Uninterrupted Flow

Reading Assignment: Chapter 5 (sections 5.1-5.3)

We are going to be discussing models of traffic flow and queuing for uninterrupted flow.

2 classifications of flow:

Interrupted:

Uninterrupted:

<u>Definitions</u>:

Flow (Q) –

Volume (V) –

Capacity (Q_{cap}) –

Time mean speed -

$$\overline{u_t} = \frac{\sum_{i=1}^{n} u_i}{n}$$

u = spot speed
(arithmetic mean)

Ex.

Veh	Speed (mph)
1	30
2	40
3	50
4	60

Space mean speed –

$$\overline{u_s} = \frac{l}{\bar{t}}$$

$$\bar{t} = \frac{1}{n}\sum_{i=1}^{n} t_i$$

Ex. (Same as previous, with fixed 1 mile distance).

Veh	Speed (mph)	Travel Time
1	30	0.033333
2	40	0.025000
3	50	0.020000
4	60	0.016667

For traffic models, we will use space mean speed!

Free speed (U_f) -

Headway (h) –

Space headway (s) –

density **(***K***)** -

If there are *P* vehicles in a section of roadway with length l miles, then

$$K = \frac{P}{l} \text{ veh/mi}$$

jam density **(**K_j**)** -

Basic Relationships:

1. $Q = \dfrac{3600}{\bar{h}}$

2. $Q = KU$ *** *Basic equation of traffic fl*

****Make sure UNITS on Q and K match!!! (vph or vph/lane)

Example: (5.4 in textbook)
Assume you are observing traffic in a single lane of a highway at a specific location. You measure the average headway and average spacing of passing vehicles as 3 seconds and 150 ft, respectively. Calculate the flow, average speed, and density of the traffic stream in this lane.

Example: (5.2 in textbook)
A section of highway has a speed-flow relationship of the form $q = au^2 + bu$. It is known that at capacity (which is 2900 veh/h) the space-mean speed of traffic is 30 mph. Determine the speed when the flow is 1400 veh/h and the free-flow speed.

Basic Traffic Flow Models:

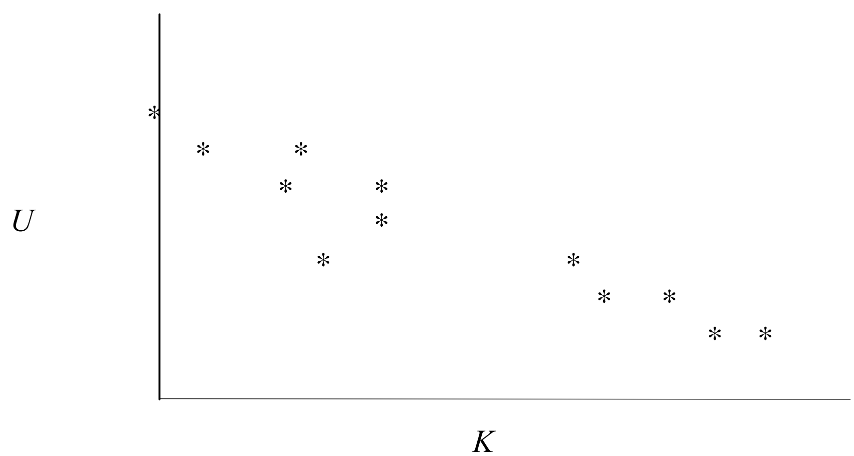
Macroscopic –

Microscopic –

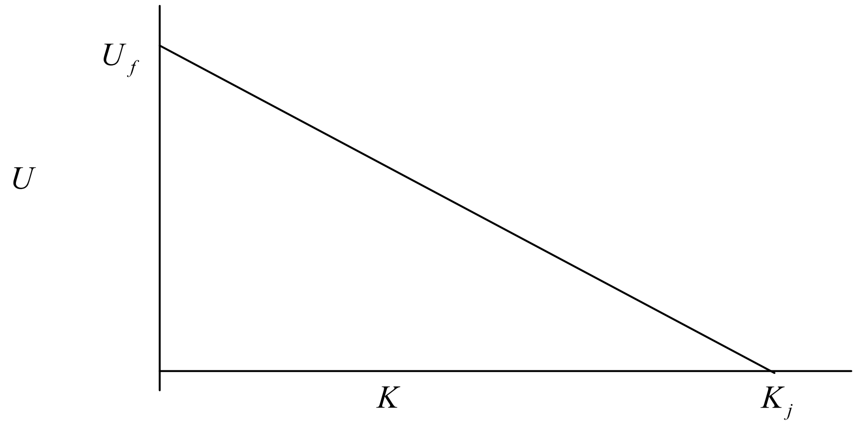
We will be discussing the most common macroscopic model (Greenshields).

LINEAR SPEED-DENSITY MODEL

Greenshields hypothesized that a linear relationship exists between speed and density, implying flow/density and flow/speed relationships are parabolic. Greenshields model is useful for either light or dense traffic.



Greenshields assumed that the relationship between U and K actually was linear, and created a mathematical (linear regression model: y = a + bx → $U = U_f - \left(\frac{U_f}{K_j}\right)K$) model to that effect. The graphical representation of this model is shown below.



Linear Speed-Density Model

The endpoints of this curve are determined by U_f and K_j.

$$\boxed{U = U_f\left(1 - \frac{K}{K_j}\right)}$$

This is the basic equation of the linear model.

We can get another useful relationship out of this if we multiply both sides of the equation by K.

$$KU = U_f\left(K - \frac{K^2}{K_j}\right)$$

Since $Q = KU$:

$$Q = U_f\left(K - \frac{K^2}{K_j}\right)$$

Divide the last equation by U_f.

$$\frac{Q}{U_f} = K - \frac{K^2}{K_j}$$

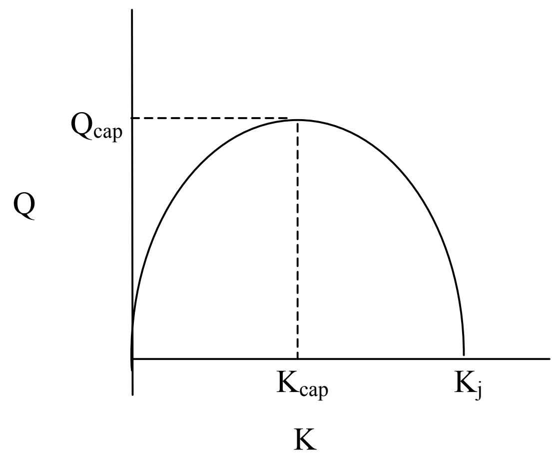
Multiply both sides of this by K_j.

$$\frac{QK_j}{U_f} = (K_j)K - K^2$$

$$K^2 - (K_j)K + \frac{QK_j}{U_f} = 0$$

$$\boxed{K = \frac{K_j \pm \sqrt{K_j^2 - \frac{4QK_j}{U_f}}}{2}}$$

FLOW-DENSITY MODEL



Variation of Flow Rate With Density

Note that there is an upper limit on flow rate. We designate this as Q_{cap}. This maximum flow rate is actually <u>capacity</u>. If we define K_{cap} as the density corresponding to the maximum flow rate, Q_{cap}, then

$$\boxed{K_{cap} = \frac{K_j}{2}}$$

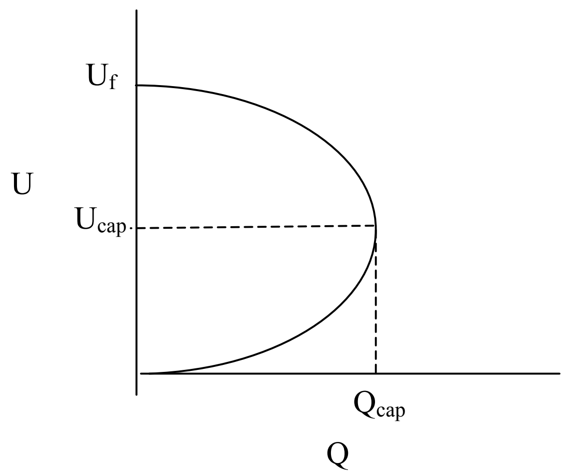
<u>for the linear model</u> (*only!*) because of symmetry.

SPEED-FLOW MODEL

Just as we developed a quadratic equation for writing density as a function of Q, we can also develop one for space mean speed as a function of Q. It is as follows:

$$\boxed{U = \frac{U_f \pm \sqrt{U_f^2 - \frac{4QU_f}{K_j}}}{2}}$$

This, too, will be parabolic when plotted.



Variation of Speed With Flow Rate

We note the existence of Q_{cap} on this plot also. Defining U_{cap} to be the space mean speed *at which capacity occurs*, we see that

$$\boxed{U_{cap} = \frac{U_f}{2}}$$

because of symmetry.

Making use of the expressions for K_{cap} and U_{cap}, we can use the $Q = KU$ relationship to get a simple equation for capacity, Q_{cap}.

$$Q_{cap} = K_{cap} U_{cap} = \left(\frac{K_j}{2}\right)\left(\frac{U_f}{2}\right)$$

This can be shortened to:

$$\boxed{Q_{cap} = \frac{U_f K_j}{4}}$$

Keep in mind that with the exception of $Q = KU$, *all of these equations are good for the* *<u>linear speed-density model **only**!</u>*

Example:

Given the data set of speeds and densities attached, determine the linear regression model for the data set. Determine values of U_f, K_j, Q_{cap}, U_{cap}, K_{cap}. How well does this model fit the data?

Linear Regression Model : $y = a + bx$

a.) Greenshields Model $\quad \overset{y}{u} = \overset{a}{uf} - \underbrace{(uf/kj)}_{b}\overset{x}{k}$

Speed, U (mph) yi	Density, k (vpm) xi	xiyi	xi^2
53.2	20	1064	400
48.1	27	1298.7	729
44.8	35	1568	1225
40.1	44	1764.4	1936
37.3	52	1939.6	2704
35.2	58	2041.6	3364
34.1	60	2046	3600
27.2	64	1740.8	4096
20.4	70	1428	4900
17.5	75	1312.5	5625
14.6	82	1197.2	6724
13.1	90	1179	8100
11.2	100	1120	10000
8	115	920	13225

n	14
sum yi	404.8
sum xi	892
sum xiyi	20619.8
sum xi^2	66628
y	28.91429
x	63.71429

$$a = \frac{1}{n}\sum_{i=1}^{n} y_i - \frac{b}{n}\sum_{i=1}^{n} x_i = \bar{y} - b\bar{x}$$

$$b = \frac{\sum_{i=1}^{n} x_i y_i - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2}$$

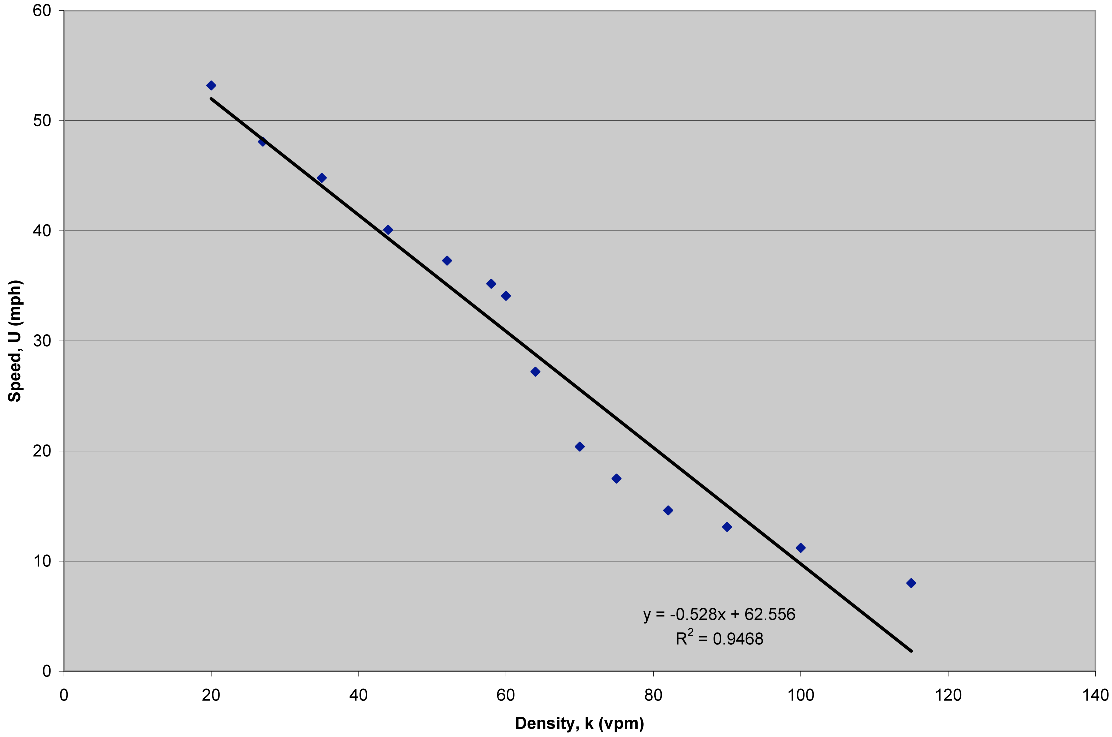
$$R^2 = \frac{\sum_{i=1}^{n}(Y_i - \bar{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

(Coefficient of determination – measure of suitability of an estimated regression function.)

n = # observations

Y_i = value of y computed from regression equation

y_i = value of ith observation of y.



60
50
40
30
20
10
0
Speed, U (mph)
0
20
40
60
80
100
120
140
Density, k (vpm)
y = -0.528x + 62.556
R2 = 0.9468

Recap of Greenshields Formuals:

$$\boxed{U = U_f\left(1 - \frac{K}{K_j}\right)}$$

$$\boxed{K = \frac{K_j \pm \sqrt{K_j^2 - \frac{4QK_j}{U_f}}}{2}}$$

$$\boxed{U = \frac{U_f \pm \sqrt{U_f^2 - \frac{4QU_f}{K_j}}}{2}}$$

$$\boxed{K_{cap} = \frac{K_j}{2}}$$

$$\boxed{U_{cap} = \frac{U_f}{2}}$$

$$\boxed{Q_{cap} = \frac{U_f K_j}{4}}$$

*Remember- these equations are valid for linear speed-density model only!!!

Q = KU ***always true- basic equation of traffic flow.

*The only time you use the (+) sign on density (and thus (–) on space mean speed) is when DEMAND EXCEEDS CAPACITY AND THE LOCATION IS IMMEDIATELY UPSTREAM OF A BOTTLENECK. The majority of the time we will use (-) for density and therefore (+) for space mean speed.

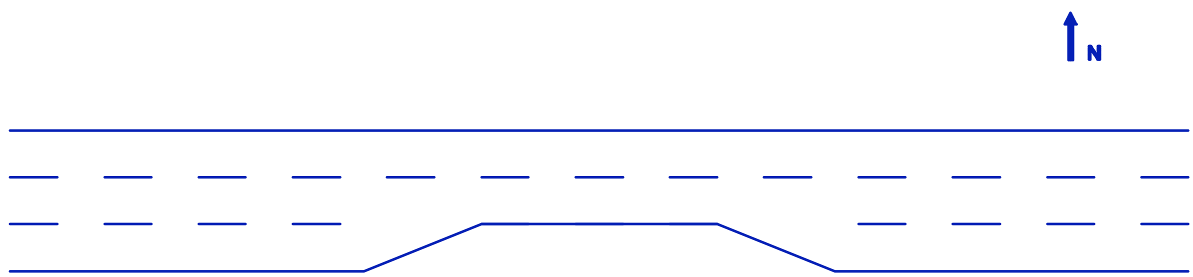
SAMPLE Q-K-U PROBLEMS

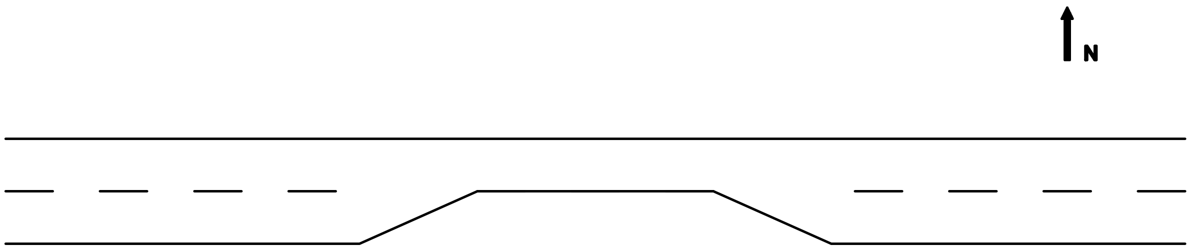
During the AM peak period when total demand is 5100 vehicles per hour, Dr. Lipinski's pickup truck stalls in the right lane of eastbound I-240, a six-lane freeway. Assuming a linear speed-density model with $U_f = 60$ mph and $K_j = 120$ veh/lane-mi, find the following:

i. Speed and density downstream of the blockage.
ii. Speed, flowrate, and density immediately upstream of the blockage.

Suppose it takes 30 minutes to get Dr. Lipinski's pickup moving again.

iii. Estimate the maximum queue length and the number of vehicles in it at that time.
iv. How long does it take for the queue to dissipate?





Given: U_f = 60 mph
K_j = 120 veh/mi-lane
Total eastbound demand = 1200 vph (off peak), then:
= 2500 vph (for 30 min. in peak), then:
= 1200 after peak

a. Is this model realistic?
b. What are the speeds upstream, at, and downstream of the lane reduction in the off-peak?
c. What are the speeds a "long way" upstream, immediately upstream, at, and downstream of the lane reduction during the peak?
d. If there is a queue at the end of the peak, how long is it?
e. If there is a queue at the end of the peak, how long does it take for it to dissolve?
f. How many vehicles experienced queueing?