Rules for Rivet-Hole Deductions in Tension Members

Simple Formulas for Equal-Stress and Equal-Area Methods Derived—How They Compare in Practical Application

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THERE has recently been considerable discussion in regard to the proper deductions for staggered rivet holes in tension members, but it appears that no satisfactory conclusions have been reached. The writer desires to make some further observations on the subject, in the hope that the definite and simple working rules proposed herein may aid in securing general acceptance of the method of fractional deductions.

The so-called theoretical method of allowing for the effect of staggered rivets, proposed by the writer in *Engineering News* of April 23, 1908, p. 465, assumes in effect that transverse and zigzag sections are equal in strength when the maximum tension, considering the effect of shear, is the same in both sections (equal-stress method). Another rule in common use assumes the equality of strength when the transverse and zigzag sections are of equal area (equal-area method). The former requires the greater stagger for equal deductions.

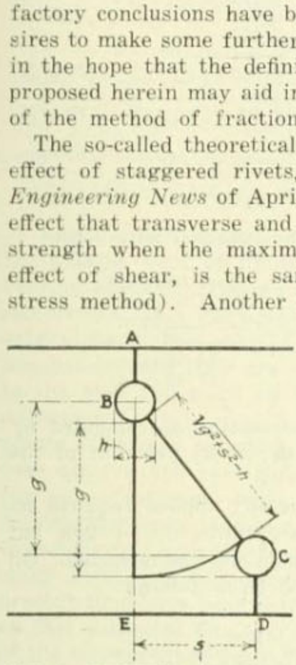


FIG. 1—MINIMUM STAGGER OF RIVET HOLES

There has been some controversy as to whether the equal-stress method is in accordance with experimental results. Edward Godfrey, in your issue of Aug. 31, 1922, p. 366, contends that this rule gives excessively large deductions, and that tests made by him prove the correctness of the equal-area rule. These claims are controverted by Prof. C. R. Young in your issue of Sept. 21, 1922. It seems worth while therefore to show by examples just what the difference in the two rules amounts to, expressed in percentages of the gross section.

Whichever of these two rules may be considered correct, there is no question as to the desirability of making fractional rivet hole deductions when the stagger is less than that required for zero deduction. It will be shown that this can be done with equal facility by either method; a simple formula can be written in either case, a single diagram will suffice for any and all sizes of rivets, and if desired the deductions may be found by a simple graphical method.

Consider first the equal-area rule. In Fig. 1 the net section along the diagonal is $\sqrt{g^2 + s^2} - h$, in which g is the gage, s the stagger and h diameter of rivet holes. If the net area of the transverse section *ABE* is equal to that of the diagonal section *ABCD*, it is evident that this quantity must be equal to g, whence

$$s = \sqrt{2gh + h^2} \qquad (1)$$

This is the well-known formula given in handbooks to show the relation between s and g required to maintain net section. On the basis of the fractional-hole method of computing the allowance for staggered rivets, as advocated by T. A. Smith and Professor Young, the value of s given by Eq. (1) is that value for which no deduction from the transverse section is to be made.

It can be shown that under the equal-stress hypothesis Eq. (1) gives that value of s for which the deduction is half of a rivet hole, instead of zero as in the case of the equal-area rule.

Let w be the width of strip to be deducted from the transverse section on account of a staggered rivet hole. It is evident that when s is equal to zero, the width w is equal to h. As s increases, w decreases, slowly at first and then more rapidly. It seems reasonable to assume that the decrement of w, as s increases, varies as the square of s, hence we may write

$$w = h - cs^2, \qquad (2)$$

in which c is a constant.

Therefore if in accordance with the above assumption we substitute $h/2$ for w and $\sqrt{2gh + h^2}$ for s in Eq. (2), we obtain the equation

$$h/3 = h - c\,(2gh + h^2),$$

whence

$$c = \frac{1}{4g + 2h}$$

and

$$w = h - \frac{s^2}{4g + 2h}$$

Since the term $2h$ in the denominator is comparatively small we may for practical purposes simplify the above formula by omitting it, thus reducing the equation to

$$w = h - \frac{s^2}{4g} \qquad (3)$$

This formula gives the width to be deducted on account of a staggered rivet and it is recommended for practical use. Although much simpler than Professor Young's formula, or the substitute therefor proposed by the writer in your issue of July 6, 1922, it gives results in remarkably close agreement with the so-called theoretical formula as derived by Professor Young.

Proceeding in a similar manner the equal-area rule gives the following formula for fractional deductions:

$$w = h - \frac{s^2}{2g + h}$$

Substituting for h in the denominator of the last term an average value of 1, we have

$$w = h - \frac{s^2}{2g + 1} \qquad (4)$$

This is the formula proposed for use in making fractional deductions by the equal-area rule.

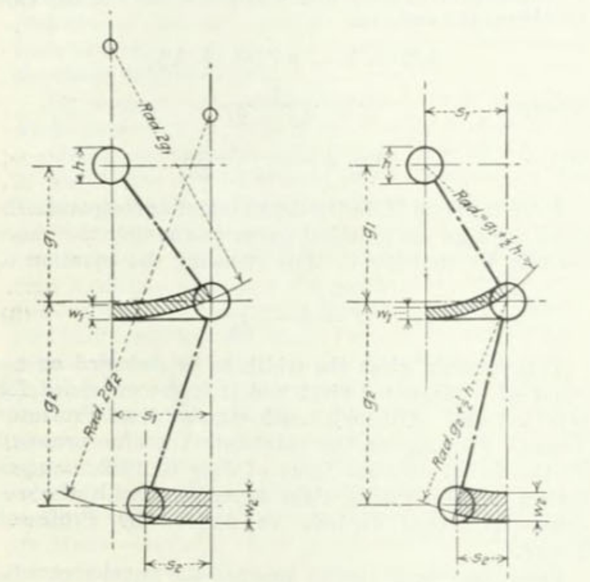
Comparing equations (3) and (4), it will be seen that they give practically the same deductions for small values of s; but as s increases, the negative term increases almost twice as fast in the equal-area formula as in the equal-stress formula. Either formula will give consistent results. The maximum difference in the deduction for one hole by the two formulas is nearly half a rivet hole, but it will usually be less than this.

The width w to be deducted in any case can be determined by a simple graphical method. Thus in Fig. 2 the shaded strip w, is the deduction for the rivet having stagger s_1 and gage g_1. The width w_1 is that portion of the diameter h intercepted by an arc of radius $2g_1$ in the position shown. Fig. 3 shows a similar construction for the equal-area method, the arc being concentric with the last rivet and having a radius equal to $g + \frac{1}{2}h$.

A single diagram giving values of the last term of formula (3) or (4), the decrement of h, will suffice for any diameter of rivet, thus making the method of fractional deductions more convenient in practical use than if a separate diagram were required for each size of rivet.

Some examples will now be given showing the difference in net section by the two rules, using equations (3) and (4) to compute the deductions.

First take the case of a 12 x ½-in. plate having two rivet holes on a transverse line, located 1½ in. from each edge, with a staggered rivet on the center line. Assuming ⅞-in. rivets, h is equal to 1. The gage g is equal to 4.5 in. The maximum deduction along the



FIGS. 2 AND 3—RIVET-HOLE DEDUCTIONS BY GRAPHICS

transverse line $= 2h = 2$. The following table shows the deductions along the zigzag line through the staggered hole for various staggers; also the difference in net areas expressed as percentages of the gross area:

	Deductions Along Zigzag Line Computed by		Maximum Deductions By		Differences	
Stagger s in Inches	Equal-Stress Formula	Equal-Area Formula	Equal-Stress Method	Equal-Area Method	Width in Inches	Per Cent Of Gross Width
1.0	0.94	0.90	2.88*	2.80	0.08	0.6
1.5	0.87	0.78	2.74	2.56	0.18	1.5
2.0	0.78	0.60	2.56	2.20	0.36	3.0
2.25	0.72	0.49	2.44	2.00	0.44	3.7
2.5	0.65	0.38	2.30	2.00	0.30	2.5
3.0	0.50	0.10	2.00	2.00	.....	.....
3.5	0.18	0.00	2.00	2.00	.....	.....
4.0	0.11	0.00	2.00	2.00	.....	.....

* 1.00 + 0.94 + 0.94 = 2.88

In this case the maximum difference is 3.7 per cent of the gross section.

Consider now the built-up section shown in Fig. 4. By the equal-stress formula (3) the net section is computed thus:

	Gage g	Stagger s			
Angles,	4½	1½	(2 x 1⅛—0.13) x ½ x 4		4.24 sq.in.
30 in. x ¾ in. plates,	First hole		1⅛—0	1.125	
	2	3	do.—1⅛		
and	3¼	3	do.—0.69		
or	5¾	0	do.—0	1.125	
	5	3	do.—0.45	0.675	
	5	0	do.—0	1.125	
	5	3	do.—0.45	0.675	
	5¾	0	do.—0	1.125	
				5.850 x ¾ x 2	8.78 sq.in.
18 in. x ½ in. plates				3.60 x ½ x 2	3.60 sq.in.
Total deductions					16.62 sq.in.

Net section, 82.0—16.62 = 65.38 sq.in.

By the equal-area formula (4) the deductions are as follows:

Angles	4.04 sq.in.
30-in. plates	7.66 sq.in.
18-in. plates	2.86 sq.in.
Total	14.56 sq.in.
Net section	67.44 sq.in.

Hence the difference in net section as computed by the two formulas is 2.06 sq.in., or 2½ per cent of the gross section.

The equal-area rule as generally applied requires no deduction if the stagger is as great as $\sqrt{h^2 + 2gh}$, but for any lesser stagger a full hole must be deducted. On this basis the deduction would be as follows:

Angles, 8 holes out	4.50 sq.in.
30-in. plates, 12 holes out	10.12 sq.in.
18-in. plates, 8 holes out	4.50 sq.in.
Total	19.12 sq.in.

This is a greater deduction than that obtained by either of the fractional-deduction formulas. This shows the importance of making fractional deductions rather than counting out the entire hole or none.

The differences in results by formulas (3) and (4) are so small that there seems to be no prospect of conclusively establishing by experiment the correctness of

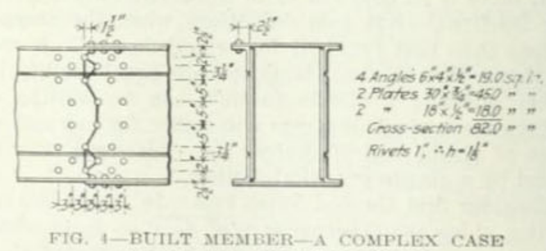


FIG. 4—BUILT MEMBER—A COMPLEX CASE OF COMPUTING NET SECTION

either formula as compared with the other. The equal-stress rule is preferred by the writer because it has some theoretical basis and is somewhat more conservative than the equal-area rule. On the other hand it requires much less reduction of area, or much smaller staggers for the same reduction, than does the common rule requiring 30 per cent excess diagonal area for equal transverse and diagonal strength.