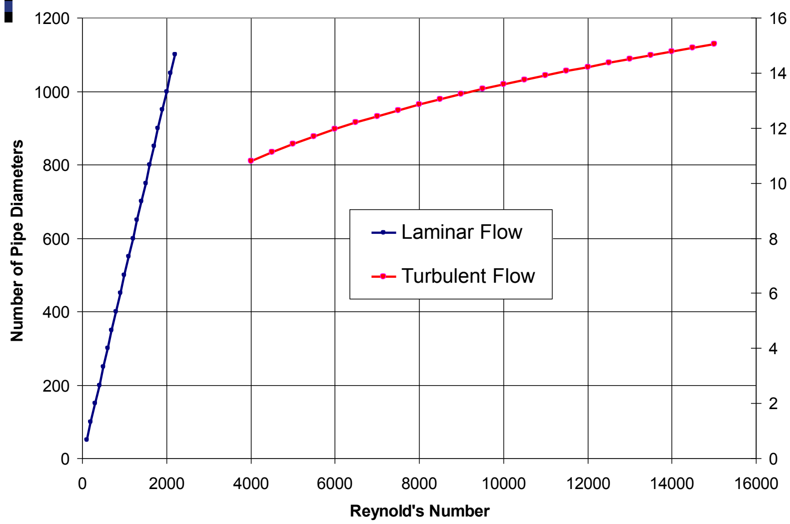




# Example Problem

- A concentric circular annulus conveys benzene at 0.2 $m^3/s$. The annulus is made up of two tubes. The inside diameter of the outer tube is 20 cm and the outside diameter of the inner tube is 10 cm. Determine the hydraulic diameter and the Reynolds number.

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## Homework 17-1

- Propylene glycol flows from a tank into a 2-nominal, schedule 80 pipe. Measurements indicate that the flow becomes fully developed 25 diameters downstream.
- Determine the volume flow rate through the pipe in cubic meters per second (a) if laminar conditions exist and (b) if turbulent conditions exist.



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## Homework 17-2

- A rectangular duct is 1 in. tall and is to be constructed so that its effective diameter equals its hydraulic diameter. What is the required width of the duct (or can’t this be done)?

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# Homework 17-3

- An annular duct is formed by an outer tube with an inside diameter of 10 cm and an inner tube with an outside diameter of 5 cm. Calculate (a) effective diameter, (b) hydraulic diameter, and (c) hydraulic radius.