Civil Engineering Hydraulics

# Conservation of Energy


One man's "magic" is another man's engineering. "Supernatural" is a null word.

**Robert A. Heinlein**

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# Conservation of Energy

- We are going to step over section 3.4 in the text for a bit and go on to section 3.5
- We will come back to section 3.4
- Now we will move from the conversation of mass to the conservation of energy
- In order to do this, we first need to define the terms we will use to describe energy and energy input and output

# Conservation of Energy

- When we add or remove energy from a system, we normally do not have the ability to get all the energy into or out of the system
- Some things are better than others and getting the job done and this is known as the efficiency of energy transfer
- it describes how well the machinery can take energy out of or provide energy to a flow



# Conservation of Energy

- In our work with fluid flow, we can
  - add energy to a flow through a pump and
  - extract energy using a turbine
- There may be different names for these machines but fundamentally that is what each comes down to

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## Conservation of Energy

- We aren't going to concern ourselves in this class with why some of the energy transfers are more efficient than others
- We will just accept that they are
- Later we give a numerical value to the effeciency



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## Conservation of Energy

- Now we have to look at just how we describe the energy of a flow
- This will require a very short trip back to Physics and Dynamics

# Conservation of Energy

- Any mass may be considered to have two types of energy
  - Kinetic
  - Potential



# Conservation of Energy

- Kinetic energy is the energy of motion and is defined as the mass times velocity of the mass squared divided by 2

$$\frac{mV^2}{2}$$

## Conservation of Energy

- Potential energy is the energy of position and is measured relative to some reference height as

$$mgh$$



## Conservation of Energy

- So the energy of a mass is the sum of the kinetic and potential energy

$$mgh + \frac{mV^2}{2}$$

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## Conservation of Energy

- A fluid mass also has the ability to do work (energy) through its pressure component

$$mgh + \frac{mV^2}{2}$$



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## Conservation of Energy

- We can develop this by looking at the units of energy from the potential or kinetic energy expressions
- We will use the potential energy expression to look at the unit of energy

$$mgh = mass \times \frac{length}{time^2} \times length$$

$$mgh = \frac{ml^2}{t^2}$$

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## Conservation of Energy

- Pressure is force per unit of area

$$p = \frac{F}{A} = \frac{mass \times \dfrac{length}{time^2}}{length^2} = \frac{m}{lt^2}$$

$$mgh = \frac{ml^2}{t^2}$$



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## Conservation of Energy

- If we multiply the pressure term by the volume that the mass occupies we will have units of energy

$$p = \frac{F}{A} = \frac{mass \times \dfrac{length}{time^2}}{length^2} = \frac{m}{lt^2}$$

$$p\cancel{V} = \frac{m}{lt^2} l^3 = \frac{ml^2}{t^2}$$

## Conservation of Energy

- So we can describe the energy of a mass of fluid as the sum of the three energy terms

$$mgh + \frac{mV^2}{2} + p\cancel{V}$$



## Conservation of Energy

- Now we can divide all the terms through by the mass to get an energy per unit of mass

$$gh + \frac{V^2}{2} + \frac{p\cancel{V}}{m}$$

## Conservation of Energy

- The ratio of the volume to the mass is the specific volume or the inverse of the mass density

$$gh + \frac{V^2}{2} + \frac{p}{\rho}$$



## Conservation of Energy

- This is the complete description of the energy per unit of mass at a point in fluid in reference to some elevation

$$gh + \frac{V^2}{2} + \frac{p}{\rho}$$

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## Conservation of Energy

- The second two terms in the expression are properties of the fluid itself and are not related to any reference plane

$$gh + \frac{V^2}{2} + \frac{p}{\rho}$$



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## Conservation of Energy

- The first term is only relative to a reference plane and can change if we change that plane

$$gh + \frac{V^2}{2} + \frac{p}{\rho}$$

## Conservation of Energy

- The first term is the potential energy
- The second if the kinetic energy
- The third is the flow energy

$$gh + \frac{V^2}{2} + \frac{p}{\rho}$$



## Conservation of Energy

- Remember that this is a description of the energy per unit mass not the total energy of the flow

$$gh + \frac{V^2}{2} + \frac{p}{\rho}$$

## Conservation of Energy

- If we look at two points in the same flow, 1 and 2, we can consider the energy at each of these points

$$gh_1 + \frac{V_1^2}{2} + \frac{p_1}{\rho}$$

$$gh_2 + \frac{V_2^2}{2} + \frac{p_2}{\rho}$$



## Conservation of Energy

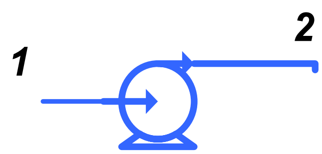
- If there is no addition or subtraction of energy from the flow
- Energy is conserved between the two points

$$gh_1 + \frac{V_1^2}{2} + \frac{p_1}{\rho} = gh_2 + \frac{V_2^2}{2} + \frac{p_2}{\rho}$$

# Conservation of Energy

- If we have a pump between the two points, we add energy to the flow

$$gh_1 + \frac{V_1^2}{2} + \frac{p_1}{\rho}$$



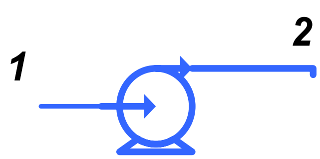

$$gh_2 + \frac{V_2^2}{2} + \frac{p_2}{\rho}$$





# Conservation of Energy

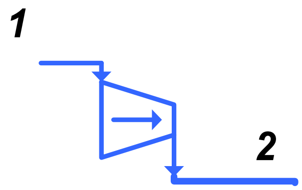
- Then the energy expression between the two points is




$$gh_1 + \frac{V_1^2}{2} + \frac{p_1}{\rho} + e_{pump} = gh_2 + \frac{V_2^2}{2} + \frac{p_2}{\rho}$$

## Conservation of Energy

- If we have a turbine between points 1 and 2, energy is removed from the flow and the energy expression would become




$$gh_1 + \frac{V_1^2}{2} + \frac{p_1}{\rho} - e_{turbine} = gh_2 + \frac{V_2^2}{2} + \frac{p_2}{\rho}$$



## Bernoulli's Equation

- One of the most important expressions in fluid mechanics is Bernoulli's Equation
- While it has general usage, it actually was defined for a limited case
  - Regions of steady, incompressible flow
  - Net frictional forces are negligible

## Bernoulli's Equation

- When these conditions are violated such as in rapidly varying flow conditions or when frictional forces are considerable, the assumptions on which Bernoulli's equation are based are no longer valid



## Bernoulli's Equation

- The form of the expression says that at any point in a streamline the sum of the kinetic, potential, and pressure energy is a constant.

$$gh + \frac{V^2}{2} + \frac{p}{\rho} = constant$$

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## Bernoulli's Equation

- A streamline is a line which is everywhere tangent to the velocity of the flow.
- A pathline is the trajectory that an imaginary small point would follow if it followed the flow of the fluid in which it was embedded.

$$gh + \frac{V^2}{2} + \frac{p}{\rho} = constant$$



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## Bernoulli's Equation

- If points 1 and 2 are on the same streamline and all the other conditions for the Bernoulli equation hold true the relationship between the energy at the two points is given by

$$gh_1 + \frac{V_1^2}{2} + \frac{p_1}{\rho} = gh_2 + \frac{V_2^2}{2} + \frac{p_2}{\rho}$$

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## Bernoulli's Equation

- Formal expression
- The sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow when the compressibility and frictional effects are negligible

$$gh + \frac{V^2}{2} + \frac{p}{\rho} = constant$$



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## Bernoulli's Equation

- If we multiply the Bernoulli's equation by the mass density of the fluid, we get the equation is pressure form

$$gh + \frac{V^2}{2} + \frac{p}{\rho} = constant$$

$$\rho gh + \frac{\rho V^2}{2} + p = c\rho$$

## Bernoulli's Equation

- Each of the terms has a label

$$\rho g h \rightarrow \text{hydrostatic pressure}$$

$$\frac{\rho V^2}{2} \rightarrow \text{dynamic pressure}$$

$$p \rightarrow \text{static pressure}$$



## Bernoulli's Equation

- Usually, the h variable in the hydrostatic pressure term is represented by the variable z which is the height above some reference datum plane

$$\rho g h = \rho g z \rightarrow \text{hydrostatic pressure}$$

$$\frac{\rho V^2}{2} \rightarrow \text{dynamic pressure}$$

$$p \rightarrow \text{static pressure}$$

## Bernoulli's Equation

- The sum of these three pressure terms is known as the total pressure

$$\rho gh = \rho gz \rightarrow \text{hydrostatic pressure}$$

$$\frac{\rho V^2}{2} \rightarrow \text{dynamic pressure}$$

$$p \rightarrow \text{static pressure}$$



## Bernoulli's Equation

- The sum of the static and dynamic pressures are known as the stagnation pressure

$$\frac{\rho V^2}{2} + p = p_{stagnation}$$

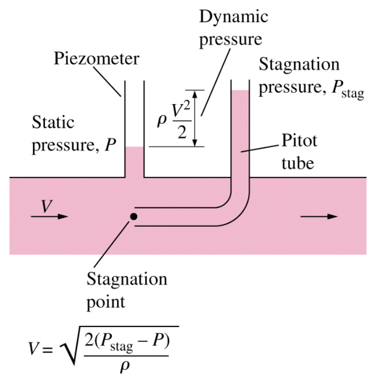
$$\rho gh = \rho gz \rightarrow \text{hydrostatic pressure}$$

$$\frac{\rho V^2}{2} \rightarrow \text{dynamic pressure}$$

$$p \rightarrow \text{static pressure}$$

## Bernoulli's Equation

- The Pitot tube (Henri Pitot a French hydraulic engineer) is used for measuring the stagnation pressure in a flow




$$\frac{\rho V^2}{2} + p = p_{stagnation}$$

$\rho gh = \rho gz \rightarrow$ hydrostatic pressure

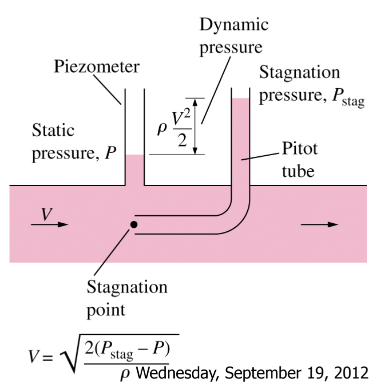
$\frac{\rho V^2}{2} \rightarrow$ dynamic pressure

$p \rightarrow$ static pressure



## Bernoulli's Equation

- If we have a manometer or some other device for measuring static pressure, we can determine the velocity of the flow from the reading on the Pitot tube.




$$\frac{\rho V^2}{2} + p = p_{stagnation}$$

$\rho gh = \rho gz \rightarrow$ hydrostatic pressure

$\frac{\rho V^2}{2} \rightarrow$ dynamic pressure

$p \rightarrow$ static pressure

# Problem 5-43

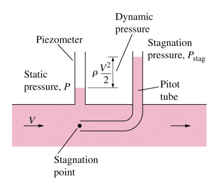
- **5–43** A piezometer and a Pitot tube are tapped into a 3-cm diameter horizontal water pipe, and the height of the water columns are measured to be 20 cm in the piezometer and 35 cm in the Pitot tube (both measured from the top surface of the pipe). Determine the velocity at the center of the pipe.

$$\frac{\rho V^2}{2} + p = p_{stagnation}$$

$\rho g h = \rho g z \rightarrow$ hydrostatic pressure

$\frac{\rho V^2}{2} \rightarrow$ dynamic pressure

$p \rightarrow$ static pressure








# Problem 5-43

We know the height of the column of water for both the static pressure P and the Dynamic pressu we can convert these heights of water into pressures using the mass density of water and the graviational constant.

$\rho_{water} := 1000 \frac{kg}{m^3}$ $\quad h_{piezometer} := 20cm$ $\quad h_{pitot} := 35cm$

$kPa \equiv 1000Pa$

$P_{static} := \rho_{water} \cdot g \cdot h_{piezometer}$ $\quad P_{static} = 1.961\,kPa$

$P_{stagnation} := \rho_{water} \cdot g \cdot h_{pitot}$ $\quad P_{stagnation} = 3.432\,kPa$

$V := \sqrt{\frac{2 \cdot (P_{stagnation} - P_{static})}{\rho_{water}}}$ $\quad V = 1.72 \frac{m}{s}$

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# Homework 10-1

**3.49** A venturi meter is a device placed in a pipeline and is calibrated to give the volume rate of liquid through it as a function of pressure drop (see Figure P3.49). For a flow rate of 4 $ft^3/s$ through the $16 \times 8$ in. meter shown, determine the reading $\Delta h$ on the manometer. Assume that the liquid in the meter and in the manometer is water.

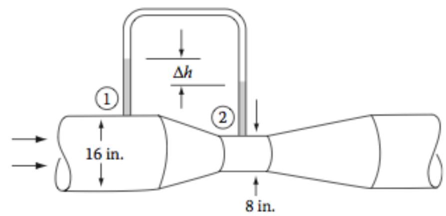


**FIGURE P3.49**



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# Homework 10-2

**3.51** Water flows steadily through the system shown in Figure P3.51. The 4-in. section leads to a 3-in. throat, followed by a divergent section and, finally, a nozzle whose exit diameter is 2 in. An air-over-water manometer is connected between the 4-in. and 3-in. sections. Determine the expected reading $\Delta h$ on the manometer if the velocity at the exit is 8 ft/s.

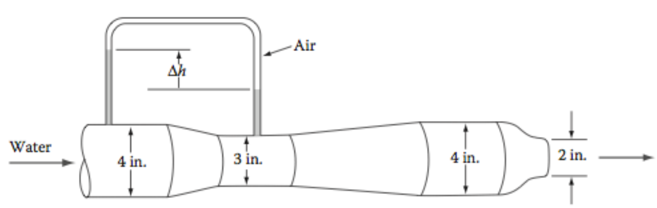


**FIGURE P3.51**

# Homework 10-3

**3.52** An open-top tank is discharging glycerine through an opening onto which an elbow has been placed, as in Figure P3.52. Determine the height of the glycerine jet, assuming frictionless flow and a tank diameter that is much greater than the exit pipe diameter.

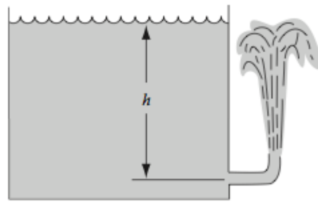


**FIGURE P3.52**

*Assume that the velocity at the top of the water is equal to 0 (tank diameter much greater than the exit pipe diameter). Solution will be in terms of h.*