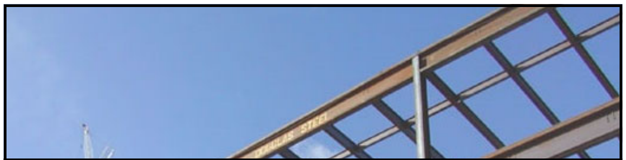


Chapter 4
Compression Members

1

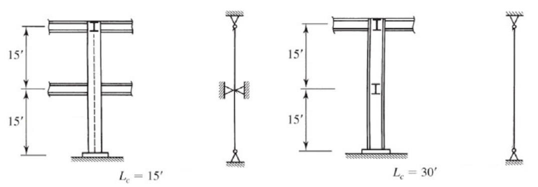
Chapter 4 – More on Effective Length

- We introduced the concept of ***effective length*** in Section 4.2, "Column Theory."
- All ***compression members*** are treated as pin-ended regardless of the actual end conditions, but with an effective length L_c that may differ from the actual length.
- With this modification, the load capacity of compression members is a function of only the ***slenderness ratio*** and ***modulus of elasticity***.
- For a given material, the ***load capacity*** is a function of the ***slenderness ratio*** only.

2

Chapter 4 – More on Effective Length

- If a ***compression member*** is supported differently with respect to each of its ***principal axes***, the ***effective length*** will be different for the two directions.
- Consider a ***W-shape*** is used as a column and is braced by horizontal members in two perpendicular directions at the top.

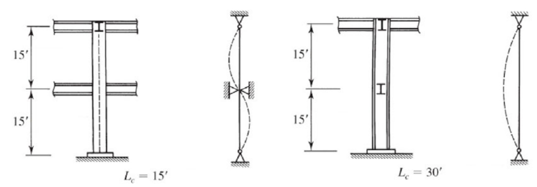


(a) Minor Axis Buckling (b) Major Axis Buckling

3

Chapter 4 – More on Effective Length

- If a ***compression member*** is supported differently with respect to each of its ***principal axes***, the ***effective length*** will be different for the two directions.
- These members ***prevent translation*** of the column in all directions, but the connections, permit ***small rotations*** to take place.

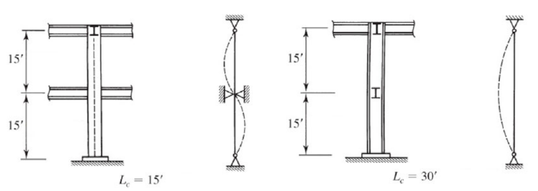


(a) Minor Axis Buckling (b) Major Axis Buckling

4

Chapter 4 – More on Effective Length

- Under these conditions, the member can be treated as ***pin-connected*** at the top and bottom.
- A ***rigid, or fixed, condition is very difficult*** to achieve, and ordinary connections will usually closely approximate a ***hinge or pin connection***.

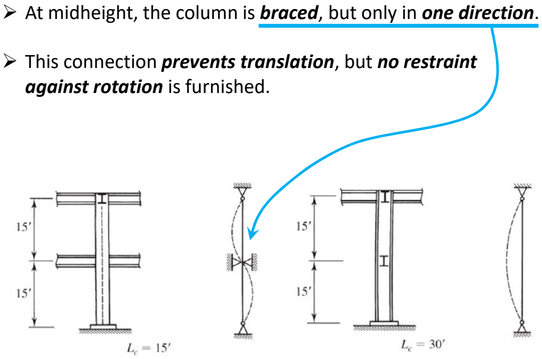


(a) Minor Axis Buckling (b) Major Axis Buckling

5

Chapter 4 – More on Effective Length

- At midheight, the column is ***braced***, but only in ***one direction***.
- This connection ***prevents translation***, but ***no restraint against rotation*** is furnished.

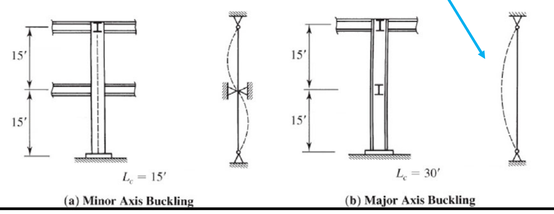


(a) Minor Axis Buckling (b) Major Axis Buckling

6

Chapter 4 – More on Effective Length

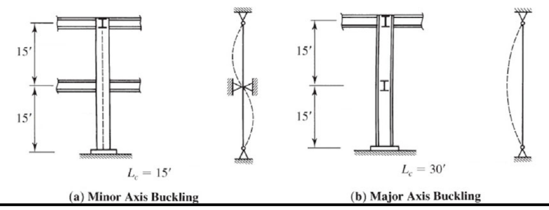
- This brace prevents translation perpendicular to the ***weak axis of the cross section*** but provides no restraint perpendicular to the strong axis.
- If the member were to buckle ***about the major axis***, the ***effective length*** would be 30 *ft*.



7

Chapter 4 – More on Effective Length

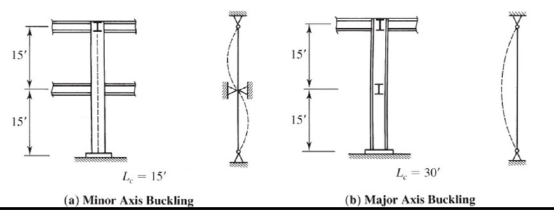
- Buckling about the ***minor axis*** would have to be in the ***second buckling mode***, corresponding to an effective length of 15 *ft*.
- Because its strength decreases with increasing L_c/r, a column will buckle in the direction corresponding to the largest ***slenderness ratio***.



8

Chapter 4 – More on Effective Length

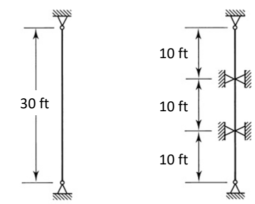
- Therefore, L_{cx}/r_x must be compared **with** L_{cy}/r_y
- In this example, the ratio 30 *ft*(12 *in/ft*)/r_x must be compared with 15 *ft*(12 *in/ft*)/r_y
- The ***largest ratio*** would be used for the computing the ***axial compressive strength***.



9

Chapter 4 – More on Effective Length

- **Example 4-10**: A **W12 x 58**, 30 *ft* long, is pinned at both ends and braced in the weak direction at the third points. **A992** steel (F_y = 50 *ksi*) is used. Determine the available compressive strength.



10

Chapter 4 – More on Effective Length

- **Example 4-10**: A **W12 x 58**, 30 *ft* long, is pinned at both ends and braced in the weak direction at the third points. **A992** steel (F_y = 50 *ksi*) is used. Determine the available compressive strength.

$r_x = 5.28in$

$r_y = 2.51in$

Table 1-1 (continued)
W-Shapes
Properties
W14–W12

Nominal Wt.	Compact Section Criteria		Axis X-X				Axis Y-Y				r_{ts}	h_o	Torsional Properties	
	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	I	S	r	Z	I	S	r	Z			J	C_w
lb/ft			in.^{4}	in.^{3}	in.	in.^{3}	in.^{4}	in.^{3}	in.	in.^{3}	in.	in.	in.^{4}	in.^{6}
72	8.99	22.6	597	97.4	5.31	108	195	32.4	3.04	49.2	3.41	11.6	2.93	6540
65	9.92	24.9	533	87.9	5.28	96.8	174	29.1	3.02	44.1	3.38	11.5	2.18	5780
58	7.82	27.0	475	78.0	5.28	86.4	107	21.4	2.51	32.5	2.81	11.6	2.10	3570
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
50	6.31	26.8	391	64.2	5.18	71.9	56.3	13.9	1.96	21.3	2.25	11.6	1.71	1880
45	7.00	29.6	348	57.7	5.15	64.2	50.0	12.4	1.95	19.0	2.23	11.5	1.26	1650
40	7.77	33.6	307	51.5	5.13	57.0	44.1	11.0	1.94	16.8	2.21	11.4	0.906	1440
35	6.31	36.2	285	45.6	5.25	51.2	24.5	7.47	1.54	11.5	1.79	12.0	0.741	879
30	7.41	41.8	238	38.6	5.21	43.1	20.3	6.24	1.52	9.56	1.77	11.9	0.457	720
26	8.54	47.2	204	33.4	5.17	37.2	17.3	5.34	1.51	8.17	1.75	11.8	0.300	607

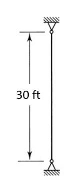
11

Chapter 4 – More on Effective Length

- **Example 4-10**: A **W12 x 58**, 30 *ft* long, is pinned at both ends and braced in the weak direction at the third points. **A992** steel (F_y = 50 *ksi*) is used. Determine the available compressive strength.

$r_x = 5.28in$

$r_y = 2.51in$



$$\frac{L_{cx}}{r_x} = \frac{30ft(12in/ft)}{5.28in} = 68.18$$

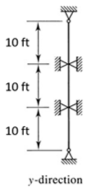
12

Chapter 4 – More on Effective Length

- **Example 4-10**: A **W12 x 58**, 30 *ft* long, is pinned at both ends and braced in the weak direction at the third points. **A992** steel (**F_y** = 50 *ksi*) is used. Determine the available compressive strength.

$r_x = 5.28in$

$r_y = 2.51in$



$$\frac{L_{cx}}{r_x} = \frac{30ft(12in/ft)}{5.28in} = 68.18$$

$$\frac{L_{cy}}{r_y} = \frac{10ft(12in/ft)}{2.51in} = 47.81$$

$\frac{L_{cx}}{r_x}$ is the larger value and controls.

13

Chapter 4 – More on Effective Length

- **Example 4-10**: A **W12 x 58**, 30 *ft* long, is pinned at both ends and braced in the weak direction at the third points. **A992** steel (**F_y** = 50 *ksi*) is used. Determine the available compressive strength.
- ***LRFD*** solution: Use **Table 4-14** with ***KL/r*** = 68.18 and interpolation values.

Also, **A_g** = 17.0 in^2 from Table 1-1 (**1-26**)

14

Chapter 4 – More on Effective Length

$$32.1 + (31.8 - 32.1)0.18 = 32.05ksi$$

$$\phi_c F_{cr} = 32.05ksi$$

$$\phi_c P_n = \phi_c F_{cr} A_g$$

$$= (32.05ksi)17.0in^2$$

$$= 544.8k$$

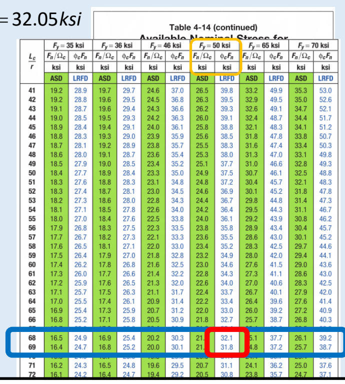


Table 4-14 (continued)

$\frac{L_c}{r}$	F_y = 35 ksi		F_y = 36 ksi		F_y = 46 ksi		F_y = 50 ksi		F_y = 65 ksi		F_y = 70 ksi	
	F_n/Ω_c	$\phi_c F_n$	F_n/Ω_c	$\phi_c F_n$	F_n/Ω_c	$\phi_c F_n$	F_n/Ω_c	$\phi_c F_n$	F_n/Ω_c	$\phi_c F_n$	F_n/Ω_c	$\phi_c F_n$
	ksi	ksi	ksi	ksi	ksi	ksi	ksi	ksi	ksi	ksi	ksi	ksi
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
41	19.2	28.9	19.7	29.7	24.6	37.0	26.5	39.8	33.2	49.9	35.3	53.0
42	19.2	28.8	19.6	29.5	24.5	36.8	26.3	39.5	32.9	49.5	35.0	52.6
43	19.1	28.7	19.6	29.4	24.3	36.6	26.2	39.3	32.6	49.1	34.7	52.1
44	19.0	28.5	19.5	29.3	24.2	36.3	26.0	39.1	32.4	48.7	34.4	51.7
45	18.9	28.4	19.4	29.1	24.0	36.1	25.8	38.8	32.1	48.3	34.1	51.2
46	18.8	28.3	19.3	29.0	23.9	35.9	25.6	38.5	31.8	47.8	33.8	50.7
47	18.7	28.1	19.2	28.9	23.8	35.7	25.5	38.3	31.6	47.4	33.4	50.3
48	18.6	28.0	19.1	28.7	23.6	35.4	25.3	38.0	31.3	47.0	33.1	49.8
49	18.5	27.9	19.0	28.5	23.4	35.2	25.1	37.7	31.0	46.6	32.8	49.3
50	18.4	27.7	18.9	28.4	23.3	35.0	24.9	37.5	30.7	46.1	32.5	48.8
51	18.3	27.6	18.8	28.3	23.1	34.8	24.8	37.2	30.4	45.7	32.1	48.3
52	18.3	27.4	18.7	28.1	23.0	34.5	24.6	36.9	30.1	45.2	31.8	47.8
53	18.2	27.3	18.6	28.0	22.8	34.3	24.4	36.7	29.8	44.8	31.4	47.3
54	18.1	27.1	18.5	27.8	22.6	34.0	24.2	36.4	29.5	44.3	31.1	46.7
55	18.0	27.0	18.4	27.6	22.5	33.8	24.0	36.1	29.2	43.9	30.8	46.2
56	17.9	26.8	18.3	27.5	22.3	33.5	23.8	35.8	28.9	43.4	30.4	45.7
57	17.7	26.7	18.2	27.3	22.1	33.3	23.6	35.5	28.6	43.0	30.1	45.2
58	17.6	26.5	18.1	27.1	22.0	33.0	23.4	35.2	28.3	42.5	29.7	44.6
59	17.5	26.4	17.9	27.0	21.8	32.8	23.2	34.9	28.0	42.0	29.4	44.1
60	17.4	26.2	17.8	26.8	21.6	32.5	23.0	34.6	27.6	41.5	29.0	43.6
61	17.3	26.0	17.7	26.6	21.4	32.2	22.8	34.3	27.3	41.1	28.6	43.0
62	17.2	25.9	17.6	26.5	21.3	32.0	22.6	34.0	27.0	40.6	28.3	42.5
63	17.1	25.7	17.5	26.3	21.1	31.7	22.4	33.7	26.7	40.1	27.9	42.0
64	17.0	25.5	17.4	26.1	20.9	31.4	22.2	33.4	26.4	39.6	27.6	41.4
65	16.9	25.4	17.3	25.9	20.7	31.2	22.0	33.0	26.0	39.2	27.2	40.9
66	16.8	25.2	17.1	25.8	20.5	30.9	21.8	32.7	25.7	38.7	26.8	40.3
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
68	16.5	24.9	16.9	25.4	20.2	30.3	21.[illegible]	32.1	[illegible]5.1	37.7	26.1	39.2
69	16.4	24.7	16.8	25.2	20.0	30.1	21.[illegible]	31.8	[illegible]4.8	37.2	25.7	38.7
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
71	16.2	24.3	16.5	24.8	19.6	29.5	20.7	31.1	24.1	36.2	25.0	37.6
72	16.1	24.2	16.4	24.7	19.4	29.2	20.5	30.8	23.8	35.7	24.7	37.1

15

Chapter 4 – More on Effective Length

- The available strengths given in the column load tables are based on the effective length with respect to the ***y***-axis.
- A procedure for using the tables with **L_{cx}** can be developed by examining how the tabular values were obtained.
- Starting with a value of **L_c**, the strength was obtained by a procedure similar to the following:
  - **L_c** was divided by **r_y** to obtain **L_c/r_y**
  - **F_n** was computed
  - The available strength **$\phi_c P_n$** for ***LRFD*** and **P_n/Ω_c** for ***ASD*** were computed

16

Chapter 4 – More on Effective Length

- The tabulated strengths are based on the values of **L_c** being equal to **L_{cy}**.
- If the capacity with respect to the *x*-axis buckling is desired, the table can be entered with:

$$L_c = \frac{L_{cx}}{r_x/r_y}$$

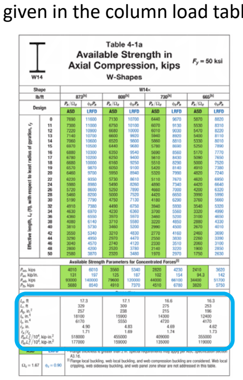
- The tabulated load will be based on:

$$\frac{L_c}{r_y} = \frac{L_{cx}/(r_x/r_y)}{r_y} = \frac{L_{cx}}{r_x}$$

17

Chapter 4 – More on Effective Length

- The ratio **r_x/r_y** is given in the column load tables for each shape listed.



18

Chapter 4 – More on Effective Length

- The ratio r_x/r_y is given in the column load tables for each shape listed.

Table 4-1a
Available Strength in Axial Compression, kips
W-Shapes
F_y = 50 ksi
W14

Shape		W14×							
lb/ft		873[h]		808[h]		730[h]		665[h]	
Design		P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
radius of gyration, r_y	0	7690	11600	7130	10700	6440	9670	5870	8820
	11	7300	11000	6750	10100	6070	9130	5530	8310
	12	7220	10900	6680	10000	6010	9030	5470	8220
	13	7140	10700	6600	9920	5940	8920	5400	8110
	14	7060	10600	6520	9800	5860	8810	5330	8010
	15	6970	10500	6440	9680	5780	8690	5250	7890
	16	6880	10300	6350	9540	5690	8560	5170	7770

Available Strength Parameters for Concentrated Forces[i]

	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
P_{wo}, kips	4010	6010	3560	5340	2820	4230	2410	3620
P_{wi}, kip/in.	131	197	125	187	102	154	94.3	142
P_{wb}, kips	93000	140000	79600	120000	44000	66100	34400	51700
P_{fb}, kips	5680	8540	4910	7370	4510	6780	3820	5750

Properties

L_p, ft	17.3	17.1	16.6	16.3
L_r, ft	329	309	275	253
A_g, in.^{2}	257	238	215	196
I_x, in.^{4}	18100	15900	14300	12400
I_y, in.^{4}	6170	5550	4720	4170
r_x/r_y	1.71	1.69	1.74	1.73
$P_{ey}L_c^2/10^4$, kip-in.^{2}	177000	159000	135000	119000

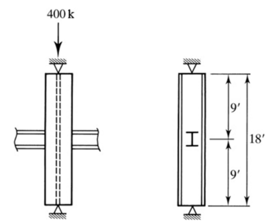
ASD: $\Omega_c = 1.67$ LRFD: $\phi_c = 0.90$

[h] Flange thickness is greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1d.
[i] Flange local buckling, web local buckling, and web compression buckling are considered. Web local crippling, web sidesway buckling, and web panel zone shear are not addressed in this table.

19

Chapter 4 – More on Effective Length

- **Example 4-11**: The compression member shown below is pinned at both ends and supported in the weak direction at midheight. A service load of 400 *k*, with equal parts dead and live load, must be supported. Use F_y = 50 *ksi* and select the lightest **W-shape**.



20

Chapter 4 – More on Effective Length

- **Example 4-11**: ***LRFD*** solution:

$$\text{Factored Load} = 1.2(200k) + 1.6(200k) = 560k$$

Assume that the weak direction controls and enter **Table 4-1a** for F_y = 50 ksi and L_c = 9 *ft.*

Table 4-1a (continued)
Available Strength in Axial Compression, kips
W-Shapes
F_y = 50 ksi
W8

Shape		W8×											
lb/ft		67		58		48		40		35		31	
Design		P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
radius of gyration, r_y	0	590	886	512	769	422	634	350	526	308	463	273	411
	6	542	815	470	706	387	581	320	481	281	423	249	374
	7	526	790	455	685	375	563	309	465	272	409	241	362
	9	488	733	422	634	347	521	285	429	251	377	222	333
	11	444	668	384	576	314	473	258	388	226	340	200	301
	12	421	633	363	546	297	447	243	366	213	321	189	283

$$\phi_c P_n = 634k$$

21

Chapter 4 – More on Effective Length

- **Example 4-11**: From **Table 1-1** (**1**-28) a **W8 x 58** has:

$$r_x = 3.65in \qquad r_y = 2.10in \qquad A_g = 17.1in$$

Check the strong axis: $\dfrac{L_{cx}}{r_x/r_y} = \dfrac{18\,ft}{3.65in/2.10in} = 10.36\,ft \;\; > 9\,ft$

Therefore, L_{cx} controls for this shape.

Enter the table with L_c = 10.34 *ft*.

22

Chapter 4 – More on Effective Length

- **Example 4-11**:

$$606 + (576 - 606)0.34 = 594.8k$$

$$\phi_c P_n = 594.8k > 560k \quad \textbf{OK}$$

Table 4-1a (continued)
Available Strength in Axial Compression, kips
W-Shapes
F_y = 50 ksi
W8

Shape		W8×											
lb/ft		67		58		48		40		35		31	
Design		P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
radius of gyration, r_y	0	590	886	512	769	422	634	350	526	308	463	273	411
	6	542	815	470	706	387	581	320	481	281	423	249	374
	7	526	790	455	685	375	563	309	465	272	409	241	362
	8	508	763	439	660	361	543	298	448	262	394	232	348
	10	467	701	403	606	331	497	272	409	239	359	211	317
	11	444	668	384	576	314	473	258	388	226	340	200	301

23

Chapter 4 – More on Effective Length

- **Example 4-11**: Try a **W10** shape

$$\text{Factored Load} = 1.2(200k) + 1.6(200k) = 560k$$

Assume that the weak direction controls and enter **Table 4-1a** for F_y = 50 ksi and L_c = 9 *ft.*

Table 4-1a (continued)
Available Strength in Axial Compression, kips
W-Shapes
F_y = 50 ksi
W10

Shape		W10×									
lb/ft		54		49		45		39		33	
Design		P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
Least radius of gyration, r_y	0	473	711	431	648	398	598	344	517	291	437
	6	446	671	407	611	363	545	313	470	263	395
	7	437	657	398	598	350	527	302	454	253	381
	9	415	624	378	568	322	485	277	416	232	348
	11	389	585	354	532	291	437	249	374	207	311
	12	375	564	341	512	274	411	234	352	194	292
	13	361	542	327	492	256	385	219	329	181	272
	14	345	519	313	471	239	359	203	306	168	253

$$\phi_c P_n = 568k$$

24

Chapter 4 – More on Effective Length

- **Example 4-11**: From **Table 1-1** (**1**-28) a **W10 x 49** has:

$r_x = 4.35 in \qquad r_y = 2.54 in \qquad A_g = 14.4 in$

Check the strong axis: $\frac{L_{cx}}{r_x/r_y} = \frac{18\,ft}{4.35in/2.54in} = 10.51\,ft \;\; > 9\,ft$

Therefore, $\boldsymbol{L_{cx}}$ controls for this shape.

Enter the table with $\boldsymbol{L_c}$ = 10.51 *ft*.

25

Chapter 4 – More on Effective Length

- **Example 4-11**:

$605 + (585 - 605)0.51 \;\; = 594.8k$

$\phi_c P_n = 594.8k > 560k$ **OK**

Table 4-1a (continued)
Available Strength in Axial Compression, kips
W-Shapes
F_y = 50 ksi — W10

Shape		W10×									
lb/ft		54		49		45		39		33	
Design		P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
Effective length, least radius of gyration, r_y	0	473	711	431	648	398	598	344	517	291	437
	6	446	671	407	611	363	545	313	470	263	395
	7	437	657	398	598	350	527	302	454	253	381
	8	427	642	388	584	337	507	290	436	243	365
	10	403	605	[illegible]	550	307	461	263	396	220	330
	11	389	585	[illegible]	532	291	437	249	374	207	311
	13	361	542	327	492	256	385	219	329	181	272
	14	345	519	313	471	239	359	203	306	168	253

26

Chapter 4 – More on Effective Length

- **Example 4-11**: Try a **W12** shape

$\text{Factored Load} = 1.2(200k) + 1.6(200k) \;\; = 560k$

Assume that the weak direction controls and enter **Table 4-1a** for $\boldsymbol{F_y}$ = 50 ksi and $\boldsymbol{L_c}$ = 9 *ft*.

Table 4-1a (continued)
Available Strength in Axial Compression, kips
W-Shapes
F_y = 50 ksi — W12

Shape		W12×									
lb/ft		58		53		50		45		40	
Design		P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
Effective length, least radius of gyration, r_y	0	509	765	467	702	437	657	392	589	350	526
	6	479	720	439	660	396	595	355	534	317	476
	7	469	705	429	646	382	574	342	515	305	459
	9	445	668	407	611	[illegible]	526	313	471	279	[illegible]
	11	416	625	380	571	314	472	281	422	250	375
	12	400	601	365	549	295	443	263	396	234	352
	13	384	577	350	526	275	413	246	369	218	328
	14	367	551	334	502	255	384	228	343	202	304
	15	349	525	318	478	236	355	210	316	187	281

$\phi_c P_n = 611k$

27

Chapter 4 – More on Effective Length

- **Example 4-11**: From **Table 1-1** (**1**-26) a **W12 x 53** has:

$r_x = 5.23 in \qquad r_y = 2.48 in \qquad A_g = 15.6 in$

Check the strong axis: $\frac{L_{cx}}{r_x/r_y} = \frac{18\,ft}{5.23in/2.48in} = 8.54\,ft \;\; < 9\,ft$

Therefore, $\boldsymbol{L_{cy}}$ controls for this shape. $\phi_c P_n = 611k$

28

Chapter 4 – More on Effective Length

- **Example 4-11**: Try a **W14** shape

$\text{Factored Load} = 1.2(200k) + 1.6(200k) \;\; = 560k$

Assume that the weak direction controls and enter ***Table 4-1a*** for $\boldsymbol{F_y}$ = 50 ksi and $\boldsymbol{L_c}$ = 9 *ft*.

Table 4-1a (continued)
Available Strength in Axial Compression, kips
W-Shapes
F_y = 50 ksi — W14

Shape		W14×													
lb/ft		82		74		68		61		53		48		43[c]	
Design		P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
Effective length, least radius of gyration, r_y	0	719	1080	653	981	599	900	536	805	467	702	422	634	374	562
	6	676	1020	614	922	562	845	503	756	421	633	380	572	339	510
	7	661	993	600	902	550	826	492	739	406	610	366	551	327	491
	9	626	940	568	854	520	782	[illegible]	699	[illegible]	557	334	502	297	447
	11	584	878	531	797	485	729	433	651	331	497	298	447	264	397
	12	562	844	510	767	466	701	416	626	310	465	279	419	247	371
	13	538	809	489	735	446	671	398	599	288	433	259	390	229	345
	14	514	772	467	701	426	640	380	571	267	401	240	360	212	318
	15	489	735	444	667	405	608	361	543	246	369	221	331	194	292

$\phi_c P_n = 699k$

29

Chapter 4 – More on Effective Length

- **Example 4-11**: Try a **W14** shape

$\text{Factored Load} = 1.2(200k) + 1.6(200k) \;\; = 560k$

The **W14 x 61** is the lightest section possible, but it is ***heavier*** than the other sections.

Table 4-1a (continued)
Available Strength in Axial Compression, kips
W-Shapes
F_y = 50 ksi — W14

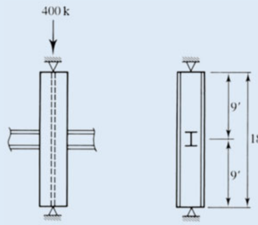
Shape		W14×													
lb/ft		82		74		68		61		53		48		43[c]	
Design		P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
Effective length, least radius of gyration, r_y	0	719	1080	653	981	599	900	536	805	467	702	422	634	374	562
	6	676	1020	614	922	562	845	503	756	421	633	380	572	339	510
	7	661	993	600	902	550	826	492	739	406	610	366	551	327	491
	9	626	940	568	854	520	782	[illegible]	699	[illegible]	557	334	502	297	447
	11	584	878	531	797	485	729	433	651	331	497	298	447	264	397
	12	562	844	510	767	466	701	416	626	310	465	279	419	247	371
	13	538	809	489	735	446	671	398	599	288	433	259	390	229	345
	14	514	772	467	701	426	640	380	571	267	401	240	360	212	318
	15	489	735	444	667	405	608	361	543	246	369	221	331	194	292

$\phi_c P_n = 699k$

30

Chapter 4 – More on Effective Length

- **Example 4-11**: The compression member shown below is pinned at both ends and supported in the weak direction at midheight. A service load of 400 *k*, with equal parts dead and live load, must be supported. Use F_y = 50 *ksi* and select the lightest ***W-shape***.



W8 x 58
W10 x 54
W12 x 53 ←
W14 x 61

Use a **W12 x 53**

31

Chapter 4 – More on Effective Length

Let's work on some problems

32

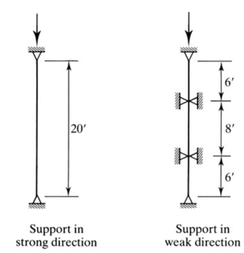
Chapter 4 – More on Effective Length

- Whenever possible, the designer should provide extra support for the weak direction of a column.
- Otherwise, the member is ***inefficient*** (i.e., it has an excess of strength in one direction).
- When L_{cx} and L_{cy} are different, L_{cy} will control unless r_x/r_y is smaller than L_{cx}/L_{cy}
- When the ***two ratios are equal***, the column has ***equal strength in both directions***.
- For most of the ***W-shapes*** in the column load tables, r_x/r_y ranges between 1.6 and 1.8, but it is as high as 3.1 for some shapes.

33

Chapter 4 – More on Effective Length

- **Example 4-12**: The column shown below is subjected to a service dead load of 140 *k* and a service live load of 420 *k*. Use **A992** steel (F_y = 50 *ksi*) and select a **W-shape**.



$L_{cx} = 20\,ft \qquad L_{cy} = 8\,ft$

The effective length L_{cx} will control whenever:

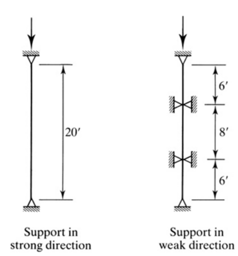
$$\frac{L_{cx}}{r_x/r_y} > L_{cy} \qquad r_x/r_y < \frac{L_{cx}}{L_{cy}}$$

$$\frac{L_{cx}}{L_{cy}} = \frac{20\,ft}{8\,ft} = 2.5$$

34

Chapter 4 – More on Effective Length

- **Example 4-12**: The column shown below is subjected to a service dead load of 140 *k* and a service live load of 420 *k*. Use **A992** steel (F_y = 50 *ksi*) and select a **W-shape**.



$L_{cx} = 20\,ft \qquad L_{cy} = 8\,ft$

So, L_{cx} will control if $r_x/r_y < 2.5$

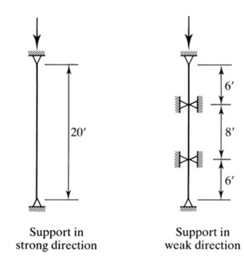
$$\frac{L_{cx}}{r_x/r_y} > L_{cy} \qquad r_x/r_y < \frac{L_{cx}}{L_{cy}}$$

$$\frac{L_{cx}}{L_{cy}} = \frac{20\,ft}{8\,ft} = 2.5$$

35

Chapter 4 – More on Effective Length

- **Example 4-12**: The column shown below is subjected to a service dead load of 140 *k* and a service live load of 420 *k*. Use **A992** steel (F_y = 50 *ksi*) and select a **W-shape**.



$L_{cx} = 20\,ft \qquad L_{cy} = 8\,ft$

Since this is true for almost every shape in the column load tables, L_{cx} probably controls in this example.

36

Chapter 4 – More on Effective Length

- **Example 4-12**: The column shown below is subjected to a service dead load of 140 *k* and a service live load of 420 *k*. Use **A992** steel (F_y = 50 *ksi*) and select a **W-shape**.

Assume r_x/r_y = 1.7

$$\frac{L_{cx}}{r_x/r_y} = \frac{20\,ft}{1.7} = 11.76 \; > L_{cy} = 8\,ft$$

Assume L_{cx} controls in this example.

37

Chapter 4 – More on Effective Length

- **Example 4-12**: The column shown below is subjected to a service dead load of 140 *k* and a service live load of 420 *k*. Use **A992** steel (F_y = 50 *ksi*) and select a **W-shape**.

LRFD solution: $P_u = 1.2D + 1.6L \; = 1.2(140k) + 1.6(420k) = 840k$

Enter the column load table with L_{cx} = 12 ft.

Assume that the strong direction controls and enter **Table 4-1a** for F_y = 50 ksi and L_c = 12 *ft.*

38

Chapter 4 – More on Effective Length

- **Example 4-12**: Try a **W8** shape for a strength > 840 *k*

Assume that the strong direction controls and enter **Table 4-1a** for F_y = 50 ksi and L_c = 12 *ft.*

There are ***no W8 sections*** with enough strength

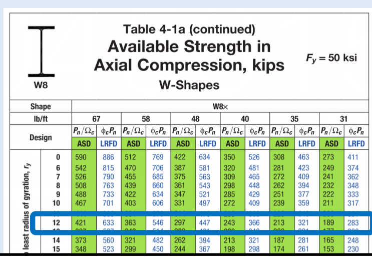


Table 4-1a (continued)
Available Strength in Axial Compression, kips
W-Shapes
F_y = 50 ksi
W8

Shape		W8×											
lb/ft		67		58		48		40		35		31	
Design		P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
least radius of gyration, r_y	0	590	886	512	769	422	634	350	526	308	463	273	411
	6	542	815	470	706	387	581	320	481	281	423	249	374
	7	526	790	455	685	375	563	309	465	272	409	241	362
	8	508	763	439	660	361	543	298	448	262	394	232	348
	9	488	733	422	634	347	521	285	429	251	377	222	333
	10	467	701	403	606	331	497	272	409	239	359	211	317
	12	421	633	363	546	297	447	243	366	213	321	189	283
	14	373	560	321	482	262	394	213	321	187	281	165	248
	15	348	523	299	450	244	367	198	298	174	261	153	230

39

Chapter 4 – More on Effective Length

- **Example 4-12**: Try a **W10** shape for a strength > 840 *k*

Assume that the strong direction controls and enter ***Table 4-1a*** for F_y = 50 ksi and L_c = 12 *ft.*

$\phi_c P_n = 940k$

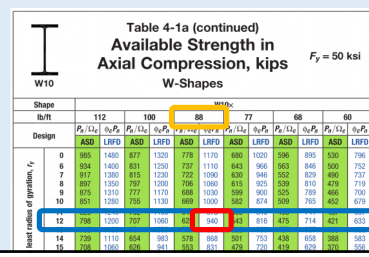


Table 4-1a (continued)
Available Strength in Axial Compression, kips
W-Shapes
F_y = 50 ksi
W10

Shape		W10×											
lb/ft		112		100		88		77		68		60	
Design		P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
least radius of gyration, r_y	0	985	1480	877	1320	778	1170	680	1020	596	895	530	796
	6	934	1400	831	1250	737	1110	643	966	563	846	500	752
	7	917	1380	815	1230	722	1090	630	946	552	829	490	737
	8	897	1350	797	1200	706	1060	615	925	539	810	479	719
	9	875	1310	777	1170	688	1030	599	900	525	789	466	700
	10	851	1280	755	1130	669	1000	582	874	509	765	452	679
	12	798	1200	707	1060	[illegible]	940	[illegible]	816	475	714	421	633
	14	739	1110	654	983	578	868	501	753	438	658	388	583
	15	708	1060	626	941	553	831	479	720	419	629	370	556

40

Chapter 4 – More on Effective Length

- **Example 4-12**: From **Table 1-1** (**1**-28) a **W10 x 88** has:

$r_x = 4.54\,in \qquad r_y = 2.63\,in \qquad A_g = 26.0\,in$

$$r_x/r_y = \frac{4.60\,in}{2.63\,in} = 1.73$$

$\phi_c P_n = 940\,k$

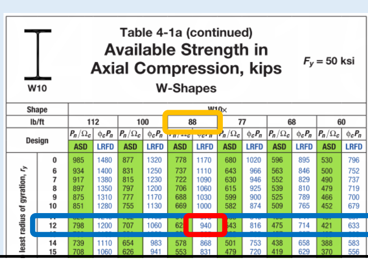


Table 4-1a (continued)
Available Strength in Axial Compression, kips
W-Shapes
F_y = 50 ksi
W10

Shape		W10×											
lb/ft		112		100		88		77		68		60	
Design		P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
least radius of gyration, r_y	0	985	1480	877	1320	778	1170	680	1020	596	895	530	796
	6	934	1400	831	1250	737	1110	643	966	563	846	500	752
	7	917	1380	815	1230	722	1090	630	946	552	829	490	737
	8	897	1350	797	1200	706	1060	615	925	539	810	479	719
	9	875	1310	777	1170	688	1030	599	900	525	789	466	700
	10	851	1280	755	1130	669	1000	582	874	509	765	452	679
	12	798	1200	707	1060	[illegible]	940	[illegible]	816	475	714	421	633
	14	739	1110	654	983	578	868	501	753	438	658	388	583
	15	708	1060	626	941	553	831	479	720	419	629	370	556

41

Chapter 4 – More on Effective Length

- **Example 4-12**: From **Table 1-1** (**1**-28) a **W10 x 88** has:

$$\text{Actual } \frac{L_{cx}}{r_x/r_y} = \frac{20\,ft}{1.73} = 11.56\,ft \; < 12\,ft$$

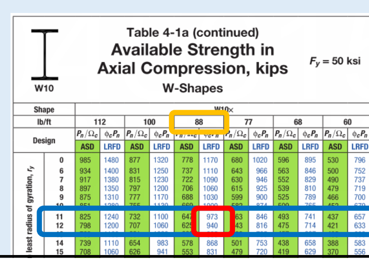


Table 4-1a (continued)
Available Strength in Axial Compression, kips
W-Shapes
F_y = 50 ksi
W10

Shape		W10×											
lb/ft		112		100		88		77		68		60	
Design		P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
least radius of gyration, r_y	0	985	1480	877	1320	778	1170	680	1020	596	895	530	796
	6	934	1400	831	1250	737	1110	643	966	563	846	500	752
	7	917	1380	815	1230	722	1090	630	946	552	829	490	737
	8	897	1350	797	1200	706	1060	615	925	539	810	479	719
	9	875	1310	777	1170	688	1030	599	900	525	789	466	700
	11	825	1240	732	1100	647	973	[illegible]	846	493	741	437	657
	12	798	1200	707	1060	625	940	[illegible]	816	475	714	421	633
	14	739	1110	654	983	578	868	501	753	438	658	388	583
	15	708	1060	626	941	553	831	479	720	419	629	370	556

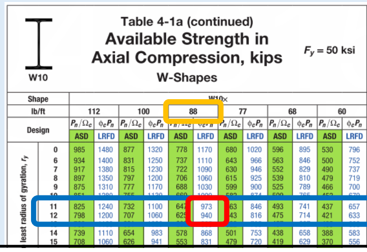
42

Chapter 4 – More on Effective Length

- **Example 4-12**: By interpolation, a **W10 x 88** has:

$$973+(940-973)0.56 \;=954.5k$$

$$\phi_c P_n = 954.5k > 840k \quad \textbf{OK}$$



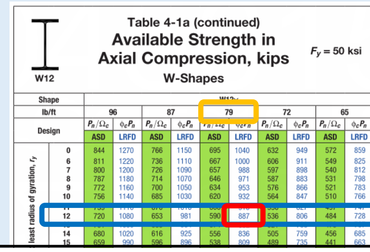
$\phi_c P_n = 954.5k$

43

Chapter 4 – More on Effective Length

- **Example 4-12**: Try a **W12** shape for a strength > 840 *k*

Assume that the strong direction controls and enter **Table 4-1a** for $\boldsymbol{F_y}$ = 50 ksi and $\boldsymbol{L_c}$ = 12 *ft.*



$\phi_c P_n = 887k$

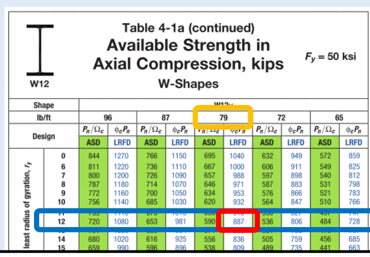
44

Chapter 4 – More on Effective Length

- **Example 4-12**: From **Table 1-1** (**1**-28) a **W12 x 79** has:

$$r_x = 5.34in \qquad r_y = 3.05in \qquad A_g = 23.2in$$

$$r_x/r_y = \frac{5.34in}{3.05in} = 1.75$$



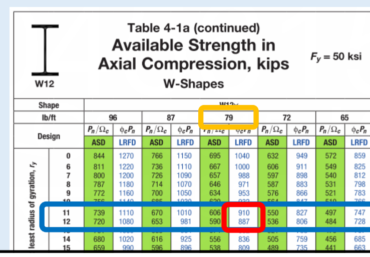
$\phi_c P_n = 887k$

45

Chapter 4 – More on Effective Length

- **Example 4-12**: From **Table 1-1** (**1**-28) a **W12 x 79** has:

$$\text{Actual } \frac{L_{cx}}{r_x/r_y} = \frac{20\,ft}{1.75} = 11.43ft \; < 12ft$$



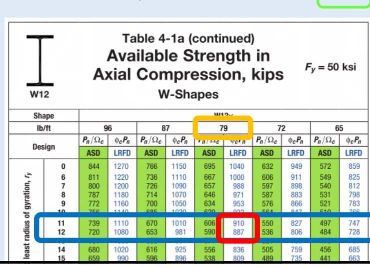
46

Chapter 4 – More on Effective Length

- **Example 4-12**: By interpolation, a **W12 x 79** has:

$$910+(887-910)0.43 = 900.1k$$

$$\phi_c P_n = 900.1k > 840k \quad \textbf{OK}$$



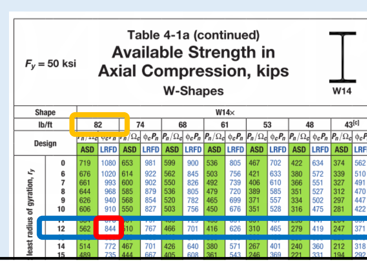
$\phi_c P_n = 900k$

47

Chapter 4 – More on Effective Length

- **Example 4-12**: Try a **W14** shape for a strength > 840 *k*

Assume that the strong direction controls and enter **Table 4-1a** for $\boldsymbol{F_y}$ = 50 ksi and $\boldsymbol{L_c}$ = 12 *ft.*



$\phi_c P_n = 884k$

48

Chapter 4 – More on Effective Length

➢ **Example 4-12**: From **Table 1-1** (**1**-28), the **W14 x 82**:

$r_x = 6.05\,in \qquad r_y = 2.48\,in \qquad r_x/r_y = \dfrac{6.05\,in}{2.48\,in} = 2.44$

Actual $\dfrac{L_{cx}}{r_x/r_y} = \dfrac{20\,ft}{2.44} = 8.198\,ft < 12\,ft$ Assume $\boldsymbol{L_c}$ = 9 *ft*.

Table 4-1a (continued)
Available Strength in Axial Compression, kips
W-Shapes
$F_y = 50$ ksi — W14

Shape	W14×													
lb/ft	82		74		68		61		53		48		43[c]	
Design	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
0	719	1080	653	981	599	900	536	805	467	702	422	634	374	562
6	676	1020	614	922	562	845	503	756	421	633	380	572	339	510
7	661	993	600	902	550	826	492	739	406	610	366	551	327	491
9	626	940	568	854	520	782	465	699	371	557	334	502	297	447
11	584	878	531	797	485	729	433	651	331	497	298	447	264	397
12	562	844	510	767	466	701	416	626	310	465	279	419	247	371
13	538	809	489	735	446	671	398	599	288	433	259	390	229	345
14	514	772	467	701	426	640	380	571	267	401	240	360	212	318
15	489	735	444	667	405	608	361	543	246	369	221	331	194	292

Effective length, L_c (ft), with respect to least radius of gyration, r_y

$\phi_c P_n = 854\,k$

49

Chapter 4 – More on Effective Length

➢ **Example 4-12**: From **Table 1-1** (**1**-28), the **W14 x 74**:

$r_x = 6.04\,in \qquad r_y = 2.48\,in \qquad r_x/r_y = \dfrac{6.04\,in}{2.48\,in} = 2.44$

Actual $\dfrac{L_{cx}}{r_x/r_y} = \dfrac{20\,ft}{2.44} = 8.198\,ft < 12\,ft$ Assume $\boldsymbol{L_c}$ = 9 *ft*.

Table 4-1a (continued)
Available Strength in Axial Compression, kips
W-Shapes
$F_y = 50$ ksi — W14

Shape	W14×													
lb/ft	82		74		68		61		53		48		43[c]	
Design	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
0	719	1080	653	981	599	900	536	805	467	702	422	634	374	562
6	676	1020	614	922	562	845	503	756	421	633	380	572	339	510
7	661	993	600	902	550	826	492	739	406	610	366	551	327	491
9	626	940	568	854	520	782	465	699	371	557	334	502	297	447
11	584	878	531	797	485	729	433	651	331	497	298	447	264	397
12	562	844	510	767	466	701	416	626	310	465	279	419	247	371
13	538	809	489	735	446	671	398	599	288	433	259	390	229	345
14	514	772	467	701	426	640	380	571	267	401	240	360	212	318
15	489	735	444	667	405	608	361	543	246	369	221	331	194	292

Effective length, L_c (ft), with respect to least radius of gyration, r_y

$\phi_c P_n = 854\,k$

50

Chapter 4 – More on Effective Length

➢ **Example 4-12**: By interpolation, a **W14 x 74** has

$\phi_c P_n = 874\,k > 840\,k$ **OK**

Table 4-1a (continued)
Available Strength in Axial Compression, kips
W-Shapes
$F_y = 50$ ksi — W14

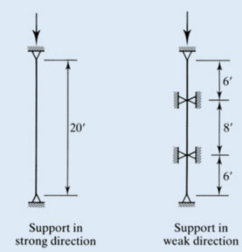
Shape	W14×													
lb/ft	82		74		68		61		53		48		43[c]	
Design	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
0	719	1080	653	981	599	900	536	805	467	702	422	634	374	562
6	676	1020	614	922	562	845	503	756	421	633	380	572	339	510
8	644	968	585	879	536	805	479	720	389	585	351	527	312	470
9	626	940	568	854	520	782	465	699	371	557	334	502	297	447
11	584	878	531	797	485	729	433	651	331	497	298	447	264	397
12	562	844	510	767	466	701	416	626	310	465	279	419	247	371
13	538	809	489	735	446	671	398	599	288	433	259	390	229	345
14	514	772	467	701	426	640	380	571	267	401	240	360	212	318
15	489	735	444	667	405	608	361	543	246	369	221	331	194	292

Effective length, L_c (ft), with respect to least radius of gyration, r_y

51

Chapter 4 – More on Effective Length

➢ **Example 4-12**: The column shown below is subjected to a service dead load of 140 *k* and a service live load of 420 *k*. Use **A992** steel ($\boldsymbol{F_y}$ = 50 *ksi*) and select a **W-shape**.



W10 x 88
W12 x 79
W14 x 74 ⬅

Use a **W14 x 74**

52

Chapter 4 – More on Effective Length

Let's work on some problems

53

Chapter 4 – More on Effective Length

Any questions?

54