Calvin: You know, Hobbes, some days even your lucky rocket ship underpants don’t help.

Parallel Pipes

- There are often cases where we find pipes laid in parallel that begin at the same junction and terminate at the same junction.
- One of the most common cases is to replace a segment of pipe with two smaller pipes.
- This may be a stopgap measure that just seems to remain in place.
Parallel Pipes

- If you have had circuits in physics or an electrical engineering class, you may recognize some of the same methods for solution that are used for solving parallel resistances.

- The fundamentals are also used in solving for water distribution systems through closed loop piping networks.
Parallel Pipes

- We will start with the simplest form of a parallel network containing two pipes

![Diagram of pipes in parallel](image)

**FIGURE 5.27** Pipes in parallel.

Parallel Pipes

- The volumetric flow rate approaching point A is known and due to conservation of mass that is the same as the flow exiting at point B

![Diagram of pipes in parallel](image)

**FIGURE 5.27** Pipes in parallel.
Parallel Pipes

- We usually know the length and cross section area of both of the pipes from A to B and we want to know how much flow is going through each pipe.

![Diagram of Parallel Pipes]

**Figure 5.27** Pipes in parallel.

Parallel Pipes

- Now the view we have of this system is shown from above so all the pipes shown are on the same elevation.

![Diagram of Parallel Pipes]

**Figure 5.27** Pipes in parallel.
Parallel Pipes

- If we neglect minor losses, all the pressure drops along the two pipes from A to B are due to friction losses.

![Diagram of parallel pipes](image)

**FIGURE 5.27** Pipes in parallel.

- The pressure at A is the same for both pipes and the pressure at B is also the same for both pipes.

![Diagram of parallel pipes](image)

**FIGURE 5.27** Pipes in parallel.
Parallel Pipes

- This means that the pressure drop along both pipes must be the same

\[ \Delta p_3 = \Delta p_2 \]

Since both of these pressure drops are due to friction we can replace both of them by the pressure loss due to friction loss terms

\[ \Delta p_3 = \Delta p_2 \]

\[ \frac{f_3 L_3}{D_3} \frac{v_3^2}{2g} = \frac{f_2 L_2}{D_2} \frac{v_2^2}{2g} \]
Parallel Pipes

○ The diameter and length of each of the pipes is known but we have unknown friction factors and velocities on each side of the expression

\[
\Delta p_3 = \Delta p_2 \\
\frac{f_3 L_3}{D_3} \frac{v_3^2}{2g} = \frac{f_2 L_2}{D_2} \frac{v_2^2}{2g}
\]

Since the friction factor is dependent on the Reynolds number and the Re is a function of the velocity, we have an expression that cannot be solved directly

\[
\Delta p_3 = \Delta p_2 \\
\frac{f_3 L_3}{D_3} \frac{v_3^2}{2g} = \frac{f_2 L_2}{D_2} \frac{v_2^2}{2g}
\]
Parallel Pipes

- Most of the solution methods assume a flow in one of the pipes (which sets the flow in the other) and then iterates the solution until the difference in pressure drops is within tolerance.

\[ \Delta p_3 = \Delta p_2 \]

\[ \frac{f_3 L_3}{D_3} \frac{v_3^2}{2g} = \frac{f_2 L_2}{D_2} \frac{v_2^2}{2g} \]

- One method that is used is to manipulate the expression so that we get an equivalent to an electrical circuit.

\[ \frac{f_3 L_3}{d_3} \frac{v_3^2}{2g} = \frac{f_2 L_2}{d_2} \frac{v_2^2}{2g} \]

\[ \frac{Q_3^2}{\left( \pi \left( \frac{d_3}{4} \right)^2 \right)^2} = \frac{Q_2^2}{\left( \pi \left( \frac{d_2}{4} \right)^2 \right)^2} \]
Parallel Pipes

One method that is used is to manipulate the expression so that we get an equivalent to an electrical circuit.

\[
\frac{Q_1^2}{\frac{\pi d_1^2}{4}} = \frac{Q_2^2}{\frac{\pi d_2^2}{4}} = \frac{Q_3^2}{\frac{\pi d_3^2}{4}}
\]

\[
f \frac{L_1}{d_1} \frac{Q_3^2}{4} = f \frac{L_2}{d_2} \frac{Q_2^2}{4}
\]

\[
Q_1^2 = \frac{f L_1}{2g \left( \frac{\pi d_1^2}{4} \right) d_1^5}
\]

\[
Q_2^2 = \frac{f L_2}{2g \left( \frac{\pi d_2^2}{4} \right) d_2^5}
\]

\[
Q_3^2 = \frac{f L_3}{2g \left( \frac{\pi d_3^2}{4} \right) d_3^5}
\]

This is similar to the division of current between two parallel resistances.

\[
Q_3 = \left( \frac{f L_3}{d_3^5} \right)^{0.5} Q_2
\]

\[
Q_3 = \left( \frac{f L_3}{d_3^5} \right)^{0.5} Q_2
\]
The difference is that \( f \) is a function of \( Q \) so this is just an estimate.

\[
Q_3^2 = \frac{f_2 L_2}{d_2^5} Q_2^2
\]

\[
Q_3 = \frac{f_3 L_3}{d_3^5} 0.5 \cdot Q_2
\]

We can start by assuming a split between the two pipes, calculate the fractions and use that as the first iteration.
Parallel Pipes

Example Problem 5.12
Equivalent Resistance

\[
Q_3^2 = \left( \frac{f_2 L_2}{d_2^5} \right) Q_2^2
\]

\[
Q_3 = \left( \frac{f_3 L_3}{d_3^5} \right)^{0.5} Q_2^{0.5}
\]

Pipe 2 3
Diameter 0.835 0.6651 ft
Area 0.5476 0.3474 ft²
Length 4500 4500 ft

Fraction of Q in pipe 2 0.5 0.5 Initial Assumption

Q 0.1500 0.1500 ft³/s

v 0.273332517 0.431778029 ft/s

Re 2.34E+04 3.68E+04

f 2.53E-02 2.30E-02

\[\Delta p = 0.242414248 \times 0.863540077 \text{lbf/ft}^2\]

\[\Delta p_2 \Delta p_3 = -0.62112383 \text{ lbf/ft}^2\]
Parallel Pipes

The fraction of total flow into pipe 2 will be $1/(1+r)$ where $r$ is the value of the ratio.

$$Q_3^2 = \left( \frac{fL_2}{d_2^5} \right) Q_2^2$$

$$Q_3 = \left( \frac{fL_2}{d_2^5} \right) 0.5 Q_2$$

<table>
<thead>
<tr>
<th>Calculated Ratio</th>
<th>5.94E-01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial 1 Fractions</td>
<td>0.6275 0.3725</td>
</tr>
<tr>
<td>$Q_1$</td>
<td>0.1882 0.1118 ft$^3$/s</td>
</tr>
<tr>
<td>$v$</td>
<td>0.343750154 0.321711042 ft$^3$/s</td>
</tr>
<tr>
<td>$Re$</td>
<td>2.93E+04 2.74E+04</td>
</tr>
<tr>
<td>$f$</td>
<td>2.40E-02 2.45E-02</td>
</tr>
<tr>
<td>$\Delta p$</td>
<td>0.237763045 0.266770858 lb/ft$^2$</td>
</tr>
<tr>
<td>$\Delta p_2$, $\Delta p_3$</td>
<td>-0.029007814 lb/ft$^2$</td>
</tr>
</tbody>
</table>

The fraction of total flow into pipe 2 will be $1/(1+r)$ where $r$ is the value of the ratio.

$$Q_3^2 = \left( \frac{fL_2}{d_2^5} \right) Q_2^2$$

$$Q_3 = \left( \frac{fL_2}{d_2^5} \right) 0.5 Q_2$$

<table>
<thead>
<tr>
<th>Calculated Ratio</th>
<th>5.61E-01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial 2 Fractions</td>
<td>0.6408 0.3592</td>
</tr>
<tr>
<td>$Q_1$</td>
<td>0.1922 0.1078 ft$^3$/s</td>
</tr>
<tr>
<td>$v$</td>
<td>0.351055343 0.31019601 ft$^3$/s</td>
</tr>
<tr>
<td>$Re$</td>
<td>2.99E+04 2.64E+04</td>
</tr>
<tr>
<td>$f$</td>
<td>2.39E-02 2.47E-02</td>
</tr>
<tr>
<td>$\Delta p$</td>
<td>0.246819714 0.250034617 lb/ft$^2$</td>
</tr>
<tr>
<td>$\Delta p_2$, $\Delta p_3$</td>
<td>-0.003214904 lb/ft$^2$</td>
</tr>
</tbody>
</table>
Parallel Pipes

The fraction of total flow into pipe 2 will be $1/(1+r)$ where $r$ is the value of the ratio.

\[
Q_3^2 = \left( \frac{f_2 L_2}{d_2^5} \right) Q_2^2 \]

\[
Q_3 = \left( \frac{f_3 L_3}{d_3^5} \right) 0.5 Q_2 
\]

Calculated Ratio 5.57E-01
Trial 3 Fractions 0.6423 0.3577
Q 0.1927 0.1073 ft/ft
v 0.351860532 0.308926807 ft/s
Re 3.00E+04 2.63E+04
f 2.39E-02 2.48E-02
Δp 0.24782779 0.248219889 lbf/ft²
Δp₂ Δp₃ -0.00039261 lbf/ft²

The fraction of total flow into pipe 2 will be $1/(1+r)$ where $r$ is the value of the ratio.

\[
Q_3^2 = \left( \frac{f_2 L_2}{d_2^5} \right) Q_2^2 \]

\[
Q_3 = \left( \frac{f_3 L_3}{d_3^5} \right) 0.5 Q_2 
\]

Calculated Ratio 5.57E-01
Trial Fractions 0.6424 0.3576
Q 0.1927 0.1073 ft/ft
v 0.351950129 0.308785576 ft/s
Re 3.00E+04 2.63E+04
f 2.39E-02 2.48E-02
Δp 0.24793951 0.248018323 lbf/ft²
Δp₂ Δp₃ -7.88132E-05 lbf/ft²
While that method works well and will be useful when you consider the Hardy-Cross method in Hydrology, we can utilize EXCEL to solve the problem.

\[
\Delta p_3 = \Delta p_2 \\
\frac{f_3 L_3}{D_3} \frac{v_3^2}{2g} = \frac{f_2 L_2}{D_2} \frac{v_2^2}{2g}
\]

Starting with what we are given in the problem

\[
\Delta p_3 = \Delta p_2 \\
\frac{f_3 L_3}{D_3} \frac{v_3^2}{2g} = \frac{f_2 L_2}{D_2} \frac{v_2^2}{2g}
\]
Example Problem 5.12
Using Goal Seek
\[ \rho = 1.94 \text{ slug/ft}^3 \]
\[ \mu = 1.90E-05 \text{ lbf/s/ft}^2 \]
\[ Q = 3.00E-01 \text{ ft}^3/\text{s} \]
Pipe 2 3
Diameter 0.835 0.6651 ft
Area 0.5476 0.3474 ft^2
\[ \varepsilon = 0.00015 \]
\[ \text{Length} = 4500 \text{ ft} \]
Fraction of Q in pipe 2 0.5 0.5
\[ Q = 0.1500 \text{ ft}^3/\text{s} \]
\[ v = 0.273922571 \text{ ft}^2/\text{s} \]
Re 2.34E+04 3.68E+04
f 2.53E-02 2.30E-02
\[ \Delta p = 0.242414248 \text{ lbf/ft}^2 \]
\[ \Delta p_2 - \Delta p_3 = 0.62112583 \text{ lbf/ft}^2 \]

Example Problem 5.12
Using Goal Seek
\[ \rho = 1.94 \text{ slug/ft}^3 \]
\[ \mu = 1.90E-05 \text{ lbf/s/ft}^2 \]
\[ Q = 3.00E-01 \text{ ft}^3/\text{s} \]
Pipe 2 3
Diameter 0.835 0.6651 ft
Area 0.5476 0.3474 ft^2
\[ \varepsilon = 0.00015 \]
\[ \text{Length} = 4500 \text{ ft} \]
Fraction of Q in pipe 2 0.671330633 0.328669367
Q 0.2014 0.0986 ft^3/s
v 0.367785226 0.283250158 ft^2/s
Re 3.14E+04 2.42E+04
f 2.37E-02 2.52E-02
\[ \Delta p = 0.408859847 \text{ lbf/ft}^2 \]
\[ \Delta p_2 - \Delta p_3 = 1.42997E-07 \text{ lbf/ft}^2 \]
Parallel Pipes

While that these methods work well with two pipes in parallel, when you have three or more pipes in parallel, since there is more than one flow partition to make, the methods we have considered will not work.

Goal seek has found a flow balance that has a very small difference between the pressure loss across the two pipes.
Parallel Piping

- There are mathematical approximations and methods that can be used to generate an approximate solution but there is not a method that I know of that can be broken down into a simplification like the two pipe problem.

Parallel Piping

- In Hydrology, you will be introduced to a method of looking at pipe networks that involved a mathematical method known as a relaxation method.
- The particular method that you will see is known as the Hardy-Cross method.
Parallel Piping

- We will look at a method in EXCEL that can generate a solution using an EXCEL tool that you may not have seen before.

- If the problem required that we add yet another pipe to the system in parallel but this time we added a 4000 ft section of 6 nominal pipe in parallel to the other two sections.
Parallel Piping

- We could just add another column to the table with the characteristics of the new pipe.

<table>
<thead>
<tr>
<th>Example Problem 5.12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tree Pipes</td>
</tr>
<tr>
<td>$\rho$</td>
</tr>
<tr>
<td>$\mu$</td>
</tr>
<tr>
<td>$Q$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pipe</th>
<th>Diameter</th>
<th>Area</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.835</td>
<td>0.5476</td>
<td>0.00015</td>
</tr>
<tr>
<td>3</td>
<td>0.6651</td>
<td>0.3474</td>
<td>0.00015</td>
</tr>
<tr>
<td>4</td>
<td>0.5054</td>
<td>0.2006</td>
<td>0.00015</td>
</tr>
</tbody>
</table>

- To set the flows we now need two fractions.
- The first will partition the flows between the original (pipe 2) and the remaining pipes.
- The second will partition the flow not going into pipe 2 between pipes 3 and 4.

$$Q_2 = \text{fraction}_1 Q$$
$$Q_3 = \text{fraction}_2 (Q - Q_2)$$
$$Q_4 = Q - Q_2 - Q_3$$
Flow Partitions in 3 Pipe Problem

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>Fractions</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Q</td>
<td>0.15</td>
<td>0.075</td>
<td>0.075</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pipe</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>0.835</td>
<td>0.6651</td>
<td>0.5054</td>
</tr>
<tr>
<td>Area</td>
<td>0.5476</td>
<td>0.3474</td>
<td>0.2006</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.00015</td>
<td>0.00015</td>
<td>0.00015</td>
</tr>
<tr>
<td>Length</td>
<td>4500</td>
<td>4500</td>
<td>4000</td>
</tr>
</tbody>
</table>

| Fractions | 0.5 | 0.5 |     |
| Q         | 0.1500 | 0.0750 | 0.0750 | ft$^3$s$^{-1}$ |

<table>
<thead>
<tr>
<th>Pipe</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v$</td>
<td>0.27392257</td>
<td>0.21588946</td>
<td>0.37387836</td>
</tr>
<tr>
<td>Re</td>
<td>2.34E+04</td>
<td>1.84E+04</td>
<td>3.19E+04</td>
</tr>
<tr>
<td>$f$</td>
<td>2.53E-02</td>
<td>2.69E-02</td>
<td>2.40E-02</td>
</tr>
<tr>
<td>$\Delta p$</td>
<td>0.1589773</td>
<td>0.13169333</td>
<td>0.41152234</td>
</tr>
<tr>
<td></td>
<td>0.55965802</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Now we cannot subtract one pressure drop from another but we can develop an expression to make sure all the pressure drops are equal.

We do this by looking at the difference between all the pairs of pipes and making sure that they are all about equal.

\[ \Delta p \]

\[ \begin{array}{ccc}
21 & 0.00984573 & 0.51815303 & 0.17400761 \text{ lbf ft}^{-2} \\
23 & =\text{ABS}(B21-C21)+\text{ABS}(B21-D21)+\text{ABS}(C21-D21) \\
\end{array} \]
Parallel Piping Problem

- Rather than using Goal Seek, we will use a feature of EXCEL called Solver which allows for the modification of more than one variable to achieve a goal.

```
20
21 Δp   0.00984573  0.51815303  0.17400761 lbf/ft²
22
23 =ABS(B21-C21)+ABS(B21-D21)+ABS(C21-D21)
24
```

Solver is part of the Data menu. You may have to add it into your EXCEL.
## Parallel Piping Problem

### Example Problem 5.12

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Example Problem 5.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Tree Pipes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>1.94 slug/ft³</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>1.90E-05 lb/ft³ s²</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>3.00E-01 ft³/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Pipe</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Diameter</td>
<td>0.835</td>
<td>0.6651</td>
<td>0.5054 ft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Area</td>
<td>0.5476</td>
<td>0.3474</td>
<td>0.2006 ft²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>e</td>
<td>0.00015</td>
<td>0.00015</td>
<td>0.00015 ft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Length</td>
<td>4500</td>
<td>4500</td>
<td>4000 ft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Fractions</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Q</td>
<td>0.1500</td>
<td>0.0750</td>
<td>0.0750 ft³/s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td>0.27392257</td>
<td>0.21588946</td>
<td>0.37387836 ft³/s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Re</td>
<td>2.34E+04</td>
<td>1.84E+04</td>
<td>3.19E+04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>f</td>
<td>2.53E-02</td>
<td>2.69E-02</td>
<td>2.40E-02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>An</td>
<td>0.1589777</td>
<td>0.13161933</td>
<td>0.411274 lb/hr ft²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Monday, October 22, 2012
Homework 21-1

- Using the iteration method we developed, solve for the flow in each pipe is the system if one pipe is 3000 ft long and is Schedule 40 nominal 3 wrought iron and the second pipe is 2800 ft long and is Schedule 40 nominal 2 ½ wrought iron. The total flow in the system is 1800 gpm. The fluid is water.

Homework 21-2

- Using the EXCEL and Goal Seek to solve for the flow in each pipe is the system if one pipe is 1200 m long and is Schedule 80 nominal 4 wrought iron and the second pipe is 1500 m long and is Schedule 80 nominal 2 wrought iron. The total flow in the system is 6.0 m³/min. The fluid is water.
Homework 21-3

* Use EXCEL and Solver to solve for the flow in each pipe in a system of three pipes. All the pipes are 2500 ft long and Schedule 40 steel. The first pipe is 1 Nominal, the second is 2 Nominal, and the third is 3 Nominal. Total flow in the system is 2500 gpm.