

1

Chapter 4 – Introduction

- **Compression members** are structural elements that are subjected only to **axial compressive forces**.
- The loads are applied **along a longitudinal axis through the centroid** of the member cross-section.
- The stress can be taken as $f = P/A$, where f is uniform over the entire cross section.

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Chapter 4 – Introduction

- This ideal state for **compression members** is never achieved.
- Some **eccentricity** of the load is inevitable.
- Bending will result, but it usually can be regarded as secondary.
- As we shall see, the **AISC Specification** equations for compression member strength account for this accidental eccentricity.

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Chapter 4 – Introduction

- The most common type of compression member occurring in structures is the **column**, a vertical member whose primary function is to support vertical loads.
- In many instances, these members are also subjected to **bending**, and in these cases, the member is a **beam-column**.
- We cover this topic in **Chapter 6**.
- Compression members are also used in **trusses** and as components of bracing systems.
- Smaller compression members not classified as columns are sometimes referred to as **struts**.

4

Chapter 4 – Introduction

- In many small structures, **column axial forces** can be easily computed from the reactions of the beams that they support or computed directly from floor or roof loads.
- This is possible if the member connections **do not transfer moment**; in other words, if the column is not part of a rigid frame.
- For **columns in rigid frames**, there are calculable bending moments as well as axial forces, and a **frame analysis** is necessary.

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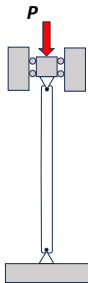
Chapter 4 – Introduction

- The **AISC Specification** provides for **three methods of analysis** to obtain the axial forces and bending moments in members of a rigid frame:
 1. Direct analysis method
 2. Effective length method
 3. First-order analysis method
- Except in very simple cases, computer software is used for the analysis.
- While the details of these three methods are beyond the scope of the present chapter, more will be said about them in **Chapter 6**.

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Chapter 4 – Column Theory

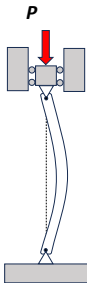
- Consider a long, slender compression member.
- If the axial load P is slowly applied, it will ultimately become large enough to cause the member to become **unstable**.



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Chapter 4 – Column Theory

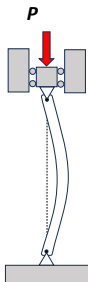
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Chapter 4 – Column Theory

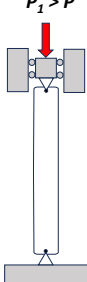
- Consider a long, slender compression member.
- If the axial load P is slowly applied, it will ultimately become large enough to cause the member to become **unstable**.
- The member is said to have **buckled**, and the corresponding load is called the **critical buckling load**.



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Chapter 4 – Column Theory


- Consider a stockier compression member.
- If the member is stockier, a larger load will be required to bring the member to the point of instability.
- For extremely stocky members, failure may occur by **compressive yielding** rather than buckling.



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Chapter 4 – Column Theory


- Consider a stockier compression member.
- If the member is stockier, a larger load will be required to bring the member to the point of instability.
- For extremely stocky members, failure may occur by **compressive yielding** rather than buckling.



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Chapter 4 – Column Theory

- Consider a stockier compression member.
- Before failure, the compressive stress P/A will be uniform over the cross section at any point along the length.
- The load at which buckling occurs is a function of **slenderness**, and for very slender members, this load could be quite small.



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Chapter 4 – Column Theory

- If the member is so **slender** (we give a precise definition of slenderness shortly) that the stress just before buckling is below the proportional limit—that is, the member is still **elastic**—the **critical buckling load** is given by:

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

where **E** is the modulus of elasticity,
I is the moment of inertia of the cross-sectional area with respect to the minor principal axis, and
L is the length of the member between points of support.

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Chapter 4 – Column Theory

- If the member is so **slender** (we give a precise definition of slenderness shortly) that the stress just before buckling is below the proportional limit—that is, the member is still **elastic**—the **critical buckling load** is given by:

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

- For this equation to be valid, the member must be elastic, and its ends must be free to rotate but not translate laterally.
- This end condition is satisfied by hinges or pins.

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Chapter 4 – Column Theory

- If the member is so **slender** (we give a precise definition of slenderness shortly) that the stress just before buckling is below the proportional limit—that is, the member is still **elastic**—the **critical buckling load** is given by:

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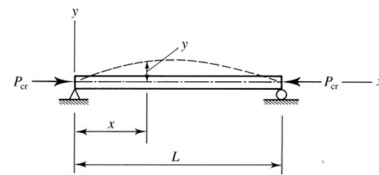


- This equation was first formulated by the Swiss mathematician **Leonhard Euler** and published in 1759.
- The critical load is referred to as the **Euler load** or the **Euler buckling load**.

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Chapter 4 – Column Theory

- For convenience, the member will be oriented with its longitudinal axis along the **x-axis**.

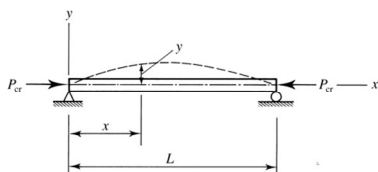


- An axial compressive load is applied and gradually increased.
- Suppose a **temporary transverse load** is applied to deflect the member into the shape indicated by the dashed line.

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Chapter 4 – Column Theory

- For convenience, the member will be oriented with its longitudinal axis along the **x-axis**.

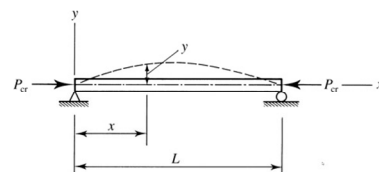


- The member will return to its original position when this temporary load is removed if the **axial load is less than the critical buckling load**.

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Chapter 4 – Column Theory

- For convenience, the member will be oriented with its longitudinal axis along the **x-axis**.



- The critical buckling load, **P_{cr}**, is defined as the **load that is just large enough to maintain the deflected shape** when the temporary transverse load is removed.

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Chapter 4 – Column Theory

- The differential equation giving the deflected shape of an elastic member subjected to bending is:

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

where y is the deflection at a point,
 M is the bending moment at a point, and
 EI has been defined

- This equation was derived by Jacob Bernoulli and independently by Euler, who specialized it for the column buckling problem



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Chapter 4 – Column Theory

- The differential equation giving the deflected shape of an elastic member subjected to bending is:

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

- At the point of buckling, the bending moment is $P_{cr}y$

$$\frac{d^2y}{dx^2} = \frac{P_{cr}y}{EI} \Rightarrow y'' = \frac{P_{cr}y}{EI}$$

- The solution to this differential equation is:

$$y = A\cos(cx) + B\sin(cx) \quad c = \sqrt{\frac{P_{cr}}{EI}}$$

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Chapter 4 – Column Theory

- The differential equation giving the deflected shape of an elastic member subjected to bending is:

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

- The boundary conditions are:

$$y(x=0) = 0 = A\cos(0) + B\sin(0) \Rightarrow A = 0$$

$$y(x=L) = 0 = B\sin(cL) \Rightarrow \sin(cL) = 0$$

$$cL = 0, \pi, 2\pi, 3\pi, \dots, n\pi$$

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Chapter 4 – Column Theory

- The differential equation giving the deflected shape of an elastic member subjected to bending is:

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

- The boundary conditions are:

$$cL = \left(\sqrt{\frac{P_{cr}}{EI}}\right)L = n\pi \Rightarrow \frac{P_{cr}}{EI}L^2 = n^2\pi^2$$

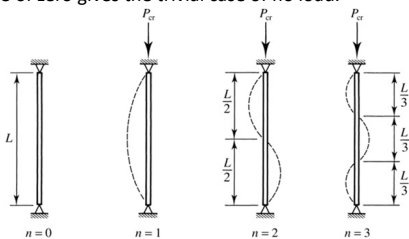
$$P_{cr} = \frac{n^2\pi^2EI}{L^2}$$

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Chapter 4 – Column Theory

- The various values of n correspond to different buckling modes; $n = 1$ represents the first mode, $n = 2$ the second, and so on.

- A value of zero gives the trivial case of no load.



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Chapter 4 – Column Theory

- For the usual case of a compression member with no supports between its ends, $n = 1$, and the Euler equation is written as:

$$P_{cr} = \frac{\pi^2EI}{L^2} = \frac{\pi^2E(Ar^2)}{L^2} = \frac{\pi^2EA}{(L/r)^2}$$

where A is the cross-sectional area, and
 r is the radius of gyration

- The ratio L/r is the **slenderness ratio** and is the measure of a member's slenderness, with large values corresponding to slender members.

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Chapter 4 – Column Theory

- If the critical load is divided by the **A**, the critical buckling stress is obtained:

$$F_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L/r)^2}$$

- At this compressive stress, **buckling** will occur about the axis corresponding to **r**
- Buckling will take place as soon as the load reaches **P_{cr}**
- The column will become **unstable** about the principal axis corresponding to the **largest slenderness ratio**.

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Chapter 4 – Column Theory

- If the critical load is divided by the **A**, the critical buckling stress is obtained:

$$F_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L/r)^2}$$

- This axis usually is the axis with the **smaller moment of inertia** (we examine exceptions to this condition later).
- Thus, the minimum **moment of inertia** and **radius of gyration** of the cross section should ordinarily be used.

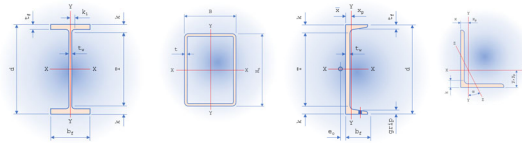
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Chapter 4 – Column Theory

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Chapter 4 – Column Theory

- **Example 4-1:** A **W10 x 30** is used as a column to support an axial compressive load of 100 k. The length is 15 ft, and the ends are pinned.

- Do not consider load or resistance factors; investigate this member for stability.

- For a **W10 x 30**: $r_{min} = r_y = 1.37 \text{ in}$ $A = 8.84 \text{ in}^2$

$$\text{Maximum } \frac{L}{r} = \frac{15 \text{ ft} \cdot (12 \text{ in / ft})}{1.37 \text{ in}} = 131.4$$

$$P_{cr} = \frac{\pi^2 EA}{(L/r)^2} = \frac{\pi^2 (29,000 \text{ ksi})(8.84 \text{ in}^2)}{(131.4)^2} = 146.57 \text{ k}$$

OK

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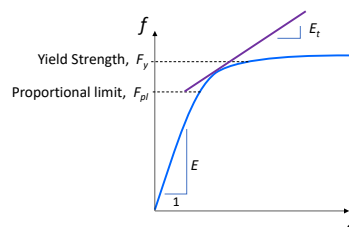
Chapter 4 – Column Theory

- Early researchers soon found that Euler's equation **did not give reliable results** for stocky, or less slender, compression members.
- The reason is that the **small slenderness** ratio for members of this type causes a large **buckling stress**.
- If the stress at which buckling occurs is **greater than the proportional limit of the material**, the relation between stress and strain is **not linear**, and the modulus of elasticity **E** can no longer be used.

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Chapter 4 – Column Theory

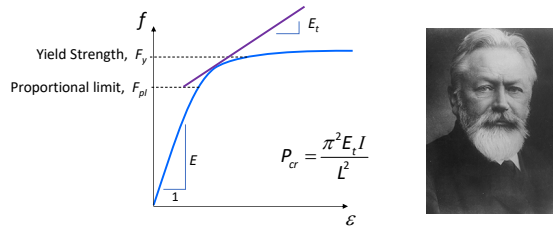
- This difficulty was resolved by Friedrich Engesser, who proposed in 1889 the use of a variable tangent modulus, **E_t**.
- Consider this **compression stress–strain** curve for a stub column.



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Chapter 4 – Column Theory

- E is not a constant for stresses greater than the proportional limit F_{pl} .
- The tangent modulus E_t is defined as the slope of the tangent to the stress-strain curve for values of f between F_{pl} and F_y .



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Chapter 4 – Column Theory

- The stress-strain curve we just discussed is different from those shown earlier for ductile steel.
- There is a pronounced region of **nonlinearity**.
- This curve is typical of a **compression test** of a short length of W-shape called a **stub column**, rather than the result of a tensile test.
- The **nonlinearity** is primarily because of the presence of residual stresses in the W-shape.
- When a hot-rolled shape cools after rolling, all elements of the cross-section do not cool at the same rate and develop **residual stresses**.

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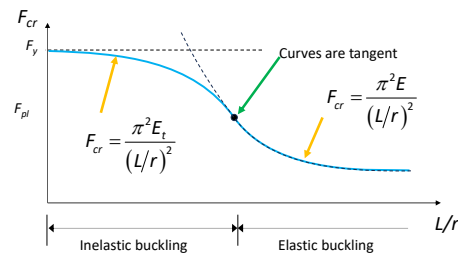
Chapter 4 – Column Theory

- Note that E_t is smaller than E , and for the same L/r , it corresponds to a smaller critical load, P_{cr} .
- Because of the variability of E_t computation of P_{cr} in the inelastic range is difficult.
- In general, a **trial-and-error approach** must be used, and a compressive stress-strain curve must be used to determine E_t for trial values of P_{cr} .
- For this reason, most design specifications, including the **AISC Specification**, contain empirical formulas for **inelastic columns**.

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Chapter 4 – Column Theory

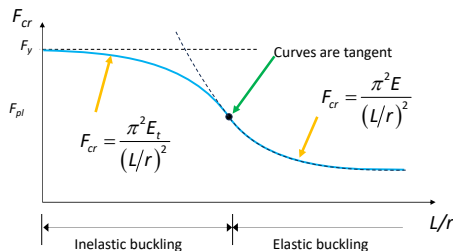
- For any material, the **critical buckling stress** can be plotted as a function of **slenderness**.



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Chapter 4 – Column Theory

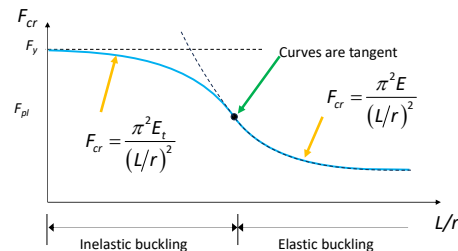
- The tangent modulus curve is tangent to the Euler curve at the point corresponding to the **proportional limit** of the material.



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Chapter 4 – Column Theory

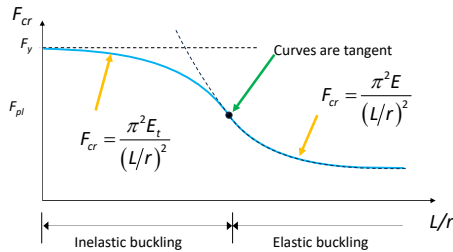
- The composite curve, called a **column strength curve**, completely describes the strength of any column of a given material.



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Chapter 4 – Column Theory

- Other than F_y , E , and E_p , which are properties of the material, the strength is a function only of the slenderness ratio L/r .



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Chapter 4 – Effective Length

- Both the **Euler** and tangent **modulus** equations are based on the following assumptions:
 1. The column is perfectly straight.
 2. The load is axial, with no eccentricity.
 3. The column is pinned at both ends.
- The first two conditions mean that there is **no bending moment in the member before buckling**.
- The requirement for pinned ends, however, is a **serious limitation**, and provisions must be made for other support conditions.

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Chapter 4 – Effective Length

- In general, the bending moment will be a function of x , resulting in a nonhomogeneous differential equation.
- The boundary conditions will be different from those in the original derivation, but the overall procedure will be the same.
- The form of the resulting equation for P_{cr} will also be the same.

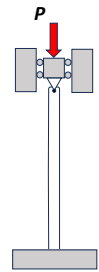
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Chapter 4 – Effective Length

- For example, consider a compression member pinned at one end and fixed against rotation and translation at the other.
- The Euler equation for this case can be derived in the same manner as before.

$$P_{cr} = \frac{2.05\pi^2 EI}{L^2}$$

$$P_{cr} = \frac{\pi^2 EI}{(0.70 L/r)^2}$$



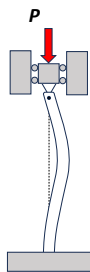
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Chapter 4 – Effective Length

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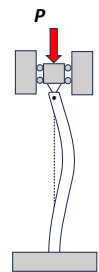
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Chapter 4 – Effective Length

- For example, consider a compression member pinned at one end and fixed against rotation and translation at the other.
- This member has the same load capacity as a column that is pinned at both ends and is only 70% as long.

$$P_{cr} = \frac{2.05\pi^2 EI}{L^2}$$

$$P_{cr} = \frac{\pi^2 EI}{(0.70 L/r)^2}$$



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Chapter 4 – Effective Length

➤ Equations for the critical buckling load will be written as:

$$P_{cr} = \frac{\pi^2 EI}{(KL/r)^2} \quad P_{cr} = \frac{\pi^2 E_t I}{(KL/r)^2}$$

where KL is the effective length and K is the effective length factor.

➤ Values of K for these and other cases can be determined with the aid of **Table C-A-7.1** in the Commentary to AISC Specification Appendix 7.

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Chapter 4 – Effective Length

TABLE C-A-7.1
Approximate Values of Effective Length Factor, K

Buckled shape of column is shown by dashed line	(a)	(b)	(c)	(d)	(e)	(f)
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.8	1.0	1.2	2.1	2.0
End condition code	Rotation fixed and translation fixed Rotation free and translation fixed Rotation fixed and translation free Rotation free and translation free Rotation fixed, horizontal translation fixed, and vertical translation free Rotation free, horizontal translation fixed, and vertical translation free					

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Chapter 4 – Effective Length

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Chapter 4 – AISC Requirements

➤ The nominal compressive strength is: $P_n = F_n A_g$

➤ For **LRFD**, $P_u \leq \phi_c P_n$

where P_u is the sum of the factored loads,
 ϕ_c is the resistance factor for compression = 0.90
 $\phi_c P_n$ is the design strength

➤ For **ASD**, $P_a \leq \frac{P_n}{\Omega_c}$

where P_a is the sum of the service loads,
 Ω_c is the safety factor for compression = 1.67
 P_n/Ω_c is the allowable strength

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Chapter 4 – AISC Requirements

➤ To get the **AISC** expressions for the nominal stress F_n , we first define the Euler load as:

$$P_e = \frac{\pi^2 EA}{(L_c/r)^2}$$

where L_c is the effective length, $L_c = KL$

$$F_e = \frac{P_e}{A} = \frac{\pi^2 E}{(L_c/r)^2}$$

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Chapter 4 – AISC Requirements

➤ To obtain the nominal stress for **elastic columns**, the elastic buckling stress, F_e , is reduced as follows to account for the effects of initial crookedness:

$$F_n = 0.877 F_e$$

➤ For **inelastic columns**, the tangent modulus equation is replaced by the exponential equation

$$F_n = \left(0.658^{F_y/F_e} \right) F_y$$

➤ Using this relationship, a direct solution for inelastic columns can be obtained, avoiding the trial-and-error approach

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Chapter 4 – AISC Requirements

- At the boundary between inelastic and elastic columns, the last two equations give the same value of F_n .
- This occurs when L_c/r is approximately $4.71\sqrt{E/F_y}$
- In summary:

$$\frac{L_c}{r} \leq 4.71\sqrt{E/F_y} \quad F_n = (0.658^{F_y/F_e})F_y$$

$$\frac{L_c}{r} > 4.71\sqrt{E/F_y} \quad F_n = 0.877F_e$$

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Chapter 4 – AISC Requirements

- The **AISC Specification** provides for separating inelastic and elastic behavior based on either the value of L_c/r or the value of the ratio F_y/F_e .

$$\frac{L_c}{r} = \sqrt{\frac{\pi^2 E}{F_e}}$$
- For:

$$\frac{L_c}{r} \leq 4.71\sqrt{E/F_y} \quad \text{or} \quad \frac{F_y}{F_e} \leq 2.25$$

$$F_n = (0.658^{F_y/F_e})F_y$$

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Chapter 4 – AISC Requirements

- The **AISC Specification** provides for separating inelastic and elastic behavior based on either the value of L_c/r or the value of the ratio F_y/F_e .

$$\frac{L_c}{r} = \sqrt{\frac{\pi^2 E}{F_e}}$$
- For:

$$\frac{L_c}{r} > 4.71\sqrt{E/F_y} \quad \text{or} \quad \frac{F_y}{F_e} > 2.25$$

$$F_n = 0.877F_e$$

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Chapter 4 – AISC Requirements

- The **AISC Specification** provides for separating inelastic and elastic behavior based on either the value of L_c/r or the value of the ratio F_y/F_e .

$$F_e = \frac{\pi^2 E}{(L_c/r)^2}$$

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Chapter 4 – AISC Requirements

- The basic requirements for compression members are covered in **Chapter E of the AISC Specification**.

CHAPTER E
DESIGN OF MEMBERS FOR COMPRESSION

The figure addresses members subjected to axial compression.

The figure is organized as follows:

- E1. General Provisions
- E2. Effective Length
- E3. Flexural Buckling of Members Without Slender Elements
- E4. Torsional and Flexural-Torsional Buckling of Single Angles and Members With Slender Elements
- E5. Single-Angle Compression Members
- E6. Built-Up Members
- E7. Members With Slender Elements

For design of members subjected to the figure, the following section apply:

- E1.02 Members subjected to combined axial compression and flexure
- E1.03 Members subjected to axial compression and torsion
- E1.04 Compression strength of built-up members
- E1.05 Compression strength of connecting elements

E1. GENERAL PROVISIONS

The design compressive strength, $\phi_c P_n$, and the allowable compressive strength, P_n/Ω_c , are determined as follows:

The nominal compressive strength, P_n , shall be the lesser value obtained based on the applicable limit state of flexural buckling, torsional buckling, and flexural-torsional buckling.

$\phi_c = 0.90$ (LRFD) $\Omega_c = 1.67$ (ASD)

TABLE USER NOTE E1.1
Selection Table for the Application of Chapter E Sections

Cross Section	Effective Length Factor, K	Limit State	Member Class	Limit State
I	0.5	FB	E1	1.0
				1.2
C	0.5	FB	E1	1.0
				1.2
H	0.5	FB	E1	1.0
				1.2
T	0.5	FB	E1	1.0
				1.2
L	0.5	FB	E1	1.0
				1.2
S	0.5	FB	E1	1.0
				1.2
Z	0.5	FB	E1	1.0
				1.2
M	0.5	FB	E1	1.0
				1.2
P	0.5	FB	E1	1.0
				1.2
R	0.5	FB	E1	1.0
				1.2
S	0.5	FB	E1	1.0
				1.2
Z	0.5	FB	E1	1.0
				1.2
M	0.5	FB	E1	1.0
				1.2
P	0.5	FB	E1	1.0
				1.2
R	0.5	FB	E1	1.0
				1.2

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Chapter 4 – AISC Requirements

- The basic requirements for compression members are covered in **Chapter E of the AISC Specification**.

E2. EFFECTIVE LENGTH

The effective length, L_c , for calculation of member slenderness, L_c/r , shall be determined in accordance with Chapter C or Appendix I.

where:

- L_c = effective length of member, in. (mm)
- r = radius of gyration, in. (mm)

E3. FLEXURAL BUCKLING OF MEMBERS WITHOUT SLENDER ELEMENTS

This section applies to members without slender compression members, as defined in Section E1.1 for elements in axial compression.

The nominal compressive strength, P_n , shall be determined based on the limit state of flexural buckling:

$$P_n = F_y A_g \quad (E3-1)$$

The nominal stress, F_n , is determined as follows:

$$F_n = F_y \quad (E3-2)$$

When $\frac{L_c}{r} > 4.71\sqrt{\frac{E}{F_y}}$ or $\frac{F_y}{F_e} > 2.25$:

$$F_n = (0.658^{F_y/F_e})F_y \quad (E3-3)$$

When $\frac{L_c}{r} > 4.71\sqrt{\frac{E}{F_y}}$ or $\frac{F_y}{F_e} > 2.25$:

$$F_n = 0.877F_e \quad (E3-4)$$

E4. TORSIONAL AND FLEXURAL-TORSIONAL BUCKLING OF SINGLE ANGLES AND MEMBERS WITHOUT SLENDER ELEMENTS

This section applies to single members and built-up members, including double angles, without slender compression members, with an unbraced length L_c and an unbraced length L_p that are less than or equal to the unbraced length L_u and the unbraced length L_p , respectively, of the member. These provisions also apply to single angles with $L_c > 4.71\sqrt{E/F_y}$ when the value of the unbraced length L_u is less than or equal to the unbraced length L_p .

The nominal compressive strength, P_n , shall be determined based on the limit state of torsional and flexural-torsional buckling:

$$P_n = F_y A_g \quad (E4-1)$$

For double angles, members connected along the shear center:

$$P_n = F_y A_g \quad (E4-2)$$

For double angles, members connected along the shear center when the axis of symmetry is in the plane of the member:

$$P_n = F_y A_g \left[1 - \frac{0.00078(L_c/r)^2}{F_y/F_e} \right] \quad (E4-3)$$

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Chapter 4 – AISC Requirements

- **AISC Equations E3-2 and E3-3** are condensed versions of five equations that cover five ranges of KL/r .
- These equations are based on experimental and theoretical studies that account for the effects of **residual stresses** and an initial **out-of-straightness of $L/1500$** .
- Although **AISC** does not require an upper limit on the slenderness ratio, L_c/r , an upper limit of 200 is recommended (see user note in **AISC E2**).
- This is a practical upper limit, because compression members that are any **more slender** will have little strength and **will not be economical**.

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Chapter 4 – AISC Requirements

- **Example 4-2:** A **W14 x 53** of **A992** steel ($F_y=50$ ksi and $F_u=65$ ksi) has a length of 15 ft, and the ends are pinned.
- Compute the design compressive strength for **LRFD** and the allowable compressive strength for **ASD**.

$$L_c = 1.0(15 \text{ ft})(12 \text{ in / ft}) = 180 \text{ in}$$

$$r_y = 1.92 \text{ in}$$

$$A_g = 15.6 \text{ in}^2$$

$$\frac{L_c}{r} = \frac{180 \text{ in}}{1.92 \text{ in}} = 93.75 < 200$$

OK

$$4.71 \sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} = 113.4$$

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Chapter 4 – AISC Requirements

- **Example 4-2:** A **W14 x 53** of **A992** steel ($F_y=50$ ksi and $F_u=65$ ksi) has a length of 15 ft, and the ends are pinned.

$$\frac{L_c}{r} \leq 4.71 \sqrt{\frac{E}{F_y}} \Rightarrow F_n = (0.658^{F_y/F_e}) F_y$$

$$F_e = \frac{\pi^2 E}{(L_c/r)^2} = \frac{\pi^2 (29,000 \text{ ksi})}{(93.75)^2} = 32.57 \text{ ksi}$$

$$F_n = (0.658^{50 \text{ ksi}/32.57 \text{ ksi}})(50 \text{ ksi}) = 26.30 \text{ ksi}$$

$$P_n = F_n A_g = 26.30 \text{ ksi}(15.6 \text{ in}^2) = 410.28 \text{ k}$$

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Chapter 4 – AISC Requirements

- **Example 4-2:** A **W14 x 53** of **A992** steel ($F_y=50$ ksi and $F_u=65$ ksi) has a length of 15 ft, and the ends are pinned.

- **LRFD Solution:** The design strength is:

$$\phi_c P_n = 0.90(410.28 \text{ k}) = 369.3 \text{ k}$$

- **ASD Solution:** The allowable stress is:

$$F_a = 0.6 F_{cr} = 0.6(26.30 \text{ ksi}) = 15.78 \text{ ksi}$$

- The allowable strength is:

$$P_a = F_a A_g = 15.78 \text{ ksi}(15.6 \text{ in}^2) = 246.17 \text{ k}$$

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Chapter 4 – AISC Requirements

- In the example, $r_y < r_x$, and there is excess strength in the **x-direction**.
- **Square structural tubes (HSS)** are efficient shapes for compression members because $r_y = r_x$, and the strength is the same for both axes.
- **Hollow circular shapes** are sometimes used as compression members for the same reason.

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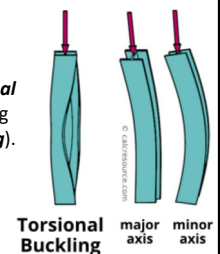
Chapter 4 – AISC Requirements

- The mode of failure considered so far is referred to as **flexural buckling**.

- When a member is subjected to **flexure**, or bending, it becomes unstable.

- For some cross-sectional configurations, the member will fail by twisting (**torsional buckling**) or by a combination of twisting and bending (**flexural-torsional buckling**).

- We consider these infrequent cases in **Section 4.8**.



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Chapter 4 – AISC Requirements

Let's work on some problems



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Chapter 4 - Structural Steel Design

Any questions?



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