Civil Engineering Hydraulics

Specific Energy

What's the problem with twin witches?
You never know which witch is which.

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Hydraulic Grade Lines

- We often want a visual representation of how the energy changes as we move downstream in a flow.
- This may be to show how friction losses and minor losses change the energy or to compare the energy at different points in the flow.

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Hydraulic Grade Lines

- The hydraulic grade line is a plot of pressure changes with distance.
- This is more useful when we are looking at systems that are flowing full since the pressure is typically atmospheric (or surface pressure) in an open-channel flow system.

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Energy Grade Lines

- The energy grade line is a plot of the sum of the potential and kinetic energy as a function of distance.
- The units of energy are typically in terms of head so the expression for the energy would be

$$E = \frac{v^2}{2g} + z$$

Specific Energy Lines

- For any channel of a constant width (and actually for any channel shape) we can rewrite the expression for energy by substituting Q/A for the velocity

$$E = \frac{v^2}{2g} + z$$

$$E = \frac{\left(\frac{Q}{A}\right)^2}{2g} + z$$

Specific Energy Lines

- If it is possible to write A as a function of z, for any Q, we can calculate the specific energy at a number of depths

$$E = \frac{v^2}{2g} + z$$

$$E = \frac{\left(\frac{Q}{A}\right)^2}{2g} + z$$

Specific Energy Lines

- For example, if we have a rectangular channel with a top width, b_t, we can plot the specific energy for different depths.

$$E = \frac{v^2}{2g} + z$$

$$E = \frac{\left(\frac{Q}{b_t z}\right)^2}{2g} + z$$

Specific Energy Lines

- Example 7.1:Consider a rectangular channel of width 8 m.
  - Plot a family of specific-energy lines for Q = 0, 10, 20, 40, and 80 m^3/s.
  - Draw the locus of critical depth points.
  - Plot Q as a function of critical depth.

$$E = \frac{\left(\frac{Q}{b_t z}\right)^2}{2g} + z$$

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Specific Energy Lines

- For consistency, we use E as the independent variable and z as the dependent variable.

$$E = \frac{\left(\frac{Q}{b_t z}\right)^2}{2g} + z$$

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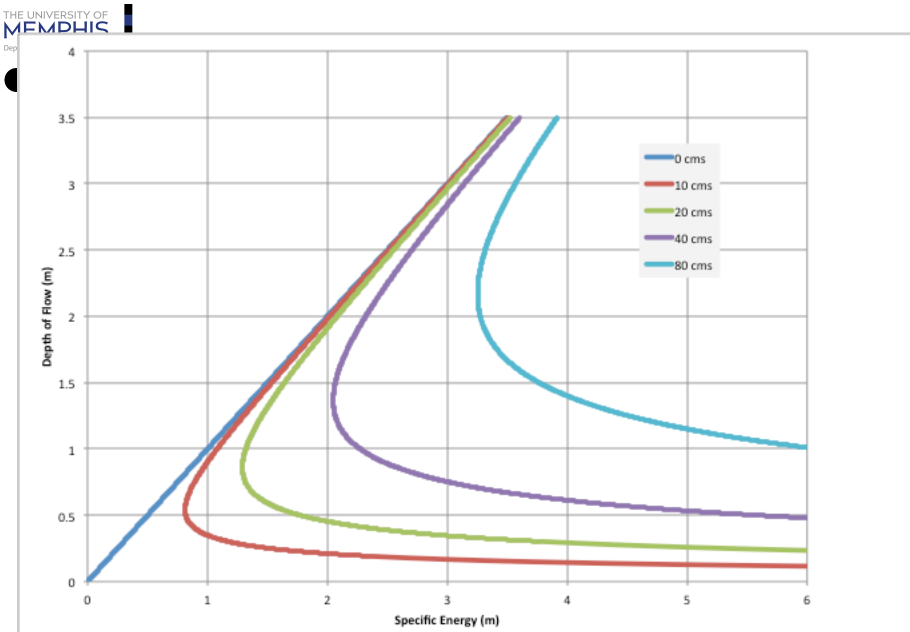

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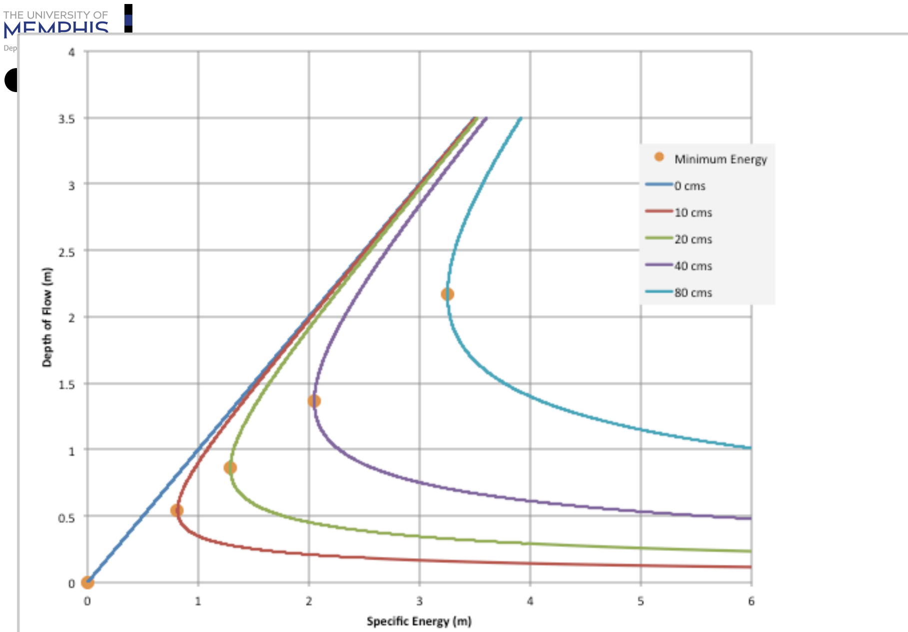
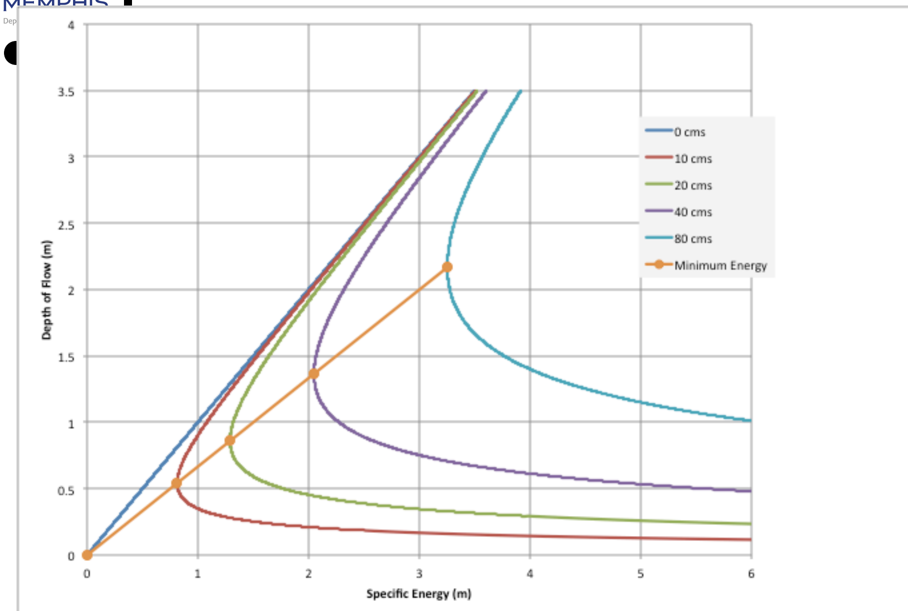
Specific Energy Lines

- The critical depth points will be those points where the specific energy is at a minimum
- This would be the leftmost point on each curve.

$$E = \frac{\left(\frac{Q}{b_t z}\right)^2}{2g} + z$$





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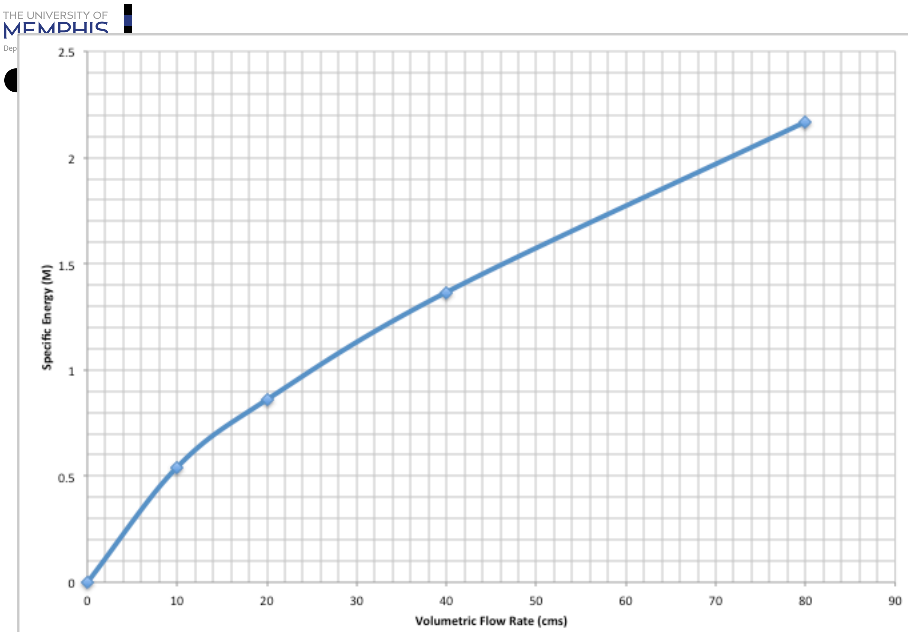

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Specific Energy

- If we look at the Froude number at each of these flows

$$Fr = \frac{v}{\sqrt{gz_m}} = \frac{v}{\sqrt{gd}}$$

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Specific Energy

- If we look at the Froude number at each of these flows

Depth	Specific Energy	Flow	Froude Number
0	0	0	
0.542	0.8131	10	1.000
0.86	1.2907	20	1.001
1.366	2.0489	40	1.000
2.168	3.2524	80	1.000

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Specific Energy

- When the Froude number is equal to 1 (approximately) the depth is the critical depth and the specific energy is at a minimum.

Depth	Specific Energy	Flow	Froude Number
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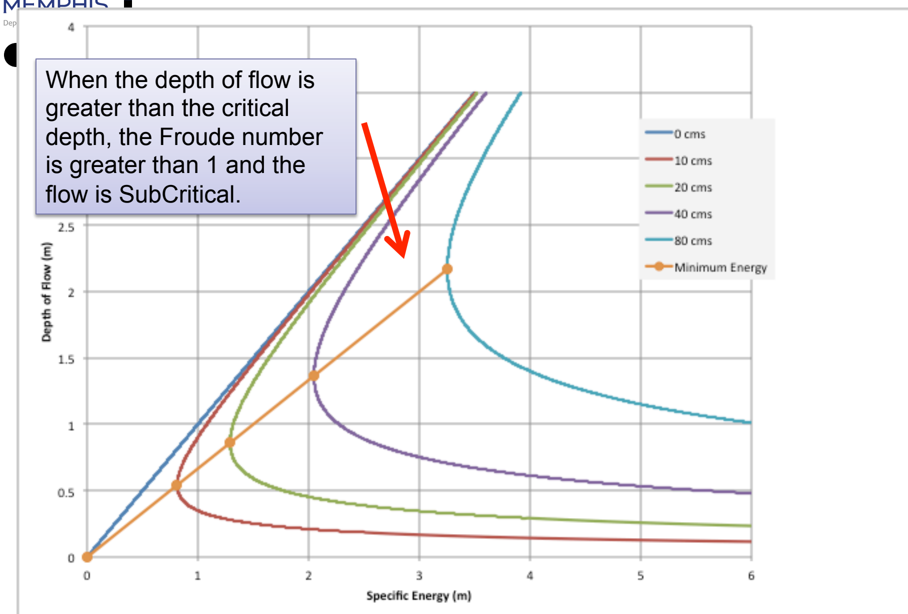
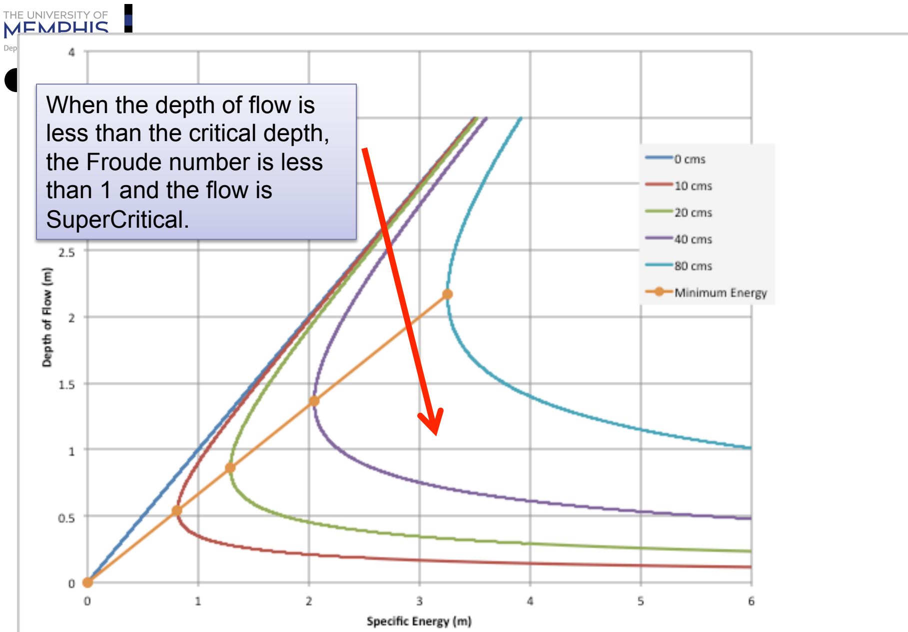
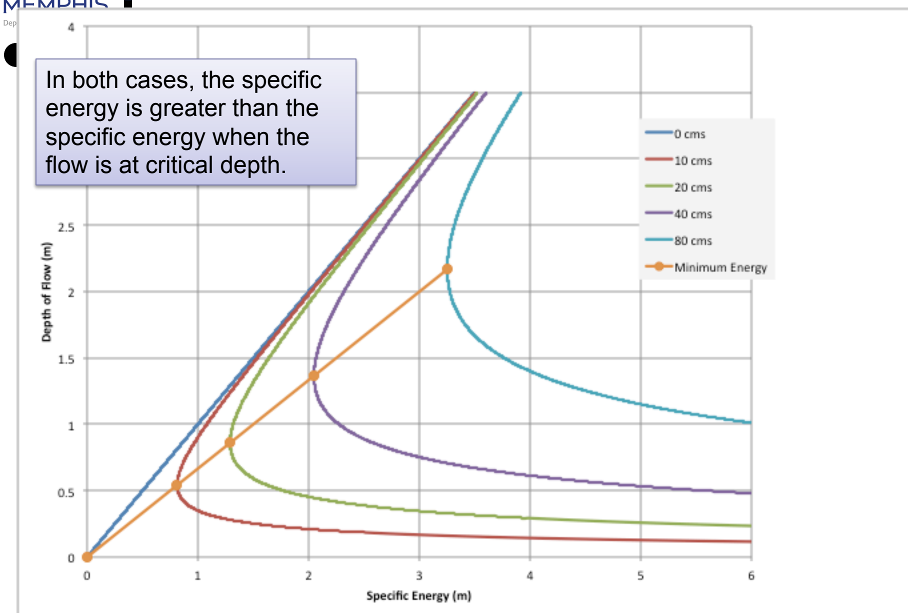

When the depth of flow is greater than the critical depth, the Froude number is greater than 1 and the flow is SubCritical.
Depth of Flow (m)
Specific Energy (m)
0 cms
10 cms
20 cms
40 cms
80 cms
Minimum Energy

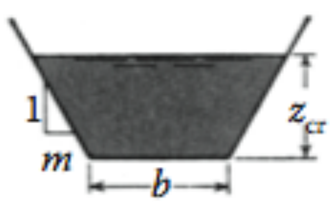

When the depth of flow is less than the critical depth, the Froude number is less than 1 and the flow is SuperCritical.
Depth of Flow (m)
Specific Energy (m)
0 cms
10 cms
20 cms
40 cms
80 cms
Minimum Energy



Example 7.3

- A trapezoidal channel with a bottom width of 15 ft has sides with slopes 1/m = 2. For a flow rate of 60 ft^3/s, determine the critical depth.

The slope would be a 2:1 slope the way we would usually write it.



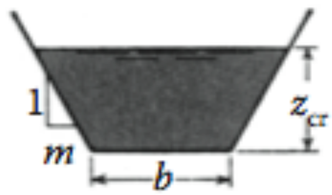
Example 7.3

- We need to write an expression for the area in terms of z.

$$A = zb + 2\left(\frac{1}{2}\frac{z}{2}z\right)$$

$$A = zb + \frac{z^2}{2}$$

$$z_m = z + b$$



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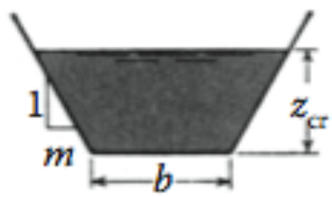
Example 7.3

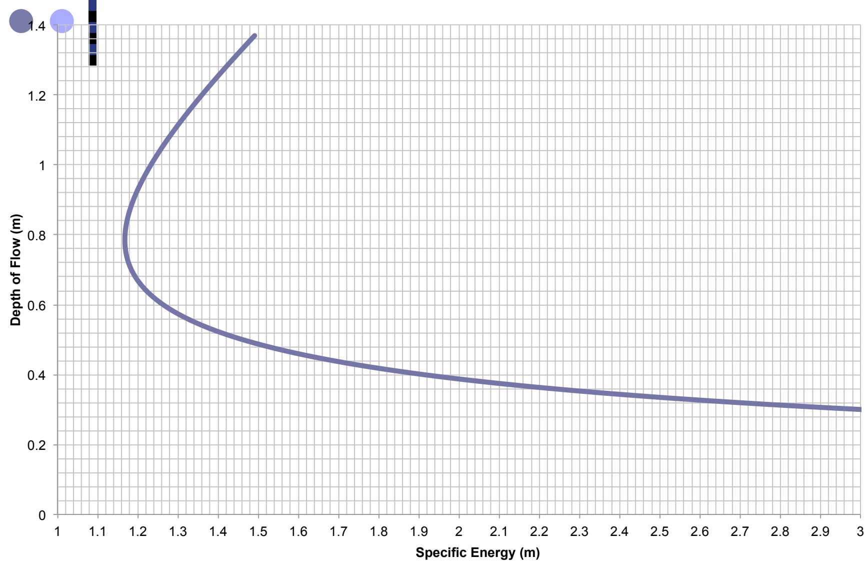
- Substituting into the term for specific energy

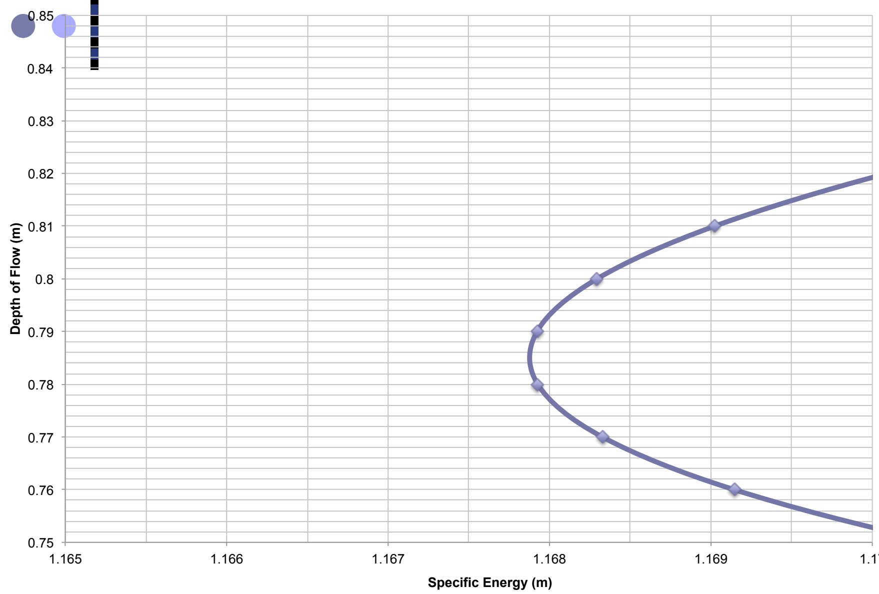
$$E = \frac{\left(\dfrac{Q}{bz + \dfrac{z^2}{2}}\right)^2}{2g} + z_m$$

Example 7.3

- There are multiple roots for z.
- Plot z with respect to E.





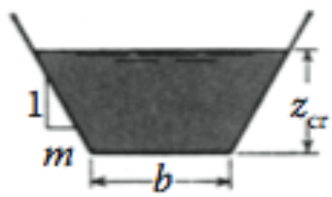


Example 7.3

- A solution for z_{cr} would be 0.7852 ft.
- You can check by calculating the Froude number at this depth.

Example 7.3

- If you want to try using Goal Seek, try finding a z so that the Froude number is equal to 1.
- Don’t try setting the specific energy to 0.



Useful Formulas for Circular Cross Sections

Formulas

$$\theta = 2\cos^{-1}[1 - 2(y/D)]$$

$$A = (D^2/8)(\theta - \sin\theta)$$

$$T = D\sin(\theta/2)$$

$$V = Q/A$$

$$(Q^2T)/(gA^3) = 1$$

Problem 7.18

- Consider a circular channel of diameter 2 m carrying water.
  - Plot a family of specific-energy lines for Q = 0, 2, 5, 10, and 20 m^3/s.
  - Draw the locus of critical depth points.
  - Plot Q versus critical depth.

  .

Flow in Open Channel – Flow Conditions

- While we used the Froude Number to calculate the critical flow conditions, the Reynolds number is still used to characterize the flow as laminar, transitional, or turbulent

Flow in Open Channel – Flow Conditions

- The calculation of the Reynolds Number for Open Channel flow is a bit different than for closed conduits.

$$\mathrm{Re} = \frac{\rho V R_h}{\mu}$$

Flow in Open Channel – Flow Conditions

- For our purposes, we will assume that transition occurs at a Reynolds number of 1000.
- Most common open-channel flows are turbulent.

$$\mathrm{Re} = \frac{\rho V R_h}{\mu}$$

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Homework 23-1

- Consider a triangular channel of m=0.4.
  - Plot a family of specific-energy lines for Q = 0, 4, 8, 12, and 16 ft^3/s.
  - Draw the locus of critical depth points.
  - Plot Q versus critical depth.

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Homework 23-2

- A triangular channel with sides having m=1.2 conveys water at 50 ft^3/s. Determine the critical depth and the corresponding critical velocity.

Homework 23-3

- A trapezoidal channel with b=4 m and m=1.2 conveys water at 30 m^3/s. Determine the critical depth and the corresponding critical velocity.