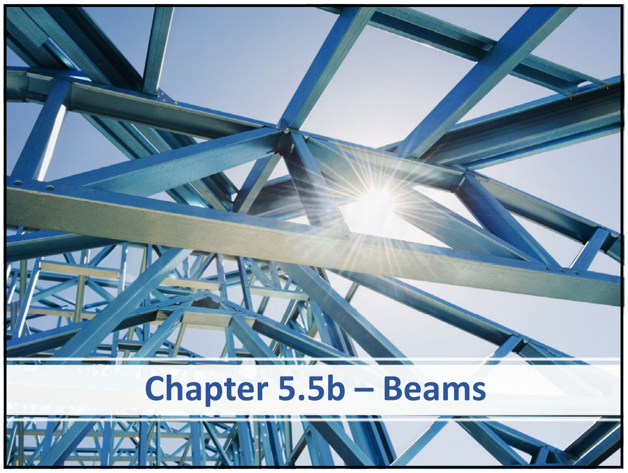
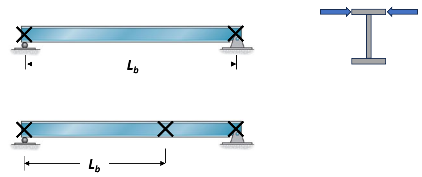


1

Chapter 5 – Compact Shapes

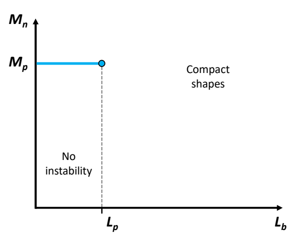
- The moment strength of ***compact shapes*** is a function of the unbraced length, L_b, defined as the distance between points of lateral support, or bracing.
- In this course, we indicate points of lateral support with an "×," as shown below.



2

Chapter 5 – Compact Shapes

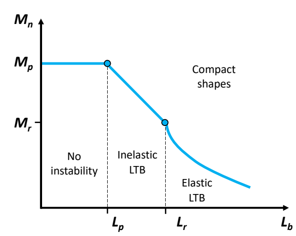
- The relationship between the nominal strength, M_n, and the unbraced length L_b is shown below.
- If the L_b is not greater than L_p, to be defined presently, the beam is considered to have full lateral support, and $M_n = M_p$.



3

Chapter 5 – Compact Shapes

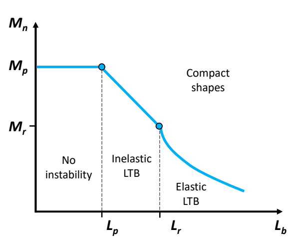
- If L_b is greater than L_p but less than or equal to the parameter L_r, the strength is based on ***inelastic LTB***.
- If L_b is greater than L_r, the strength is based on ***elastic LTB***.



4

Chapter 5 – Compact Shapes

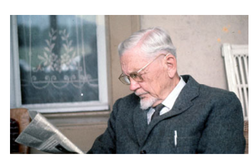
- The equation for the ***theoretical elastic lateral-torsional buckling*** strength can be found in Theory of Elastic Stability (Timoshenko and Gere, 1961).



5

Chapter 5 – Compact Shapes

- ***Stepan Prokopovich Timoshenko*** (1878-1972) is considered to be the father of modern engineering mechanics.
- A founding member of the Ukrainian Academy of Sciences, Timoshenko wrote seminal works in the areas of engineering mechanics, elasticity, and strength of materials, many of which are still widely used today.



6

Chapter 5 – Compact Shapes

- ***Stepan Prokopovich Timoshenko*** (1878-1972) is considered to be the father of modern engineering mechanics.

1. *Applied Elasticity, with J. M. Lessells, D. Van Nostrand Company, 1925*
2. *Vibration Problems in Engineering, D. Van Nostrand Company, 1st Ed. 1928, 2nd Ed. 1937, 3rd Ed. 1955 (with D. H. Young)*
3. *Strength of Materials, Part I, Elementary Theory and Problems, D. Van Nostrand Company, 1st Ed. 1930, 2nd Ed. 1940, 3rd Ed. 1955*
4. *Strength of Materials, Part II, Advanced Theory and Problems, D. Van Nostrand Company, 1st Ed. 1930, 2nd Ed. 1941, 3rd Ed. 1956*
5. *Theory of Elasticity, McGraw-Hill Book Company, 1st Ed. 1934, 2nd Ed. 1951 (with J. N. Goodier), 3rd Ed. 1970 (with J.N. Goodier)*
6. *Elements of Strength of Materials, D. Van Nostrand Co., 1st Ed. 1935, 2nd Ed. 1940, 3rd Ed. 1949 (with G.H. MacCullough), 4th Ed. 1962 (with D.H. Young)*
7. *Theory of Elastic Stability, McGraw-Hill Book Company, 1st Ed. 1936, 2nd Ed. 1961 (with J. M. Gere)*
8. *Engineering Mechanics, with D.H. Young, McGraw-Hill Book Company, 1st Ed. 1937, 2nd Ed. 1940, 3rd. Ed. 1951, 4th Ed. 1956*
9. *Theory of Plates and Shells, McGraw-Hill Book Company, 1st Ed. 1940, 2nd Ed. 1959 (with S. Woinowsky-Krieger)*
10. *Theory of Structures, with D. H. Young, McGraw-Hill Book Company, 1st Ed. 1945, 2nd Ed. 1965*
11. *Advanced Dynamics, with D. H. Young, McGraw-Hill Book Company, 1948*
12. *History of The Strength of Materials, McGraw-Hill Book Company, 1953*
13. *Engineering Education in Russia, McGraw-Hill Book Company, 1959*
14. *Mechanics of Materials, with J. M. Gere, 1st edition, D. Van Nostrand Company, 1972*

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Chapter 5 – Compact Shapes

- With some changes, the nominal moment strength is

$$M_n = F_{cr} S_x$$

where F_{cr} is the elastic buckling stress and is given by

$$F_{cr} = \frac{\pi}{L_b S_x}\sqrt{E I_y G J + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$$

where I_y is the moment of inertia about the weak axis, in^4
G is the shear modulus = 11,200 *ksi*,
J is the torsional constant, in^4,
C_w is the warping constant, in^6

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Chapter 5 – Compact Shapes

- This equation being valid if the bending moment within the unbraced length is uniform (nonuniform moment is accounted for with a factor C_b, which is explained later).

- The ***AISC Specification*** gives a different, but equivalent, form for the elastic buckling stress F_{cr}.

$$M_n = F_{cr} S_x \le M_p \qquad \textit{AISC Equation F2-3}$$

$$F_{cr} = \frac{C_b \pi^2 E}{\left(L_b / r_{ts}\right)^2}\sqrt{1 + 0.078\frac{Jc}{S_x h_0}\left(\frac{L_b}{r_{ts}}\right)^2} \qquad \textit{AISC Equation F2-4}$$

where C_b is a ***lateral-torsional buckling modification factor*** factor to account for nonuniform bending within the unbraced length L_b

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Chapter 5 – Compact Shapes

- Other relationships are:

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x} \qquad \textit{AISC Equation F2-7}$$

c = 1.0 for doubly symmetric I shapes

$$c = \frac{h_0}{2}\sqrt{\frac{I_y}{C_w}} \quad \text{for channels} \qquad \textit{AISC Equation F2-8a}$$

$h_0 = d - t_f$ distance between flange centroids

AISC Equation F2-8b

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Chapter 5 – Compact Shapes

- If the moment when ***lateral-torsional buckling occurs*** is greater than the ***moment corresponding to first yield***, the strength is based on ***inelastic*** behavior.

- The moment corresponding to first yield is: $M_r = 0.7 F_y S_x$

- Here, the yield stress F_y has been reduced by 30% to account for the effect of residual stress.

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Chapter 5 – Compact Shapes

- The boundary between elastic and inelastic behavior will be for an unbraced length of L_r, which is the value of L_b obtained from ***AISC Equation F2-4*** when F_{cr} is set equal to $0.7F_y$ with C_b = 1.0.

$$L_r = 1.95 r_{ts}\frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_x h_0} + \sqrt{\left(\frac{Jc}{S_x h_0}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}}$$

AISC Equation F2-6

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Chapter 5 – Compact Shapes

- As with columns, inelastic buckling of beams is more complicated than elastic buckling, and empirical formulas are often used.
- The following equation is used by AISC: ***AISC Equation F2-2***

$$M_n = C_b\left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$

where the $\mathbf{0.7F_yS_x}$ term is the yield moment adjusted for residual stress, and

$$L_p = 1.76 r_y\sqrt{\frac{E}{F_y}}$$ ***AISC Equation F2-5***

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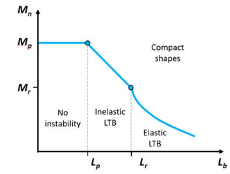
Chapter 5 – Compact Shapes

- ***Summary of Nominal Flexural Strength*** - The nominal bending strength for compact ***I*** and ***C***-shaped sections can be summarized as follows:

AISC Equation F2-5

$$L_p = 1.76 r_y\sqrt{\frac{E}{F_y}}$$



AISC Equation F2-6

$$L_r = 1.95 r_{ts}\frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_xh_0} + \sqrt{\left(\frac{Jc}{S_xh_0}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}}$$

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Chapter 5 – Compact Shapes

- ***Summary of Nominal Flexural Strength*** - The nominal bending strength for compact ***I*** and ***C***-shaped sections can be summarized as follows:

For $L_b \le L_p$ $\quad M_n = M_p$ ***AISC Equation F2-1***

For $L_p < L_b \le L_r$ $\quad M_n = C_b\left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$

AISC Equation F2-2

For $L_b > L_r$ $\quad M_n = F_{cr}S_x \le M_p$ ***AISC Equation F2-3***

$$F_{cr} = \frac{C_b\pi^2E}{\left(L_b r_{ts}\right)^2}\sqrt{1 + 0.078\frac{Jc}{S_xh_0}\left(\frac{L_b}{r_{ts}}\right)^2}$$ ***AISC Equation F2-4***

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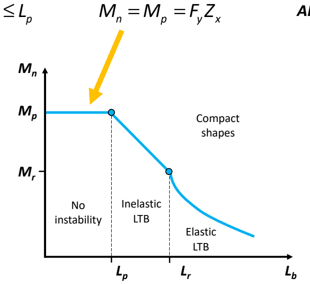
Chapter 5 – Compact Shapes

- ***Summary of Nominal Flexural Strength*** - The nominal bending strength for compact ***I*** and ***C***-shaped sections can be summarized as follows:

For $L_b \le L_p$ $\quad M_n = M_p = F_yZ_x$ ***AISC Equation F2-1***



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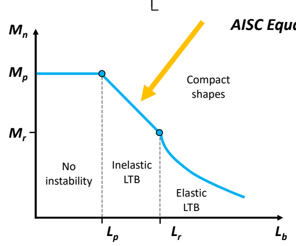
Chapter 5 – Compact Shapes

- ***Summary of Nominal Flexural Strength*** - The nominal bending strength for compact ***I*** and ***C***-shaped sections can be summarized as follows:

For $L_p < L_b \le L_r$ $\quad M_n = C_b\left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$

AISC Equation F2-2



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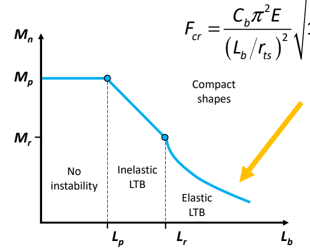
Chapter 5 – Compact Shapes

- ***Summary of Nominal Flexural Strength*** - The nominal bending strength for compact ***I*** and ***C***-shaped sections can be summarized as follows:

For $L_b > L_r$ $\quad M_n = F_{cr}S_x \le M_p$ ***AISC Equation F2-3***

$$F_{cr} = \frac{C_b\pi^2E}{\left(L_b/r_{ts}\right)^2}\sqrt{1 + 0.078\frac{Jc}{S_xh_0}\left(\frac{L_b}{r_{ts}}\right)^2}$$

AISC Equation F2-4



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Chapter 5 – Compact Shapes

- **Example 5.4**: Determine the flexural strength of a **W14 x 82** of ***A572 Grade 50*** steel (F_y = 50 *ksi*; F_u = 65 *ksi*) subject to:
  a) Continuous lateral support.
  b) An unbraced length of 25 *ft* with C_b = 1.0.
  c) An unbraced length of 35 *ft* with C_b = 1.0.
- First, determine whether this shape is ***compact***, ***noncompact***, or ***slender***:

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Chapter 5 – Compact Shapes

- **Example 5.4**:

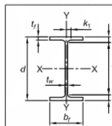

Table 1-1 (continued)
W-Shapes
Dimensions

Shape	Area, A	Depth, d		Web Thickness, t_w		$\frac{t_w}{2}$	Flange Width, b_f		Flange Thickness, t_f		Distance k k_{des}	k_{det}	k_1	T	Workable Gage
	in.^{2}	in.		in.		in.	in.		in.		in.	in.	in.	in.	in.
W14×132	38.8	14.7	14⅝	0.645	⅝	5/16	14.7	14¾	1.03	1	1.63	2 5/16	1 9/16	10	5½
×120	35.3	14.5	14½	0.590	9/16	5/16	14.7	14⅝	0.940	15/16	1.54	2¼	1½	↓	↓
×109	32.0	14.3	14⅜	0.525	½	¼	14.6	14⅝	0.860	⅞	1.46	2 3/16	1½		
×99[f]	29.1	14.2	14⅛	0.485	½	¼	14.6	14⅝	0.780	¾	1.38	2 1/16	1 7/16		
×90[f]	26.5	14.0	14	0.440	7/16	¼	14.5	14½	0.710	11/16	1.31	2	1 7/16	↓	↓
W14×82	24.0	14.3	14¼	0.510	½	¼	10.1	10⅛	0.855	⅞	1.45	1 11/16	1 1/16	10⅞	5½
×74	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		
×68	20.0	14.0	14	0.415	7/16	¼	10.0	10	0.720	¾	1.31	1 9/16	1 1/16		
×61	17.9	13.9	13⅞	0.375	⅜	3/16	10.0	10	0.645	⅝	1.24	1½	1	↓	↓

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Chapter 5 – Compact Shapes

- **Example 5.4**: $\frac{b_f}{2t_f} = 5.92$ $\frac{h}{t_w} = 22.4$ $Z_x = 139\ in^3$

Table 1-1 (continued)
W-Shapes
Properties
W14

Nominal Wt.	Compact Section Criteria $\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	Axis X-X I	S	r	Z	Axis Y-Y I	S	r	Z	r_{ts}	h_o	Torsional Properties J	C_w
lb/ft			in.^{4}	in.^{3}	in.	in.^{3}	in.^{4}	in.^{3}	in.	in.^{3}	in.	in.	in.^{4}	in.^{6}
132	7.15	17.7	1530	209	6.28	234	548	74.5	3.76	113	4.23	13.7	12.3	25500
120	7.80	19.3	1380	190	6.24	212	495	67.5	3.74	102	4.20	13.6	9.37	22700
109	8.49	21.7	1240	173	6.22	192	447	61.2	3.73	92.7	4.17	13.4	7.12	20200
99	9.34	23.5	1110	157	6.17	173	402	55.2	3.71	83.6	4.14	13.4	5.37	18000
90	10.2	25.9	999	143	6.14	157	362	49.9	3.70	75.6	4.10	13.3	4.06	16000
82	5.92	22.4	881	123	6.05	139	148	29.3	2.48	44.8	2.85	13.4	5.07	6710
74	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
68	6.97	27.5	722	103	6.01	115	121	24.2	2.46	36.9	2.80	13.3	3.01	5380
61	7.75	30.4	640	92.1	5.98	102	107	21.5	2.45	32.8	2.78	13.3	2.19	4710

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Chapter 5 – Compact Shapes

- **Example 5.4**: Check for compactness.

$$\frac{b_f}{2t_f} = 5.92 \qquad 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000ksi}{50ksi}} = 8.43$$

$$\frac{b_f}{2t_f} < 0.38\sqrt{\frac{E}{F_y}}$$ The flange is ***compact***

$$\frac{h}{t_w} = 22.4 \qquad 3.76\sqrt{\frac{E}{F_y}} = 3.76\sqrt{\frac{29{,}000ksi}{50ksi}} = 90.55$$

$$\frac{h}{t_w} < 3.76\sqrt{\frac{E}{F_y}}$$ The web is ***compact***

The web is ***compact*** for all shapes in the Manual for $F_y \le 70$ *ksi*

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Chapter 5 – Compact Shapes

- **Example 5.4**: Because the beam is compact and laterally supported, the nominal flexural strength is:

$$M_n = M_p = F_y Z_x = 50ksi\left(123 in^3\right) = 6{,}950 k\,in = 579.17 k\,ft$$

For ***LRFD***:

$$\phi_b M_n = 0.90\left(579.17 kft\right) = 521.25 k\,ft$$

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Chapter 5 – Compact Shapes

- **Example 5.4**: $S_x = 123\ in^3$ $r_y = 2.48\ in$ $r_{ts} = 2.85 in$ $h_o = 13.4 in$ $J = 5.07 in^4$

Table 1-1 (continued)
W-Shapes
Properties
W14

Nominal Wt.	Compact Section Criteria $\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	Axis X-X I	S	r	Z	Axis Y-Y I	S	r	Z	r_{ts}	h_o	Torsional Properties J	C_w
lb/ft			in.^{4}	in.^{3}	in.	in.^{3}	in.^{4}	in.^{3}	in.	in.^{3}	in.	in.	in.^{4}	in.^{6}
132	7.15	17.7	1530	209	6.28	234	548	74.5	3.76	113	4.23	13.7	12.3	25500
120	7.80	19.3	1380	190	6.24	212	495	67.5	3.74	102	4.20	13.6	9.37	22700
109	8.49	21.7	1240	173	6.22	192	447	61.2	3.73	92.7	4.17	13.4	7.12	20200
99	9.34	23.5	1110	157	6.17	173	402	55.2	3.71	83.6	4.14	13.4	5.37	18000
90	10.2	25.9	999	143	6.14	157	362	49.9	3.70	75.6	4.10	13.3	4.06	16000
82	5.92	22.4	881	123	6.05	139	148	29.3	2.48	44.8	2.85	13.4	5.07	6710
74	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
68	6.97	27.5	722	103	6.01	115	121	24.2	2.46	36.9	2.80	13.3	3.01	5380
61	7.75	30.4	640	92.1	5.98	102	107	21.5	2.45	32.8	2.78	13.3	2.19	4710

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Chapter 5 – Compact Shapes

- **Example 5.4**: An unbraced length of 25 *ft* with C_b = 1.0.

$$L_b = 25ft(12in/ft) = 300in$$

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} = 1.76(2.48in)\sqrt{\frac{29{,}000ksi}{50ksi}} = 105.12in$$

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_x h_0} + \sqrt{\left(\frac{Jc}{S_x h_0}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}}$$

$$= 1.95(2.85in)\frac{29{,}000ksi}{0.7(50ksi)}\sqrt{\frac{5.07in^4}{(123in^3)13.4in} + \sqrt{\left(\frac{5.07in^4}{(123in^3)13.4in}\right)^2 + 6.76\left(\frac{0.7(50ksi)}{29{,}000ksi}\right)^2}}$$

$$= 398.00in \qquad \boxed{L_p < L_b \le L_r}$$

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Chapter 5 – Compact Shapes

- **Example 5.4**: An unbraced length of 25 *ft* with C_b = 1.0.
- The following equation is used by AISC: ***AISC Equation F2-2***

$$M_n = C_b\left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$

$$= 1.0\left[6{,}950kin - \left(6{,}950kin - 0.7(50ksi)(123in^3)\right)\left(\frac{300in - 105.12in}{398.0in - 105.12in}\right)\right]$$

$$= 5{,}190.0kin \; = 432.50kft \quad \le M_p = 579.17kft$$

For ***LRFD***:

$$\phi_b M_n = 0.90(432.50kft) = \boxed{389.25kft}$$

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Chapter 5 – Compact Shapes

- **Example 5.4**: An unbraced length of 35 *ft* with C_b = 1.0.

$$L_b = 35ft(12in/ft) = 420in \qquad L_r = 398.00in \qquad \boxed{L_b > L_r}$$

For $L_b > L_r$ $\qquad M_n = F_{cr}S_x \le M_p$ ***AISC Equation F2-3***

$$F_{cr} = \frac{C_b\pi^2E}{\left(L_b/r_{ts}\right)^2}\sqrt{1 + 0.078\frac{Jc}{S_xh_0}\left(\frac{L_b}{r_{ts}}\right)^2} \qquad \textbf{\textit{AISC Equation F2-4}}$$

$$= \frac{1.0\pi^2(29{,}000ksi)}{(420in/2.85in)^2}\sqrt{1 + 0.078\left(\frac{5.07in^4}{(123in^3)13.4in}\right)\left(\frac{420in}{2.85in}\right)^2}$$

$$= 38.84ksi$$

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Chapter 5 – Compact Shapes

- **Example 5.4**: An unbraced length of 35 *ft* with C_b = 1.0.

$$M_n = F_{cr}S_x = 38.84ksi(123in^3) = 4{,}039.86kin = 336.66kft$$

For ***LRFD***:

$$\phi_b M_n = 0.9(336.66kft) = \boxed{302.99kft}$$

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Chapter 5 – Compact Shapes

Let's work on some problems

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Chapter 5 – Beams

Any questions?

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