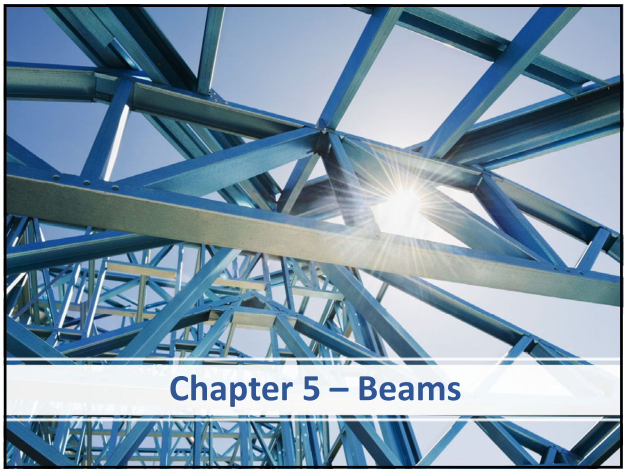

Chapter 5 – Beams

1

Chapter 5 – Introduction

- ***Beams*** are structural members that support transverse loads and are therefore subjected primarily to flexure, or bending.

2

Chapter 5 – Introduction

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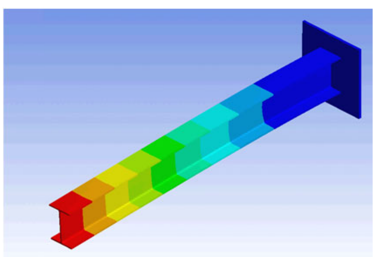
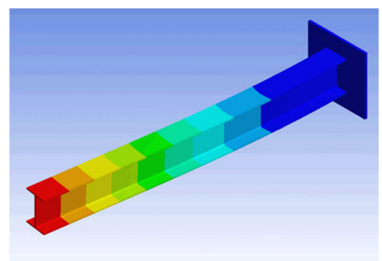


3

Chapter 5 – Introduction

- ***Beams*** are structural members that support transverse loads and are therefore subjected primarily to flexure, or bending.



4

Chapter 5 – Introduction

- If a substantial amount of ***axial load*** is also present, the member is referred to as a ***beam–column*** (beam–columns are considered in **Chapter 6**).

5

Chapter 5 – Introduction

- Although some degree of ***axial load*** will be present in any structural member, in many practical situations this ***effect is negligible,*** and the member can be treated as a beam.
- Beams are usually thought of as being ***oriented horizontally*** and ***subjected to vertical loads***, but that is not necessarily the case.
- A structural member is considered to be a ***beam*** if it is loaded so as ***to cause bending***.

6

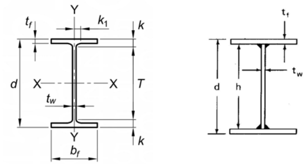
Chapter 5 – Introduction

- Commonly used cross-sectional shapes include the ***W, S***, and ***M*** shapes.
- ***Channel*** shapes are sometimes used, as are beams built up from ***plates***, in the form of ***I*** or box shapes.
- For reasons to be discussed later, ***doubly symmetric shapes such as the standard rolled W, M, and S shapes are the most efficient.***

7

Chapter 5 – Introduction

- Below are two types of beam cross sections: a hot-rolled doubly symmetric ***I*** shape and a welded doubly symmetric built-up ***I*** shape.



- The hot-rolled ***I*** shape is the one most commonly used for beams.
- Welded shapes usually fall into the category classified as ***plate girders***.

8

Chapter 5 – Introduction

- For ***flexure*** (shear will be covered later), the required and available strengths are moments. For ***LRFD***, it can be written as:

$$M_u \le \phi_b M_n$$

where M_u is the required moment strength

ϕ_b is the resistance factor for bending = 0.90

M_n is the nominal moment strength

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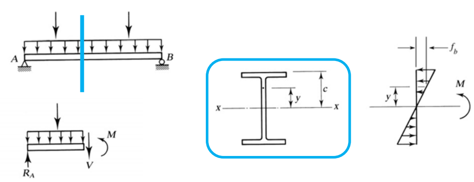
Chapter 5 – Bending Stress

- To be able to determine the nominal moment strength M_n, we must examine the ***behavior of beams*** from very small loads to the point of collapse.
- Consider the beam below, which is oriented so that bending is about the major principal axis (for an **I** shape, it will be the ***x-x*** axis.



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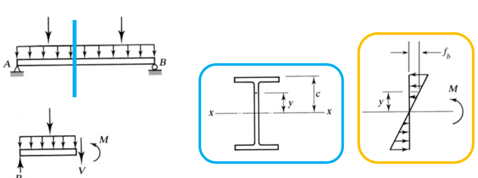
Chapter 5 – Bending Stress

- To be able to determine the nominal moment strength M_n, we must examine the ***behavior of beams*** from very small loads to the point of collapse.
- For a ***linear elastic material*** and ***small deformations***, the distribution of bending stress is shown below, with the stress assumed to be uniform across the width of the beam.



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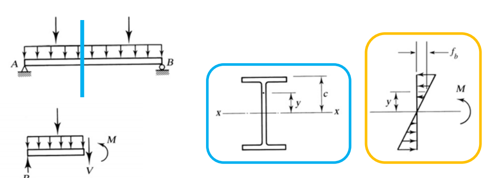
Chapter 5 – Bending Stress

- To be able to determine the nominal moment strength M_n, we must examine the ***behavior of beams*** from very small loads to the point of collapse.
- Initially, we will consider ***<u>bending only</u>***.
- Shear is considered separately in **Section 5.8**.



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Chapter 5 – Bending Stress

- From mechanics of materials, the stress at any point can be found from the flexure formula:

$$f_b = \frac{My}{I_x}$$

where M is the bending moment,

y is the perpendicular distance from the neutral plane to the point of interest, and

I_x is the moment of inertia of the area with respect to the neutral axis.

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Chapter 5 – Bending Stress

- From mechanics of materials, the stress at any point can be found from the flexure formula:

$$f_b = \frac{My}{I_x}$$

- For a ***homogeneous material***, the neutral axis coincides with the centroidal axis.
- We also assume a ***linear distribution of strains*** from top to bottom.
- Which in turn assumes that cross sections that are ***plane before bending remain plane after bending***.

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Chapter 5 – Bending Stress

- From mechanics of materials, the stress at any point can be found from the flexure formula:

$$f_b = \frac{My}{I_x}$$

- In addition, the beam cross-section must have a vertical axis of symmetry, and the ***loads must be in the longitudinal plane containing this axis.***
- Beams that ***do not satisfy these criteria*** are considered in **Section 5.15**.

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Chapter 5 – Bending Stress

- The maximum stress will occur at the ***extreme fiber***, where y is maximum.
- The ***maximum compressive stress*** is in the top fiber.
- The ***maximum tensile stress*** is in the bottom fiber.
- If the neutral axis is an axis of symmetry, these ***two stresses will be equal in magnitude***.

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Chapter 5 – Bending Stress

- The maximum stress equation becomes:

$$f_{\max} = \frac{Mc}{I_x} = \frac{M}{I_x/c} = \frac{M}{S}$$

where c is the perpendicular distance from the neutral axis to the extreme fiber, and

S is the elastic section modulus of the cross-section.

- For an ***unsymmetrical cross-section***, S_x will have two values: one for the top extreme fiber and one for the bottom.
- Values of S_x for standard rolled shapes are tabulated in the dimensions and properties tables in the **Manual**.

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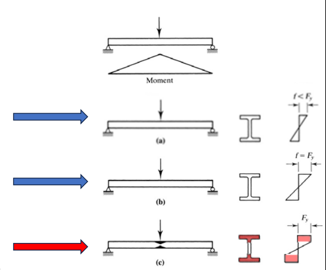
Chapter 5 – Bending Stress

- Consider a simply supported beam with a concentrated load at midspan at successive stages of loading.
- Stress less than yielding F_y
- Stress at yielding F_y
- Yielding begins, the stress on the cross-section will no longer be ***linear***



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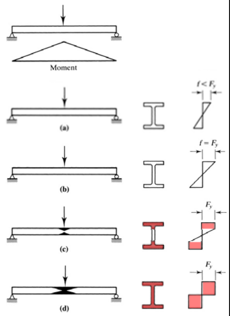
Chapter 5 – Bending Stress

- Consider a simply supported beam with a concentrated load at midspan at successive stages of loading.
- Stress less than yielding F_y
- Stress at yielding F_y
- Yielding will progress from the ***extreme fiber toward the neutral axis***.



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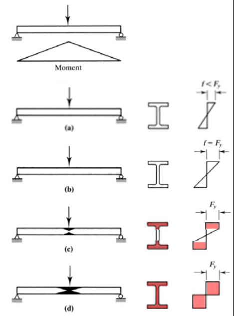
Chapter 5 – Bending Stress

- Consider a simply supported beam with a concentrated load at midspan at successive stages of loading.
- Stress less than yielding F_y
- Stress at yielding F_y
- The change from ***stage b*** to ***stage d*** is an addition of 10 to 20% of the yield moment, M_y, for W shapes.



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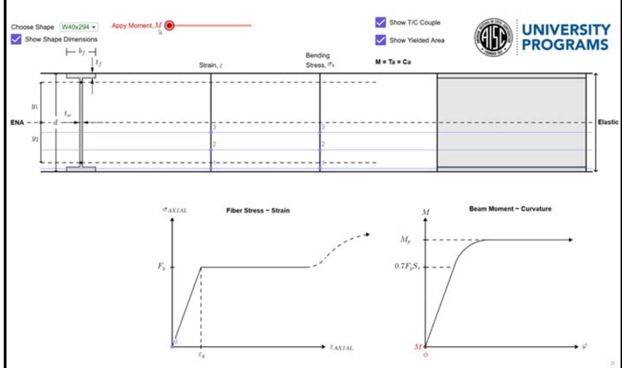
Chapter 5 – Bending Stress



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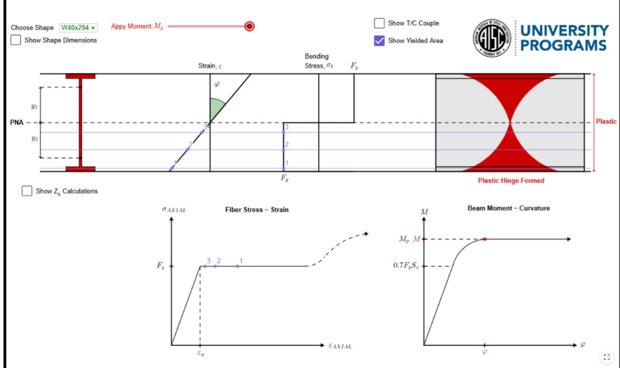
Chapter 5 – Bending Stress



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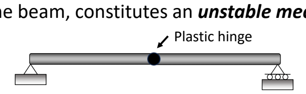
Chapter 5 – Bending Stress

- When ***stage d*** has been reached, any further increase in the load will cause ***collapse***.
- A ***plastic hinge*** is said to have formed at the center of the beam, and this ***hinge***, along with the actual hinges at the ends of the beam, constitutes an ***unstable mechanism***.



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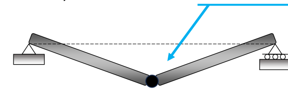
Chapter 5 – Bending Stress

- When ***stage d*** has been reached, any further increase in the load will cause ***collapse***.
- A ***plastic hinge*** is said to have formed at the center of the beam, and this ***hinge***, along with the actual hinges at the ends of the beam, constitutes an ***unstable mechanism***.



- Structural analysis based on a consideration of collapse mechanisms is called ***plastic analysis***.

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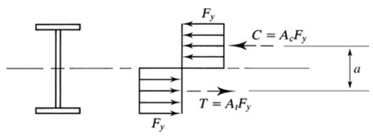
Chapter 5 – Bending Stress

- The ***plastic moment capacity***, which is the moment required to form the plastic hinge, can easily be computed.



- Consider the stress resultants shown above, where A_c is the cross-sectional area subjected to compression, and A_t is the area in tension.

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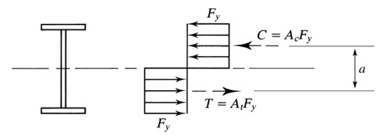
Chapter 5 – Bending Stress

- The ***plastic moment capacity***, which is the moment required to form the plastic hinge, can easily be computed.



- These are the areas above and below the ***plastic neutral axis***, which is ***not necessarily*** the same as the ***elastic neutral axis***.

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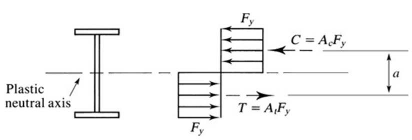
Chapter 5 – Bending Stress

- The ***plastic moment capacity***, which is the moment required to form the plastic hinge, can easily be computed.



- From the equilibrium of forces, ***C*** = ***T***, then:

$$A_c F_y = A_t F_y \quad \Rightarrow \quad A_c = A_t$$

- The ***plastic neutral axis*** divides the cross-section into ***two equal areas***.

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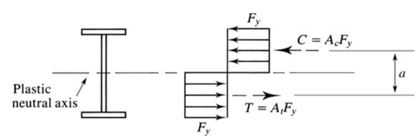
Chapter 5 – Bending Stress

- The ***plastic moment capacity***, which is the moment required to form the plastic hinge, can easily be computed.



- From the equilibrium of forces, ***C*** = ***T***, then:

$$A_c F_y = A_t F_y \quad \Rightarrow \quad A_c = A_t$$

- For shapes that are ***symmetrical*** about the axis of bending, the ***elastic and plastic neutral axes are the same***.

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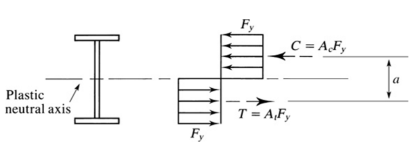
Chapter 5 – Bending Stress

- The ***plastic moment capacity***, which is the moment required to form the plastic hinge, can easily be computed.



- The plastic moment, M_p, is the resisting couple formed by the two equal and opposite forces:

$$M_p = F_y(A_c)a = F_y(A_t)a \quad = F_y\left(\frac{A}{2}\right)a \quad = F_y Z$$

where ***A*** is the total cross-sectional area, and
Z is the ***plastic section modulus*** = **½*Aa***

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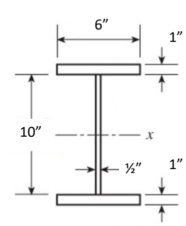
Chapter 5 – Bending Stress

- **Example 5.1**: For the built-up shape below, determine:
(a) the elastic section modulus ***S***, the yield moment M_y, and
(b) the plastic section modulus ***Z*** and the plastic moment M_p.

- Bending is about the ***x***-axis, and the steel is **A572 Grade 50** with F_y = 50 *ksi*; F_u = 65 *ksi*.

- Because of symmetry, the elastic neutral axis (the ***x-axis***) is located at the mid-depth of the cross-section (the location of the centroid).



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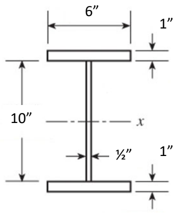
Chapter 5 – Bending Stress

- **Example 5.1**: For the built-up shape below, determine:
  (a) the elastic section modulus ***S***, the yield moment M_y, and
  (b) the plastic section modulus ***Z*** and the plastic moment M_p.
- Bending is about the ***x***-axis, and the steel is **A572 Grade 50** with F_y = 50 *ksi*; F_u = 65 *ksi*.
- The moment of inertia of the cross-section can be found by using the ***parallel axis theorem.***



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Chapter 5 – Bending Stress

- **Example 5.1**: For the built-up shape below, determine:
  (a) the elastic section modulus ***S***, the yield moment M_y, and
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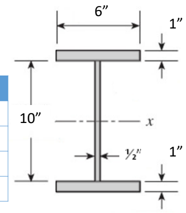
- Bending is about the ***x***-axis, and the steel is **A572 Grade 50** with F_y = 50 *ksi*; F_u = 65 *ksi*.

$$I_x = \frac{bh^3}{12}$$

Component	I_x (in⁴)	A (in²)	d (in)	$I = I_x + Ad^2$
Flange				
Flange				
Web				
Σ				



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Chapter 5 – Bending Stress

- **Example 5.1**: For the built-up shape below, determine:
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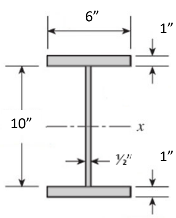
- Bending is about the ***x***-axis, and the steel is **A572 Grade 50** with F_y = 50 *ksi*; F_u = 65 *ksi*.

$$I_x = \frac{bh^3}{12}$$

Component	I_x (in⁴)	A (in²)	d (in)	$I = I_x + Ad^2$
Flange	0.500	6.0	5.5	182.000
Flange	0.500	6.0	5.5	182.000
Web	41.667	5.0	0.0	41.667
Σ		17.0		405.667



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Chapter 5 – Bending Stress

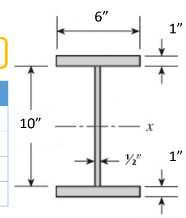
- **Example 5.1**: For the built-up shape below, determine:
  (a) the elastic section modulus ***S***, the yield moment M_y, and
  (b) the plastic section modulus ***Z*** and the plastic moment M_p.

$$S = \frac{I}{c} = \frac{405.667\,in^4}{1\,in + 10\,in/2} = \boxed{67.611\,in^3}$$

$$M_y = F_y S = 50\,ksi\left(67.611\,in^3\right)$$

$$= 3{,}380.6\,k\,in = \boxed{281.7\,kft}$$

Component	I_x (in⁴)	A (in²)	d (in)	$I = I_x + Ad^2$
Flange	0.500	6.0	5.5	182.000
Flange	0.500	6.0	5.5	182.000
Web	41.667	5.0	0.0	41.667
Σ		17.0		405.667



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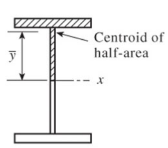
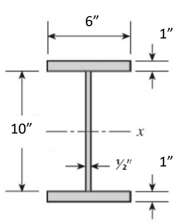
Chapter 5 – Bending Stress

- **Example 5.1**: For the built-up shape below, determine:
  (a) the elastic section modulus ***S***, the yield moment M_y, and
  (b) the plastic section modulus ***Z*** and the plastic moment M_p.
- Because this shape is symmetrical about the ***x-axis***, this axis divides the cross-section into equal areas and is therefore the plastic neutral axis.





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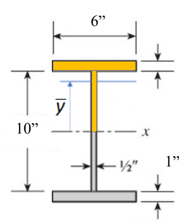
Chapter 5 – Bending Stress

- **Example 5.1**: For the built-up shape below, determine:
  (a) the elastic section modulus ***S***, the yield moment M_y, and
  (b) the plastic section modulus ***Z*** and the plastic moment M_p.
- The centroid of the top half-area can be found by the principle of moments.

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{39.25\,in^3}{8.5\,in^2} = 4.618\,in$$

$$a = 2\bar{y} = 2\left(4.618\,in\right) = 9.235\,in$$

Component	A (in²)	y (in)	Ay (in³)
Flange			
Web			
Σ			



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Chapter 5 – Bending Stress

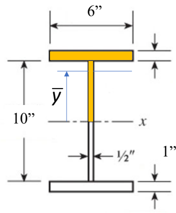
- **Example 5.1**: For the built-up shape below, determine:
  (a) the elastic section modulus ***S***, the yield moment M_y, and
  (b) the plastic section modulus ***Z*** and the plastic moment M_p.

- The centroid of the top half-area can be found by the principle of moments.

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{39.25 in^3}{8.5 in^2} = 4.618 in$$

$$a = 2\bar{y} = 2(4.618 in) = 9.235 in$$

Component	A (in²)	y (in)	Ay (in³)
Flange	6.0	5.5	33.0
Web	2.5	2.5	6.25
∑	8.5		39.25



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Chapter 5 – Bending Stress

- **Example 5.1**: For the built-up shape below, determine:
  (a) the elastic section modulus ***S***, the yield moment M_y, and
  (b) the plastic section modulus ***Z*** and the plastic moment M_p.

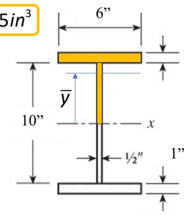
- The centroid of the top half-area can be found by the principle of moments.

$$Z = a\left(\frac{A}{2}\right) = 9.235 in\left(\frac{17 in^2}{2}\right) = 78.5 in^3$$

$$M_p = F_y Z = 50 ksi(78.5 in^3)$$

$$= 3{,}925 k in = 327.08 kft$$

Component	A (in²)	y (in)	Ay (in³)
Flange	6.0	5.5	33.0
Web	2.5	2.5	6.25
∑	8.5		39.25



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Chapter 5 – Bending Stress

- **Example 5.2**: Compute the plastic moment, M_p, for a **W12 x 50** of **A922** with F_y = 50 *ksi*; F_u = 65 *ksi*.

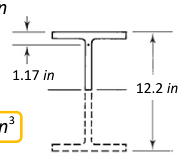
- From the dimensions and properties tables in Part 1 of the Manual – **Table 1-1** (**1**-26): ***A*** = 14.6 *in²* ***d*** = 12.2 *in*

- The centroid for the half-area can be found for a **WT6 x 25**. From the dimensions and properties tables in Part 1 of the Manual – **Table 1-8** (**1**-68), the distance from the outside face of the flange to the centroid is: $\bar{y} = 1.17 in$

$$a = d - 2\bar{y} = 12.2 in - 2(1.17 in)$$

$$= 9.86 in$$

$$Z = a\left(\frac{A}{2}\right) = 9.86 in\left(\frac{14.6 in^2}{2}\right) = 71.98 in^3$$



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Chapter 5 – Bending Stress

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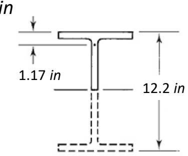
$$M_p = F_y Z = 50 ksi(71.98 in^3)$$

$$= 3{,}599.0 k in = 299.92 kft$$



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Chapter 5 – Bending Stress

Let’s work on some problems

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Chapter 5 – Beams

Any questions?

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