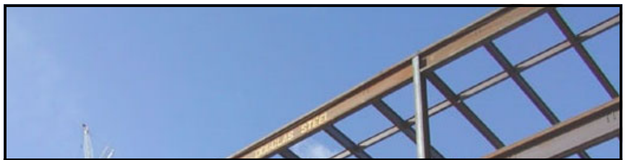


Chapter 4
Compression Members

1

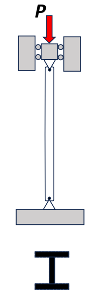
Chapter 4 – Flexural-Torsional

- When an axially loaded compression member becomes unstable overall (that is, not locally unstable), it can buckle in one of three ways:

1. ***Flexural buckling***. It is a deflection caused by bending, or flexure, about the axis corresponding to the largest slenderness ratio.

   This is usually the minor principal axis—the one with the smallest radius of gyration.

   Compression members with any type of cross-sectional configuration can fail in this way.



2

Chapter 4 – Flexural-Torsional

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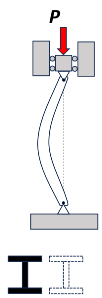

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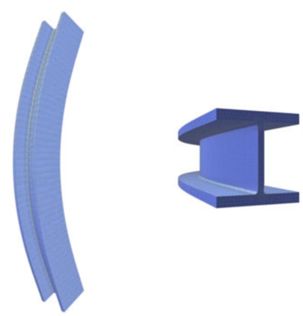
Chapter 4 – Flexural-Torsional



4

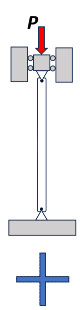
Chapter 4 – Flexural-Torsional

- When an axially loaded compression member becomes unstable overall (that is, not locally unstable), it can buckle in one of three ways:

2. ***Torsional buckling.*** This type of failure occurs when the member twists about its longitudinal axis.

   It can occur only with doubly symmetrical cross sections with very slender cross-sectional elements.

   Standard hot-rolled shapes are not susceptible to torsional buckling.



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Chapter 4 – Flexural-Torsional

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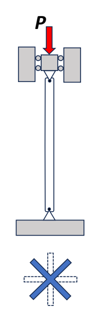

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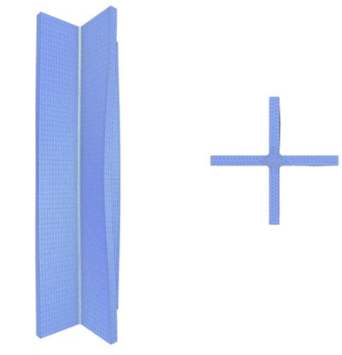
Chapter 4 – Flexural-Torsional



7

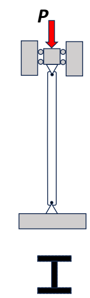
Chapter 4 – Flexural-Torsional

- When an axially loaded compression member becomes unstable overall (that is, not locally unstable), it can buckle in one of three ways:

  3. ***Flexural-torsional buckling.*** A combination of flexural buckling and torsional buckling causes this type of failure.

     The member bends and twists simultaneously.

     This type of failure can occur only with unsymmetrical cross sections.



8

Chapter 4 – Flexural-Torsional

- When an axially loaded compression member becomes unstable overall (that is, not locally unstable), it can buckle in one of three ways:

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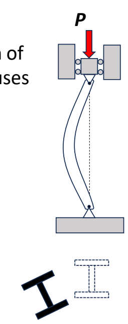

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Chapter 4 – Flexural-Torsional

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Chapter 4 – Flexural-Torsional

- The ***AISC Specification*** requires an analysis of torsional or flexural-torsional buckling when appropriate.
- The analysis is based on determining a value of F_e that can be used with ***AISC Equations E3-2 and E3-3*** to determine the nominal stress, F_n.
- The stress F_e can be defined as the elastic buckling stress corresponding to the controlling mode of failure, whether ***flexural, torsional, or flexural-torsional***.

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Chapter 4 – Flexural-Torsional

- The equations for F_e given in **AISC E4** are based on the well-established theory of elastic stability.
- For doubly symmetrical shapes (torsional buckling)

$$F_e = \left[\frac{\pi^2 E C_w}{(L_{cz})^2} + GJ\right]\frac{1}{I_x + I_y}$$ ***AISC Equation E4-2***

- For singly symmetrical shapes (flexural-torsional buckling)

$$F_e = \frac{F_{ey} + F_{ez}}{2H}\left[1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right]$$ ***AISC Equation E4-3***

where ***y*** is the axis of symmetry.

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Chapter 4 – Flexural-Torsional

- The equations for F_e given in ***AISC E4*** are based on the well-established theory of elastic stability.
- For shapes with no axis of symmetry (flexural-torsional buckling),

$$(F_e - F_{ex})(F_e - F_{ey})(F_e - F_{ez}) - F_e^2 (F_e - F_{ey})\left(\frac{x_0}{\bar{r}_0}\right)^2 - F_e^2 (F_e - F_{ex})\left(\frac{y_0}{\bar{r}_0}\right)^2$$

AISC Equation E4-4

This last equation is a cubic; F_e is the smallest root.

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Chapter 4 – Flexural-Torsional

- In the above equations, the ***z-axis*** is the longitudinal axis.
- The previously undefined terms in these three equations are defined as:

C_w	is the warping constant (in^6)
$L_{cz} = K_z L$	is the effective length for torsional buckling (in)
G	is the shear modulus = 11,200 *ksi*
J	is the torsional constant (in^4)

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Chapter 4 – Flexural-Torsional

- In the above equations, the ***z-axis*** is the longitudinal axis.
- The previously undefined terms in these three equations are defined as:

$$F_{ex} = \frac{\pi^2 E}{(L_{cx}/r_x)^2}$$ ***AISC Equation E4-5***

$$F_{ey} = \frac{\pi^2 E}{(L_{cy}/r_y)^2}$$ ***AISC Equation E4-6***

where ***y*** is the axis of symmetry for singly symmetrical shapes

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Chapter 4 – Flexural-Torsional

- In the above equations, the ***z-axis*** is the longitudinal axis.
- The previously undefined terms in these three equations are defined as:

$$F_{ez} = \left[\frac{\pi^2 E C_w}{(L_{cz})^2} + GJ\right]\frac{1}{A_g \bar{r}_0^2}$$ ***AISC Equation E4-7***

$$H = 1 - \frac{x_0^2 + y_0^2}{\bar{r}_0^2}$$ ***AISC Equation E4-8***

where z is the longitudinal axis and x_0, y_0 are the coordinates of the shear center of the cross section with respect to the centroid (in inches).

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Chapter 4 – Flexural-Torsional

- In the above equations, the ***z-axis*** is the longitudinal axis.
- The ***shear center*** is the point on the cross-section through which a transverse load on a beam must pass if the member is to ***bend without twisting***.

$$\bar{r}_0^2 = x_0^2 + y_0^2 + \frac{I_x + I_y}{A_g}$$ ***AISC Equation E4-9***

where $\bar{r}_0$ is the polar radius of gyration about the shear center, (*in*.)

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Chapter 4 – Flexural-Torsional

- Values of the constants used in the equations for F_e can be found in the dimensions and properties tables in **Part 1** of the Manual.

Shape	Constants
W, M, S, HP, WT, MT, ST	J, C_w, (AISC shapes database gives values of $\bar{r}_0$ and H for WT, MT, and ST shapes)
C	J, C_w, $\bar{r}_0$, and H
MC, Angles	J, C_w, $\bar{r}_0$, H (AISC shapes database gives values of $\bar{r}_0$ and H for WC and angle shapes)
Double Angles	$\bar{r}_0$, H (J and C_w are double the values for single angles)

- The Manual does not give the constants $\bar{r}_0$ and H for tees, although they are given in the **AISC shapes database** (AISC 2022c).

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Chapter 4 – Flexural-Torsional

- Values of the constants used in the equations for F_e can be found in the dimensions and properties tables in **Part 1** of the Manual.

Shape	Constants
W, M, S, HP, WT, MT, ST	***J, C_w***, (AISC shapes database gives values of $\bar{r}_0$ and ***H*** for WT, MT, and ST shapes)
C	***J, C_w***, $\bar{r}_0$, and ***H***
MC, Angles	***J, C_w***, $\bar{r}_0$, ***H*** (AISC shapes database gives values of $\bar{r}_0$ and ***H*** for WC and angle shapes)
Double Angles	$\bar{r}_0$, ***H*** (***J*** and ***C_w*** are double the values for single angles)

- However, they are easily computed if x_0 and y_0 are known.

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Chapter 4 – Flexural-Torsional

- Values of the constants used in the equations for F_e can be found in the dimensions and properties tables in **Part 1** of the Manual.

Shape	Constants
W, M, S, HP, WT, MT, ST	***J, C_w***, (AISC shapes database gives values of $\bar{r}_0$ and ***H*** for WT, MT, and ST shapes)
C	***J, C_w***, $\bar{r}_0$, and ***H***
MC, Angles	***J, C_w***, $\bar{r}_0$, ***H*** (AISC shapes database gives values of $\bar{r}_0$ and ***H*** for WC and angle shapes)
Double Angles	$\bar{r}_0$, ***H*** (***J*** and ***C_w*** are double the values for single angles)

- Since x_0 and y_0 are the coordinates of the shear center with respect to the centroid of the cross section, the location of the shear center must be known.

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Chapter 4 – Flexural-Torsional

- Values of the constants used in the equations for F_e can be found in the dimensions and properties tables in **Part 1** of the Manual.

Shape	Constants
W, M, S, HP, WT, MT, ST	***J, C_w***, (AISC shapes database gives values of $\bar{r}_0$ and ***H*** for WT, MT, and ST shapes)
C	***J, C_w***, $\bar{r}_0$, and ***H***
MC, Angles	***J, C_w***, $\bar{r}_0$, ***H*** (AISC shapes database gives values of $\bar{r}_0$ and ***H*** for WC and angle shapes)
Double Angles	$\bar{r}_0$, ***H*** (***J*** and ***C_w*** are double the values for single angles)

- For a tee shape, it is located at the intersection of the centerlines of the flange and the stem.

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Chapter 4 – Flexural-Torsional

- The need for a torsional buckling analysis of a doubly symmetric shape will be rare.
- Similarly, the only shape with no axis of symmetry that is likely to be used as a compression member is the ***single angle***.
- The Specification provides for this shape, but we do not cover it here (**Part 4** of the **Manual** contains tables for the strength of single-angle compression members).
- For these reasons, we ***limit*** further consideration to ***flexural-torsional buckling*** of shapes with one axis of symmetry.

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Chapter 4 – Flexural-Torsional

- For singly symmetrical shapes, the flexural-torsional buckling stress, F_e, is found from ***AISC Equation E4-3***.

$$F_e = \frac{F_{ey} + F_{ez}}{2H}\left[1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{\left(F_{ey} + F_{ez}\right)^2}}\right]$$

- In this equation, ***y*** is defined as the axis of symmetry (regardless of the orientation of the member), and ***flexural-torsional buckling*** will take place only about this axis (***flexural buckling*** about this axis will not occur).
- The ***x-axis*** (the axis of no symmetry) is subject only to ***flexural buckling***.

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Chapter 4 – Flexural-Torsional

- Therefore, for singly symmetrical shapes, there are two possibilities for the strength:
  - ***flexural-torsional buckling*** about the ***y-axis*** (the axis of symmetry) or
  - ***flexural buckling*** about the ***x-axis***.
- To determine which one controls, compute the strength corresponding to each axis and use the ***smaller*** value.

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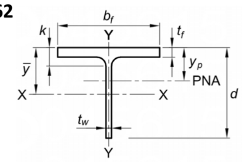
Chapter 4 – Flexural-Torsional

- **Example 4.16**: For a **WT15 x 62** of A992 steel, F_y = 50 *ksi*; F_u = 65 *ksi,* compute the compressive strength. The effective length with respect to the *x-axis* is 25 *ft* 6 *in*, the effective length with respect to the *y-axis* is 20 *ft*, and the effective length with respect to the *z-axis* is 20 *ft.*

- First, compute the ***flexural buckling*** strength for the *x-axis* (the axis of no symmetry).

From Table 1-8 (**1**-58) for a **WT15 x 62**

$$r_x = 4.66 in \qquad r_y = 2.23 in$$



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Chapter 4 – Flexural-Torsional

- **Example 4.16**:

$$\frac{L_{cx}}{r_x} = \frac{K_x L}{r_x} = \frac{1.0(25.5 ft)(12 in/ft)}{4.66 in} = 65.67$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000 ksi}{50 ksi}} = 113.43 \qquad \frac{L_{cx}}{r_x} < 4.71\sqrt{\frac{E}{F_y}}$$

Use ***AISC Equation E3-2***

$$F_e = \frac{\pi^2 E}{(L_c/r)^2} = \frac{\pi^2(29,000 ksi)}{(65.67)^2} = 66.38 ksi$$

$$F_n = \left(0.658^{50ksi/66.38ksi}\right)(50 ksi) = 36.48 ksi$$

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Chapter 4 – Flexural-Torsional

- **Example 4.16**: The nominal buckling strength is:

$$P_n = F_n A_g = 36.48 ksi(18.2 in^2) = \boxed{663.92 k}$$

The ***flexural-torsional buckling*** strength about the *y-axis*:

$$\frac{L_{cy}}{r_y} = \frac{K_y L}{r_y} = \frac{1.0(20 ft)(12 in/ft)}{2.23 in} = 107.62$$

$$F_{ey} = \frac{\pi^2 E}{(L_{cy}/r_y)^2} = \frac{\pi^2(29,000 ksi)}{(107.62)^2} = 24.71 ksi$$

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Chapter 4 – Flexural-Torsional

- **Example 4.16**: User Note in **AISC E4(c)** suggests omitting C_w and setting x_0 to zero for tees and double angles.

$$F_{ez} = \left[\frac{\pi^2 E C_w}{(L_{cz})^2} + GJ\right]\frac{1}{A_g \bar{r}_0^2} = \left[\frac{GJ}{A_g \bar{r}_0^2}\right]$$

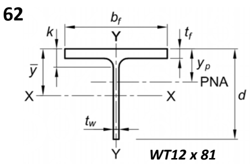
This is because the shear center of a tee is located at the intersection of the centerlines of the flange and the stem.

From Table 1-8 (**1**-58) for a **WT15 x 62**

$$\bar{y} = 3.90 in \qquad t_f = 0.93 in$$

$$I_x = 396 in^4 \qquad I_y = 90.4 in^4$$

$$A = 18.2 in^2 \qquad J = 3.98 in^4$$



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Chapter 4 – Flexural-Torsional

- **Example 4.16**: User Note in **AISC E4(c)** suggests omitting C_w and setting x_0 to zero for tees and double angles.

$$x_0 = 0 \qquad y_0 = \bar{y} - \frac{t_f}{2} = 3.90 in - \frac{0.93 in}{2} = 3.44 in$$

$$\bar{r}_0^2 = x_0^2 + y_0^2 + \frac{I_x + I_y}{A_g} = (3.44 in)^2 + \frac{396 in^4 + 90.4 in^4}{18.1 in^2} = 38.52 in^2$$

$$H = 1 - \frac{x_0^2 + y_0^2}{\bar{r}_0^2} = 1 - \frac{(3.44 in)^2}{38.52 in^2} = 0.694$$

$$F_{ez} = \left[\frac{GJ}{A_g \bar{r}_0^2}\right] = \left[\frac{11,200 ksi(3.98 in^4)}{18.1 in^2(38.52 in^2)}\right] = 63.58 ksi$$

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Chapter 4 – Flexural-Torsional

- **Example 4.16**: For singly symmetrical shapes:

$$F_e = \frac{F_{ey} + F_{ez}}{2H}\left[1 - \sqrt{1 - \frac{4 F_{ey} F_{ez} H}{(F_{ey} + F_{ez})^2}}\right] \qquad \textbf{\textit{AISC Equation E4-3}}$$

$$F_{ey} + F_{ez} = 24.71 ksi + 63.58 ksi = 88.29 ksi$$

$$F_e = \frac{88.29 ksi}{2(0.694)}\left[1 - \sqrt{1 - \frac{4(24.71 ksi)(63.58 ksi)(0.694)}{(88.29 ksi)^2}}\right]$$

$$= 21.39 ksi$$

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Chapter 4 – Flexural-Torsional

- **Example 4.16**: For singly symmetrical shapes:

$$\frac{F_y}{F_e} = \frac{50\,ksi}{21.39\,ksi} = 2.337 \qquad \frac{F_y}{F_e} > 2.25 \quad \textbf{\textit{AISC Equation E3-3}}$$

$$F_n = 0.877(21.39\,ksi) = 18.76\,ksi$$

$$P_n = F_n A_g = 18.76\,ksi(18.1\,in^2) = \boxed{341.44\,k} \quad \text{Controls}$$

Recall, the nominal buckling strength is:

$$P_n = F_n A_g = 36.48\,ksi(18.2\,in^2) = \boxed{663.92\,k}$$

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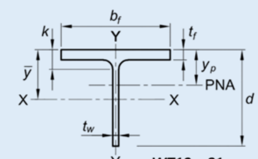
Chapter 4 – Flexural-Torsional

- **Example 4.16**: Therefore, ***flexural-torsional*** buckling strength controls, and the nominal strength is **341.44 *k***.
- ***LRFD*** design strength is: $\phi_c P_n = 0.90(341.44\,k) = \boxed{307.29\,k}$



WT12 x 81

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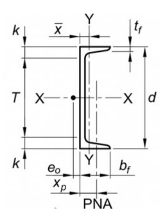
Chapter 4 – Flexural-Torsional

- **Example 4.17**: Compute the compressive strength of a **C12 x 30** of **A36** steel, F_y = 36 *ksi*; F_u = 58 *ksi*. The effective length with respect to the *x, y,* and *z* axes is 12 *ft*.
- Check the ***flexural buckling*** strength for the *y-axis* (the axis of no symmetry for a channel).

From Table 1-5 (**1**-38) for a **C12 x 30**

$$r_x = 4.29\,in \qquad r_y = 0.762\,in$$
$$A = 8.81\,in^2$$



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Chapter 4 – Flexural-Torsional

- **Example 4.17**:

$$\frac{L_{cy}}{r_y} = \frac{K_y L}{r_y} = \frac{1.0(12\,ft)(12\,in/ft)}{0.762\,in} = 188.98 < 200$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000\,ksi}{36\,ksi}} = 133.68 \qquad \boxed{\frac{L_{cy}}{r_y} > 4.71\sqrt{\frac{E}{F_y}}}$$

Use ***AISC Equation E3-3***

$$F_e = \frac{\pi^2 E}{(L_c/r)^2} = \frac{\pi^2(29{,}000\,ksi)}{(188.98)^2} = 8.01\,ksi$$

$$F_n = 0.877(8.01\,ksi) = 7.03\,ksi$$

$$P_n = F_n A_g = 7.03\,ksi(8.81\,in^2) = \boxed{61.92\,k}$$

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Chapter 4 – Flexural-Torsional

- **Example 4.17**: The ***flexural-torsional buckling*** strength about the axis of symmetry or the *x-axis*.

But in ***AISC Equation E4-3***, the axis of symmetry is called the ***y-axis***, so the ***x*** and ***y*** subscripts need to be reversed.

$$\frac{L_{cx}}{r_x} = \frac{K_x L}{r_x} = \frac{1.0(12\,ft)(12\,in/ft)}{4.29\,in} = 33.57 < 200$$

$$F_{ey} = \frac{\pi^2 E}{(L_{cy}/r_y)^2} = \frac{\pi^2(29{,}000\,ksi)}{(33.57)^2} = 254.03\,ksi$$

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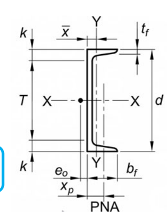
Chapter 4 – Flexural-Torsional

- **Example 4.17**: The User Note in **AISC E4(b)** states that the *x* and *y* subscripts need to be reversed.

$$F_{ez} = \left[\frac{\pi^2 E C_w}{(L_{cz})^2} + GJ\right]\frac{1}{A_g \bar{r}_0^2}$$



From Table 1-5 (**1**-38) for a **C15 x 50**

$$C_w = 151\,in^6 \qquad \bar{r}_0 = 4.54\,in \qquad H = 0.919$$

$$F_{ez} = \left[\frac{\pi^2(29{,}000\,ksi)151\,in^6}{(144\,in)^2} + 11{,}200\,ksi(0.861\,in^4)\right]\frac{1}{8.81\,in^2(4.54\,in)^2}$$

$$= 64.58\,ksi$$

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Chapter 4 – Flexural-Torsional

- **Example 4.17**:

$$F_e = \frac{F_{ey}+F_{ez}}{2H}\left[1-\sqrt{1-\frac{4F_{ey}F_{ez}H}{\left(F_{ey}+F_{ez}\right)^2}}\right] \qquad \textit{AISC Equation E4-3}$$

$$F_{ey}+F_{ez} = 254.03ksi + 64.58ksi = 318.61ksi$$

$$F_e = 1-\frac{318.61ksi}{2(0.919)}\left[1-\sqrt{1-\frac{4(254.03ksi)(64.58ksi)(0.919)}{(318.61ksi)^2}}\right]$$

$$= 62.91ksi$$

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Chapter 4 – Flexural-Torsional

- **Example 4.17**:

$$\frac{F_y}{F_e} = \frac{36ksi}{62.91ksi} = 0.572 \qquad \frac{F_y}{F_e} \le 2.25 \qquad \textit{AISC Equation E3-2}$$

$$F_n = \left(0.658^{F_y/F_e}\right)F_y$$

$$F_n = \left(0.658^{36ksi/62.91ksi}\right)(36ksi) = 28.33ksi$$

$$P_n = F_n A_g = 28.33ksi\left(8.81in^2\right) = \boxed{249.60k}$$

Recall, the nominal buckling strength is:

$$P_n = F_n A_g = 7.03ksi\left(8.81in^2\right) = \boxed{61.92k}$$ **Controls**

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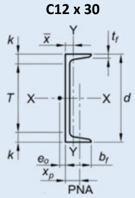
Chapter 4 – Flexural-Torsional

- **Example 4.17**: Therefore, ***flexural*** buckling strength controls, and the nominal strength is **61.92 *k***.
- ***LRFD*** design strength is: $\phi_c P_n = 0.90(61.92k) = \boxed{55.73k}$

C12 x 30



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Chapter 4 – Flexural-Torsional

Let's work on some problems

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Chapter 4 – Flexural-Torsional

Any questions?

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