

1

Chapter 5 – Shear Strength

- Beam shear strength is covered in **Chapter G of the AISC Specification, “Design of Members for Shear.”**
- Both hot-rolled shapes and welded built-up shapes are covered.
- We will discuss *hot-rolled shapes* in this course.
- The AISC provisions for hot-rolled shapes are covered in **Section G2.1.**

2

Chapter 5 – Shear Strength

- Before covering the **AISC provisions for shear strength**, we will first review some basic concepts from mechanics of materials.
- Consider the simple beam.
- At a distance x from the left end and at the neutral axis of the cross section the state of stress is as shown below.
- Because this element is located at the **neutral axis**, it is not subjected to **flexural stress**.

3

Chapter 5 – Shear Strength

- Before covering the **AISC provisions for shear strength**, we will first review some basic concepts from mechanics of materials.
- From elementary mechanics of materials, the shearing stress is:

$$f_v = \frac{VQ}{Ib}$$

f_v is the vertical and horizontal shearing stress at the point of interest,

V is the vertical shear force at the section under consideration

4

Chapter 5 – Shear Strength

- Before covering the **AISC provisions for shear strength**, we will first review some basic concepts from mechanics of materials.
- From elementary mechanics of materials, the shearing stress is:

$$f_v = \frac{VQ}{Ib}$$

Q is the first moment, about the neutral axis, of the area of the cross-section between the point of interest and the top or bottom of the cross-section

5

Chapter 5 – Shear Strength

- Before covering the **AISC provisions for shear strength**, we will first review some basic concepts from mechanics of materials.
- From elementary mechanics of materials, the shearing stress is:

$$f_v = \frac{VQ}{Ib}$$

I is the moment of inertia about the neutral axis

b is the width of the cross-section at the point of interest

6

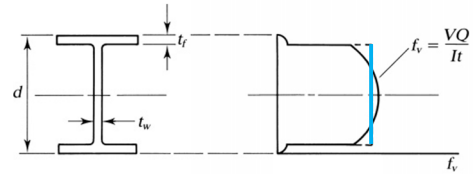
Chapter 5 – Shear Strength

- The **shear stress equation** assumes that the stress is constant across the width **b**.
- It is therefore accurate only for small values of **b**.
- For a rectangular cross-section of depth **d** and width **b**, the error for **d/b = 2** is approximately 3%.
- For **d/b = 1**, the error is 12% and for **d/b = 1/4**, it is 100%.
- For this reason, the shear stress equation cannot be applied to the **flange** of a **W-shape** in the same manner as for the **web**.

7

Chapter 5 – Shear Strength

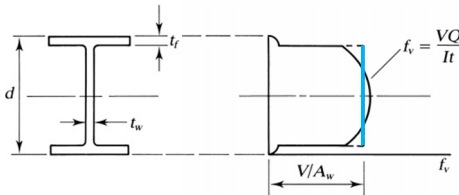
- Here is a diagram of the shearing stress distribution for a **W shape**.
- Superimposed on the actual distribution is the average stress in the web, **V/A_w**, which does not differ much from the maximum web stress.



8

Chapter 5 – Shear Strength

- Clearly, the **web will completely yield** long before the flanges begin to yield.
- Because of this, **yielding of the web** represents one of the **shear limit states**.



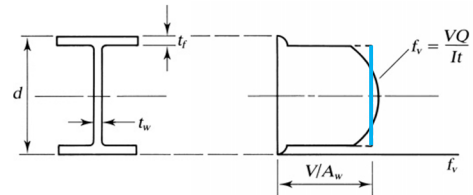
9

Chapter 5 – Shear Strength

- Taking the **shear yield stress** as 60% of the **tensile yield stress**, the equation for the stress in the web at failure as:

$$f_v = \frac{V_n}{A_w} = 0.6F_y \quad \text{where } A_w \text{ is the area of the web.}$$

$$V_n = 0.6F_y A_w \quad \text{The nominal strength corresponding to this limit state}$$



10

Chapter 5 – Shear Strength

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$$V_n = 0.6F_y A_w \quad \text{The nominal strength corresponding to this limit state}$$

- This will be the nominal strength in shear if there is **no shear buckling of the web**.
- Whether that occurs will depend on **h/t_w**, the width-to-thickness ratio of the **web**.

11

Chapter 5 – Shear Strength

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- If this ratio is too large—that is, **if the web is too slender**—the **web can buckle in shear**, either inelastically or elastically.

12

Chapter 5 – Shear Strength

- For **LRFD**, the relationship between required and available strength is: $V_u \leq \phi_v V_n$
- where V_u is the maximum shear based on the controlling combination of factored loads, and
- ϕ_v is the resistance factor for shear
- Let's look at **Chapter G** in the **AISC Specification** for more details on shear design.

13

Chapter 5 – Shear Strength

CHAPTER G
DESIGN OF MEMBERS FOR SHEAR

This chapter addresses webs of singly or doubly symmetric members subjected to shear in the plane of the web, single angles and RHS subjected to shear, and shear in the weak direction of singly or doubly symmetric shapes.

The chapter is organized as follows:

- G1. General Provisions
- G2. I-Shaped Members and Channels
- G3. Single Angles and Tees
- G4. Built-up IHS, Box Sections, and Other Singly and Doubly Symmetric Members
- G5. Round HSS
- G6. Doubly Symmetric and Singly Symmetric Members Subjected to Mean-Axis Shear
- G7. Beams and Girders with Web Openings

1. Shear Strength of Webs
The nominal shear strength, V_n , is $V_n = 0.6F_yA_wC_v$

14

Chapter 5 – Shear Strength

CHAPTER G
DESIGN OF MEMBERS FOR SHEAR

G1. GENERAL PROVISIONS

The design shear strength, $\phi_v V_n$, and the allowable shear strength, V_n/Ω_v , shall be determined as follows:

- For all provisions in this chapter except Section G2.1(a)

$$\phi_v = 0.90 \text{ (LRFD)} \quad \Omega_v = 1.67 \text{ (ASD)}$$
- The nominal shear strength, V_n , shall be determined according to Sections G2 through G7.

1. Shear Strength of Webs
The nominal shear strength, V_n , is $V_n = 0.6F_yA_wC_v$

15

Chapter 5 – Shear Strength

CHAPTER G
DESIGN OF MEMBERS FOR SHEAR

G2. I-SHAPED MEMBERS AND CHANNELS

This section addresses the determination of shear strength for I-shaped members and channels. Section G2.1 is applicable for webs with and without transverse stiffeners. Alternatively, Sections G2.2 and G2.3 are permitted to be used for webs with transverse stiffeners.

1. Shear Strength of Webs
The nominal shear strength, V_n , is $V_n = 0.6F_yA_wC_v$

16

Chapter 5 – Shear Strength

- As we will see, the values of the **resistance factor**, ϕ will depend on the **web width-to-thickness ratio**.
- Section G2.1** of the **AISC Specification** covers both beams with stiffened webs and beams with unstiffened webs.
- In most cases, **hot-rolled beams** will not have stiffeners.
- The basic strength equation is: $V_n = 0.6F_yA_wC_{v1}$
- where A_w is the area of the web $\approx dt_w$
- d is the overall depth of the beam, and
- C_{v1} is the ratio of critical **web buckling stress to shear yield stress**

17

Chapter 5 – Shear Strength

- The value of C_{v1} depends on whether the limit state is **web yielding** or **web buckling**.
- Case 1: For hot-rolled I shapes with: $\frac{h}{t_w} \leq 2.24 \sqrt{\frac{E}{F_y}}$
- The limit state is **shear yielding**, and $C_{v1} = 1.0 \quad \phi_v = 1.0$ (AISC Equation G2-2)
- Most **W shapes** with $F_y \leq 50 \text{ ksi}$ fall into this category (see User Note in AISC G2.1[a]).

User Note: All current ASTM A6/A6M W, S, and HP shapes except W44x230; W40x149, W36x135, W33x118, W30x90, W24x55, W16x26, and W12x14 (meet the criteria stated in Section G2.1(a) for $F_y = 50 \text{ ksi}$ (345 MPa)).

18

Chapter 5 – Shear Strength

- The value of C_{v1} depends on whether the limit state is **web yielding** or **web buckling**.

Case 2: For all other hot-rolled I shapes and channels: $\phi_v = 0.9$

and C_{v1} is determined as follows:

For $\frac{h}{t_w} \leq 1.10 \sqrt{\frac{k_v E}{F_y}}$ There is no web instability, and $C_{v1} = 1.0$ (AISC Equation G2-3)

For $\frac{h}{t_w} > 1.10 \sqrt{\frac{k_v E}{F_y}}$ $C_{v1} = \frac{1.10 \sqrt{k_v E / F_y}}{h/t_w}$ (AISC Equation G2-4)

where $k_v = 5.34$ (This value of k_v is for unstiffened webs)

19

Chapter 5 – Shear Strength

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User Note: $C_{v1} = 1.0$ for all ASTM A6/A6M W, S, M, and HP shapes except M12.5x12.4, M12.5x11.6, M12x11.8, M12x10.8, M12x10, M10x8, and M10x7.5, when $F_y = 50$ ksi (345 MPa).

20

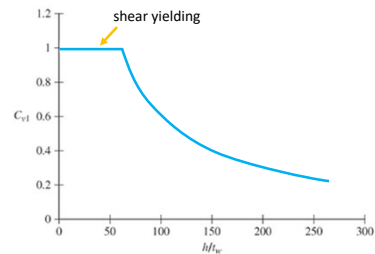
Chapter 5 – Shear Strength

- **AISC Equation G2-3** is based on the limit state of web shear yielding.
- **AISC Equation G2-4** accounts for the web buckling and post-buckling strengths.
- The **web buckling strength** is the shear that brings the web to the **point of buckling**.
- The **post-buckling strength** is the extra strength available after **web buckling has occurred**.

21

Chapter 5 – Shear Strength

- The relationship between the **web width-to-thickness ratio** and strength is shown below.

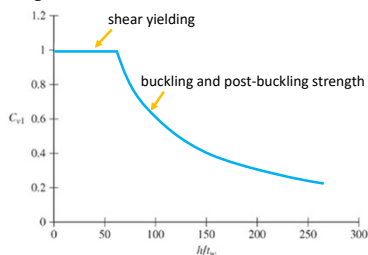


- The flat portion corresponds to web **shear yielding**

22

Chapter 5 – Shear Strength

- The relationship between the **web width-to-thickness ratio** and strength is shown below.

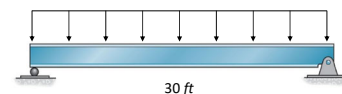


- The curved part corresponds to **buckling** and **post-buckling strength**.

23

Chapter 5 – Shear Strength

- **Example 5.7:** Check the beam in **Example 5.6** for shear.
- A simply supported beam with a span length of **30 ft** is laterally supported at its ends and is subjected to the following service loads: dead load = **0.5 k/ft** (including the weight of the beam) and live load = **1 k/ft**.
- The beam shown is a **W12 x 65** with $F_y = 50$ ksi.

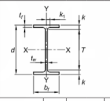


24

Chapter 5 – Shear Strength

➤ **Example 5.7:** Check for compactness. $\frac{h}{t_w} \leq 2.24 \sqrt{\frac{E}{F_y}}$

From Table 1-1 (1-26) for a **W12 x 65**



**Table 1-1 (continued)
W-Shapes
Dimensions**

Shape	Area, A		Depth, d	Web		Flange				Distance			
	in ²	in.		Thickness, t _w	t _w	Width, b _f	Thickness, t _f	k	k ₁	k ₂	r	T	Workable Gauge
W12	44.7	13.7	12.0	0.405	1/4	12.5	0.40	1.94	2.00	25/16	1 1/2	1 1/2	1 1/2
W12	39.9	13.4	12.0	0.390	1/4	12.4	0.35	1.16	1.85	2 1/2	1 1/2	1 1/2	1 1/2
W12	35.2	13.1	12.0	0.375	1/4	12.3	0.35	1.11	1.70	2	1 1/2	1 1/2	1 1/2
W12	31.2	12.9	12.0	0.360	1/4	12.2	0.35	0.900	1.50	1 1/2	1 1/2	1 1/2	1 1/2
W12	28.2	12.7	12.0	0.350	1/4	12.2	0.35	0.900	1.50	1 1/2	1 1/2	1 1/2	1 1/2
W12	25.6	12.5	12.0	0.335	1/4	12.1	0.35	0.810	1.41	1 1/2	1 1/2	1 1/2	1 1/2
W12	23.2	12.4	12.0	0.320	1/4	12.1	0.35	0.725	1.32	1 1/2	1 1/2	1 1/2	1 1/2
W12	20.8	12.2	12.0	0.305	1/4	12.0	0.35	0.640	1.20	1 1/2	1 1/2	1 1/2	1 1/2

$A_w = dt_w$

$d = 12.1in$


$t_w = 0.390in$

25

Chapter 5 – Shear Strength

➤ **Example 5.7:** Check for compactness. $\frac{h}{t_w} \leq 2.24 \sqrt{\frac{E}{F_y}}$

From Table 1-1 (1-26) for a **W12 x 65**



**Table 1-1 (continued)
W-Shapes
Properties**

Nominal WL	Compact Section Criteria		Axis X-X					Axis Y-Y					Torsional Properties	
	b _f /2t _f	h/t _w	I	S	r	Z	I	S	r	Z	J	C _w		
W12	8.22	20.7	662	107	5.34	119	216	35.8	3.05	54.3	3.43	11.7	3.84	7530
W12	8.50	21.0	669	107	5.34	120	216	35.8	3.05	54.3	3.43	11.7	3.84	7530
W12	8.78	21.3	676	107	5.34	121	216	35.8	3.05	54.3	3.43	11.7	3.84	7530
W12	9.06	21.6	683	107	5.34	122	216	35.8	3.05	54.3	3.43	11.7	3.84	7530
W12	9.34	21.9	690	107	5.34	123	216	35.8	3.05	54.3	3.43	11.7	3.84	7530
W12	9.62	22.2	697	107	5.34	124	216	35.8	3.05	54.3	3.43	11.7	3.84	7530
W12	9.90	22.5	704	107	5.34	125	216	35.8	3.05	54.3	3.43	11.7	3.84	7530
W12	10.18	22.8	711	107	5.34	126	216	35.8	3.05	54.3	3.43	11.7	3.84	7530
W12	10.46	23.1	718	107	5.34	127	216	35.8	3.05	54.3	3.43	11.7	3.84	7530
W12	10.74	23.4	725	107	5.34	128	216	35.8	3.05	54.3	3.43	11.7	3.84	7530
W12	11.02	23.7	732	107	5.34	129	216	35.8	3.05	54.3	3.43	11.7	3.84	7530
W12	11.30	24.0	739	107	5.34	130	216	35.8	3.05	54.3	3.43	11.7	3.84	7530

$\frac{h}{t_w} = 24.9$

26

Chapter 5 – Shear Strength

➤ **Example 5.7:** Compute web area:

$A_w = dt_w = 12.1in(0.390in) = 4.719in^2$

$2.24 \sqrt{\frac{E}{F_y}} = 2.24 \sqrt{\frac{29,000ksi}{50ksi}} = 54.95$ $\frac{h}{t_w} = 24.9$

Since $\frac{h}{t_w} < 2.24 \sqrt{\frac{E}{F_y}}$ The strength is governed by the **shear yielding** of the web and $C_{v1} = 1.0$ and $\phi_v = 1.0$

As pointed out in the **Specification G2 User Note**, this is the case for most **W shapes** with $F_y = 50 ksi$.

User Note: All current ASTM A6/ASTM W, S, and HP shapes except W44×230, W40×149, W36×135, W33×118, W30×90, W24×55, W16×26, and W12×14 meet the criteria stated in Section G2.1(a) for $F_y = 50 ksi$ (345 MPa).

27

Chapter 5 – Shear Strength

➤ **Example 5.7:** The nominal shear strength is:

$V_n = 0.6F_y A_w C_{v1} = 0.6(50ksi)(4.719in^2)(1.0) = 141.57k$

LRFD solution: $\phi_v V_n = 1.00(141.57k) = 141.57k$

For a simply supported, uniformly loaded beam, the maximum shear occurs at the support and is $V_u = \frac{1}{2}w_u L$


$w_u = 1.2(0.5 k/ft) + 1.6(1.0 k/ft) = 2.2k/ft$

$V_u = \frac{2.20k/ft(30ft)}{2} = 33.0k < 141.57k$ **O.K.**

28

Chapter 5 – Shear Strength

Let's work on some problems

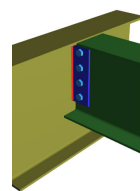
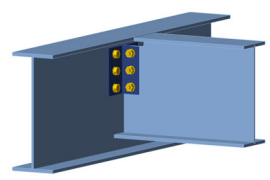


29

Chapter 5 – Block Shear

➤ **Block shear**, which was considered earlier for **tension member connections**, can occur in certain types of beam connections.

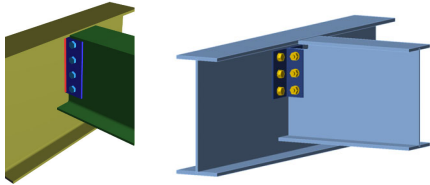
➤ Beams are usually connected to other beams so that the top flanges are at the same elevation.

30

Chapter 5 – Block Shear

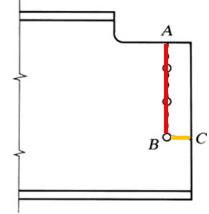
- **Block shear**, which was considered earlier in conjunction with **tension member connections**, can occur in certain types of beam connections.
- To facilitate this connection a short length of the top flange of one of the beams may be **cut away or coped**.



31

Chapter 5 – Block Shear

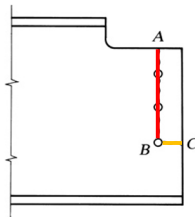
- If a coped beam is connected with bolts as shown below, segment **ABC** will tend to tear out.
- The applied load in this case will be the vertical beam reaction, so shear will occur along line **AB**.
- Tension will occur along **BC**.
- **Block shear strength** will be a limiting value of the reaction.



32

Chapter 5 – Block Shear

- We covered the computation of block shear strength in **Chapter 3**, but we will review it here.
- Failure is assumed to occur by rupture (fracture) on the **shear area** (subject to an upper limit) and rupture on the **tension area**.



33

Chapter 5 – Block Shear

- We covered the computation of block shear strength in **Chapter 3**, but we will review it here.
- **AISC J4.3**, "Block Shear Strength," gives the following equation for block shear strength:

$$R_n = \min(0.6F_u A_{nv}, 0.6F_y A_{gv}) + U_{bs} F_u A_{nt}$$

where A_{gv} is the gross area in shear (length **AB** times t_w),
 A_{nv} is the net area along the shear surface,
 A_{nt} is the net area along the tension surface, and
 U_{bs} is 1.0 when the tensile stress is uniform, or 0.5 when the tension stress is not uniform (coped beams with two lines of bolts or with a nonstandard distance from bolts to the end of the beam)

34

Chapter 5 – Block Shear

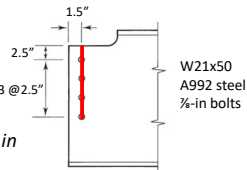
- **Example 5.8:** Determine the maximum reaction, based on block shear, that can be resisted by the beam shown below. **A992 steel** $F_y = 50 \text{ ksi}$ and $F_u = 65 \text{ ksi}$.

From Table 1-1 (1-24) for a **W21 x 50**

$$t_w = 0.380 \text{ in}$$

$$d_{hole} = d_{bolt} + \frac{1}{8} \text{ in} = \frac{7}{8} \text{ in} + \frac{1}{8} \text{ in} = 1 \text{ in}$$

$$\text{The shear areas are: } A_{gv} = t_w (2.5 \text{ in} + 3(2.5 \text{ in})) = 0.380 \text{ in}(10 \text{ in}) = 3.800 \text{ in}^2$$



35

Chapter 5 – Block Shear

- **Example 5.8:** Determine the maximum reaction, based on block shear, that can be resisted by the beam shown below. **A992 steel** $F_y = 50 \text{ ksi}$ and $F_u = 65 \text{ ksi}$.

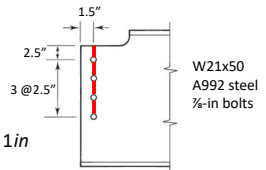
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$$\text{The shear areas are: } A_{gv} = t_w (2.5 \text{ in} + 3(2.5 \text{ in})) = 0.380 \text{ in}(10 \text{ in}) = 3.800 \text{ in}^2$$

$$A_{nv} = (0.38 \text{ in})(10 \text{ in} - 3.5(1) \text{ in}) = 2.47 \text{ in}^2$$



36

Chapter 5 – Block Shear

- **Example 5.8:** Determine the maximum reaction, based on block shear, that can be resisted by the beam shown below. **A992 steel** $F_y = 50 \text{ ksi}$ and $F_u = 65 \text{ ksi}$.

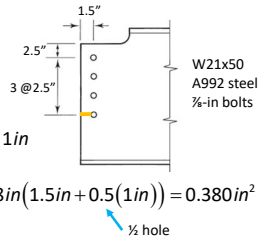
From Table 1-1 (1-24) for a **W21 x 50**

$$t_w = 0.380 \text{ in}$$

$$d_{\text{hole}} = d_{\text{bolt}} + \frac{1}{8} \text{ in} = \frac{7}{8} \text{ in} + \frac{1}{8} \text{ in} = 1 \text{ in}$$

The tension area is: $A_{nt} = 0.38 \text{ in}(1.5 \text{ in} + 0.5(1 \text{ in})) = 0.380 \text{ in}^2$

Since the block shear will occur in a coped beam with a single line of bolts, $U_{bs} = 1.0$.



37

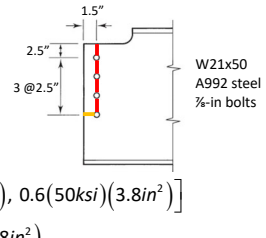
Chapter 5 – Block Shear

- **Example 5.8:** Determine the maximum reaction, based on block shear, that can be resisted by the beam shown below. **A992 steel** $F_y = 50 \text{ ksi}$ and $F_u = 65 \text{ ksi}$.

From **AISC Equation J4-5**,

$$R_n = \min(0.6F_u A_{nv}, 0.6F_y A_{gv}) + U_{bs} F_u A_{nt}$$

$$= \min[0.6(65 \text{ ksi})(2.47 \text{ in}^2), 0.6(50 \text{ ksi})(3.8 \text{ in}^2)] + 1.0(65 \text{ ksi})(0.38 \text{ in}^2)$$



38

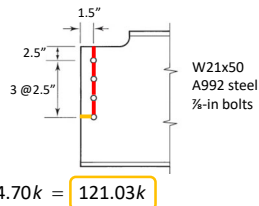
Chapter 5 – Block Shear

- **Example 5.8:** Determine the maximum reaction, based on block shear, that can be resisted by the beam shown below. **A992 steel** $F_y = 50 \text{ ksi}$ and $F_u = 65 \text{ ksi}$.

From **AISC Equation J4-5**,

$$R_n = \min(0.6F_u A_{nv}, 0.6F_y A_{gv}) + U_{bs} F_u A_{nt}$$

$$= \min[96.33 \text{ k}, 114.0 \text{ k}] + 24.70 \text{ k} = 121.03 \text{ k}$$



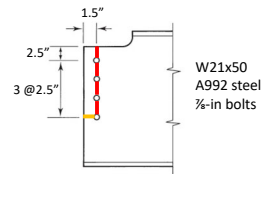
39

Chapter 5 – Block Shear

- **Example 5.8:** Determine the maximum reaction, based on block shear, that can be resisted by the beam shown below. **A992 steel** $F_y = 50 \text{ ksi}$ and $F_u = 65 \text{ ksi}$.

For **LRFD**

$$\phi R_n = 0.75(121.03 \text{ k}) = 90.77 \text{ k}$$



40

Chapter 5 – Beams

Any questions?



41