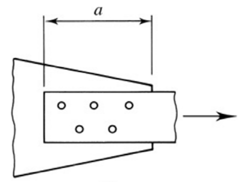
Chapter 3 Tension Members

1

Chapter 3.4 – Staggered Fasteners

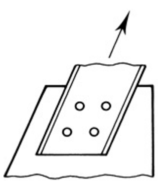
- If a tension member connection is made with bolts, the net area will be maximized if the fasteners are ***placed in a single line***.
- Sometimes space limitations, such as a limit on dimensions, necessitate using ***more than one line***.
- If so, the reduction in cross-sectional area is minimized if the fasteners are arranged in a ***staggered pattern***.

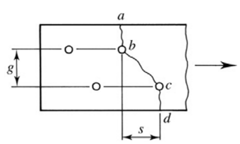


2

Chapter 3.4 – Staggered Fasteners

- Sometimes ***staggered fasteners*** are required by the geometry of a connection, such as the one shown below.
- In either case, any cross-section passing through holes will pass through fewer holes than ***if the fasteners are not staggered***.

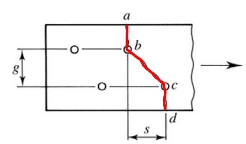




3

Chapter 3.4 – Staggered Fasteners

- If the amount of stagger is small enough, the influence of an offset hole may be felt by a nearby cross-section, and rupture along an inclined path, such as ***abcd*** is possible
- Note that ***ab*** and ***cd*** are the shortest distances to the edge of the member, as the required energy to fail the member shall be the minimum.

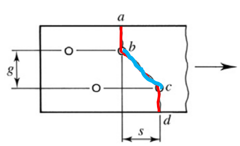


4

Chapter 3.4 – Staggered Fasteners

- In such a case, the relationship *f* = *P/A* does not apply, and stresses on the inclined portion ***bc*** are a combination of ***tensile*** and ***shearing stresses***.
- Cochrane (1922) proposed that when deducting the area corresponding to a staggered hole, ***use a reduced diameter***, ***d'*** given by:

$$d' = d - \frac{s^2}{4g}$$



5

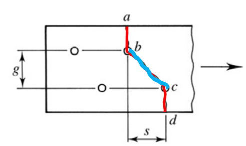
Chapter 3.4 – Staggered Fasteners

- This means that in a failure pattern consisting of both staggered and unstaggered holes, use:

  d for holes at the end of a transverse line between holes ***s*** = 0, and

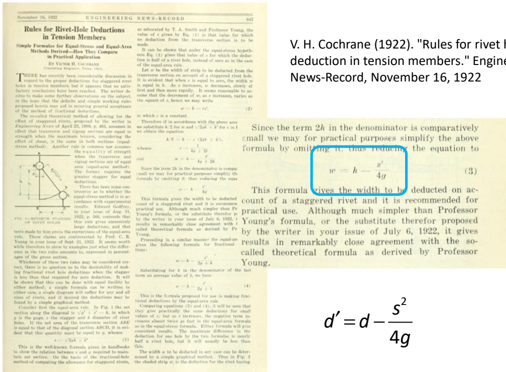
  d' for holes at the end of an inclined line between holes.

$$d' = d - \frac{s^2}{4g}$$



6

Chapter 3.4 – Staggered Fasteners


Since the term 2h in the denominator is comparatively small we may for practical purposes simplify the above formula by omitting it, thus reducing the equation to

$$w = h - \frac{s^2}{4g} \quad (3)$$

This formula gives the width to be deducted on account of a staggered rivet and it is recommended for practical use. Although much simpler than Professor Young's formula, or the substitute therefor proposed by the writer in your issue of July 6, 1922, it gives results in remarkably close agreement with the so-called theoretical formula as derived by Professor Young.

V. H. Cochrane (1922). "Rules for rivet hole deduction in tension members." Engineering News-Record, November 16, 1922

$$d' = d - \frac{s^2}{4g}$$

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Chapter 3.4 – Staggered Fasteners

- The ***AISC Specification***, in ***Section B4.3b***, uses this approach, but in a modified form.
- If the ***net area*** is treated as the product of a ***thickness*** times a ***net width***, and the diameter ***d'*** is used for all holes (since ***d'=d*** when the stagger ***s* = 0**).
- The ***net width*** in a failure line consisting of both staggered and unstaggered holes is

$$w_n = w_g - \sum d' = w_g - \sum\left(d - \frac{s^2}{4g}\right) = w_g - \sum(d) + \sum\left(\frac{s^2}{4g}\right)$$

$$A_n = A_g - \sum t(d) + \sum t\left(\frac{s^2}{4g}\right)$$

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Chapter 3.4 – Staggered Fasteners

- When more than one failure pattern is conceivable, all possibilities should be investigated, and the one corresponding to ***the smallest load capacity*** should be used.
- Note that this method will not accommodate failure patterns with lines ***parallel to the applied load***.

$$w_n = w_g - \sum d' = w_g - \sum\left(d - \frac{s^2}{4g}\right) = w_g - \sum(d) + \sum\left(\frac{s^2}{4g}\right)$$

$$A_n = A_g - \sum t(d) + \sum t\left(\frac{s^2}{4g}\right)$$

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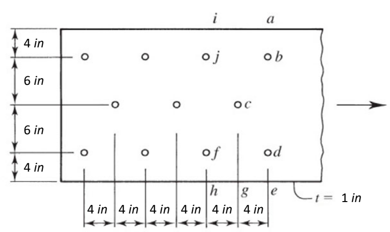
Chapter 3.4 – Staggered Fasteners

- **Example 3.6**: Compute the smallest net area for the plate shown below. The holes are for 1-in.-diameter bolts.

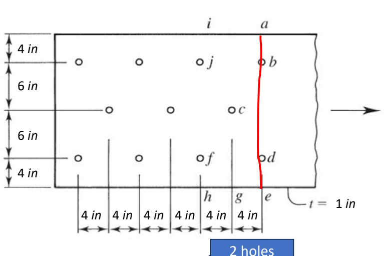


$$d_{hole} = d_{bolt} + \frac{3}{16} \quad \text{for } d_{bolt} \geq 1\,in \qquad d_{hole} = 1 + \frac{3}{16}in = 1\frac{3}{16}in$$

10

Chapter 3.4 – Staggered Fasteners

- **Example 3.6** : For line ***abde***:

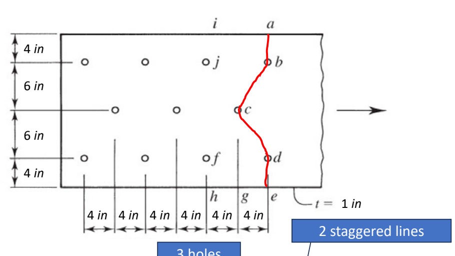


$$w_n = w_g - \sum(d) = 20in - 2(1\tfrac{3}{16}\,in) = \boxed{17.624\,in}$$

11

Chapter 3.4 – Staggered Fasteners

- **Example 3.6** : For line ***abcde***:



$$w_n = w_g - \sum(d) + \sum\left(\frac{s^2}{4g}\right) = 20\,in - 3(1\tfrac{3}{16}\,in) + 2\left(\frac{(4\,in)^2}{4(6\,in)}\right)$$

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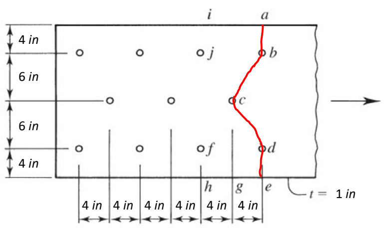
Chapter 3.4 – Staggered Fasteners

- **Example 3.6** : For line ***abcde***:



$$w_n = w_g - \sum(d) + \sum\left(\frac{s^2}{4g}\right) = \boxed{17.77\ in}$$

The first condition will give the smallest net area.

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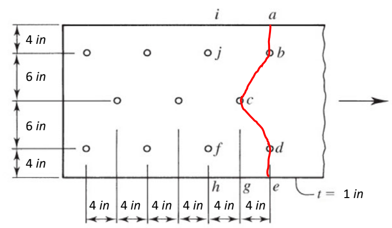
Chapter 3.4 – Staggered Fasteners

- **Example 3.6** : For line ***abcde***:



$$A_n = tw_n = (1\ in)(17.624\ in) = \boxed{17.624\ in^2}$$

The first condition will give the smallest net area.

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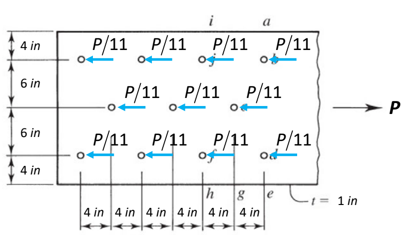
Chapter 3.4 – Staggered Fasteners

- As each fastener resists an ***equal share*** of the load (an assumption used in the design of simple connections), different potential failure lines may be subjected to different loads.
- For example, there are 11 bolts in the plate below.



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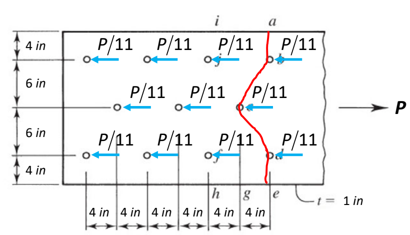
Chapter 3.4 – Staggered Fasteners

- As each fastener resists an ***equal share*** of the load (an assumption used in the design of simple connections), different potential failure lines may be subjected to different loads.
- For example, consider line ***abcde***.



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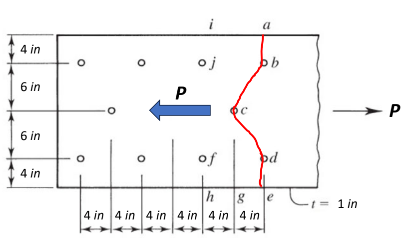
Chapter 3.4 – Staggered Fasteners

- As each fastener resists an ***equal share*** of the load (an assumption used in the design of simple connections), different potential failure lines may be subjected to different loads.
- The line ***abcde*** must resist the full load.



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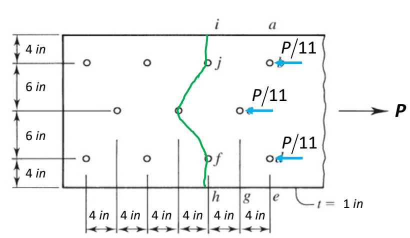
Chapter 3.4 – Staggered Fasteners

- As each fastener resists an ***equal share*** of the load (an assumption used in the design of simple connections), different potential failure lines may be subjected to different loads.
- Consider the line ***ijfh*** in the plate.



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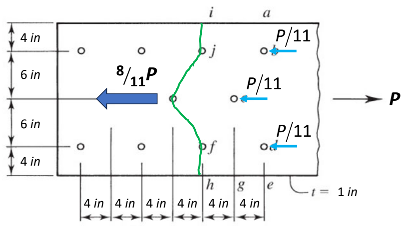
Chapter 3.4 – Staggered Fasteners

- As each fastener resists an ***equal share*** of the load (an assumption used in the design of simple connections), different potential failure lines may be subjected to different loads.
- The section ***ijfh*** will be subjected to 8/11 of the applied load.



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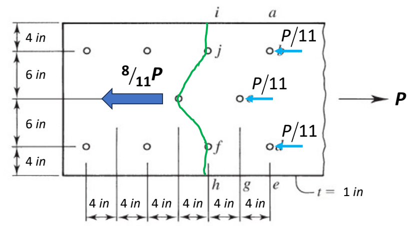
Chapter 3.4 – Staggered Fasteners

- As each fastener resists an ***equal share*** of the load (an assumption used in the design of simple connections), different potential failure lines may be subjected to different loads.
- The reason is that 3/11 of the load will have been transferred from the member before ***ijfh*** receives any load.



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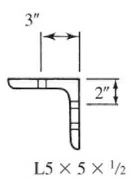
Chapter 3.4 – Staggered Fasteners

- When lines of bolts are present in more than one element of the cross section of a rolled shape, and the bolts in these lines are staggered with respect to one another, the ***reduced diameter*** equation is preferred.

$$d' = d - \frac{s^2}{4g}$$



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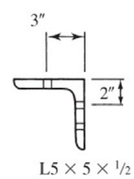
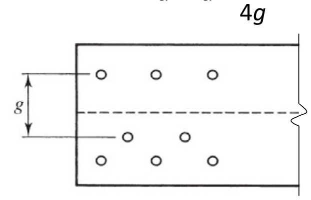
Chapter 3.4 – Staggered Fasteners

- If the shape is an angle, it can be visualized as a plate formed by "***unfolding***" the legs to identify the ***pitch*** and ***gage*** distances more clearly.

$$d' = d - \frac{s^2}{4g}$$





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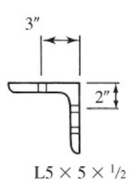
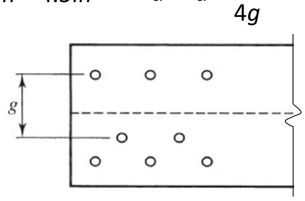
Chapter 3.4 – Staggered Fasteners

- ***AISC B4.3b*** specifies that any gage line crossing the heel of the angle be ***reduced*** by an amount that equals the ***angle thickness***.
- Thus, the distance ***g*** shown below is used in the $s^2/4g$ term

$$g = 3in + 2in - \tfrac{1}{2}in = 4.5in \qquad d' = d - \frac{s^2}{4g}$$





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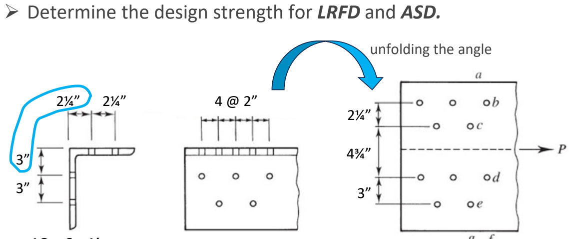
Chapter 3.4 – Staggered Fasteners

- **Example 3.7**: An angle with staggered fasteners in each leg is shown below. Use ***A572, Grade 50*** (F_Y = 50 *ksi* and F_u = 65 *ksi*) steel, and holes are for ¾ in. bolts.
- Determine the design strength for ***LRFD*** and ***ASD.***



$$g = 3in + 2\tfrac{1}{4}in - \tfrac{1}{2}in = 4\tfrac{3}{4}in$$

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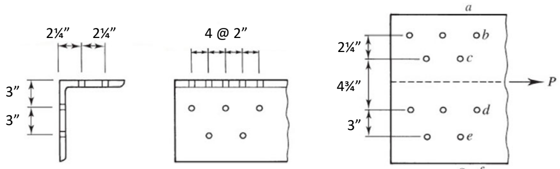
Chapter 3.4 – Staggered Fasteners

- **Example 3.7**: From **Table 1-7**, the gross area is: $A_g = 6.80 in^2$

The effective hole diameter is: $d_{hole} = d_{bolt} + \frac{1}{8}$ for $d_{bolt} < 1\,in$

$$= \frac{3}{4}in + \frac{1}{8}in = \frac{7}{8}in$$



L8 x 6 x ½

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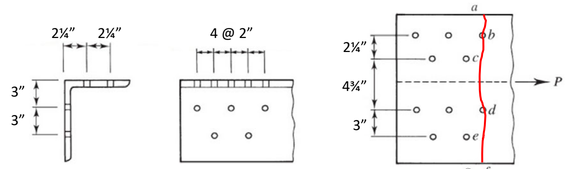
Chapter 3.4 – Staggered Fasteners

- **Example 3.7**: For line ***abdf***, the net area is:

$$A_n = A_g - \sum t_w (d \text{ or } d')$$

$$= 6.80\,in^2 - \left(\tfrac{1}{2}in\right)\left(\tfrac{7}{8}in\right)(2\,holes) = \boxed{5.93 in^2}$$



L8 x 6 x ½

26

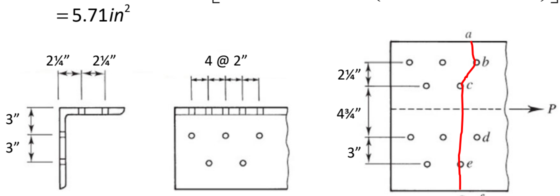
Chapter 3.4 – Staggered Fasteners

- **Example 3.7**: For line ***abceg***, the net area is:

$$A_n = A_g - \sum t_w (d \text{ or } d')$$

$$= 6.80\,in^2 - \left(\tfrac{1}{2}in\right)\left[\left(\tfrac{7}{8}in\right)(2\,holes) + \left(\left(\tfrac{7}{8}in\right) - \frac{(2in)^2}{4(2.25in)}\right)\right]$$

$$= 5.71 in^2$$



L8 x 6 x ½

27

Chapter 3.4 – Staggered Fasteners

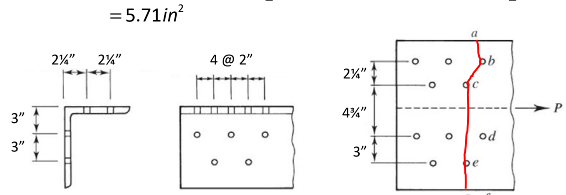
- **Example 3.7**: An alternative approach for ***abceg***, the net area is:

$$A_n = A_g - \sum t_w (d \text{ or } d')$$

3 holes, 1 staggered length

$$= 6.80\,in^2 - \left(\tfrac{1}{2}in\right)\left[\left(\tfrac{7}{8}in\right)(3\,holes) - \frac{(2in)^2}{4(2.25in)}\right]$$

$$= 5.71 in^2$$



L8 x 6 x ½

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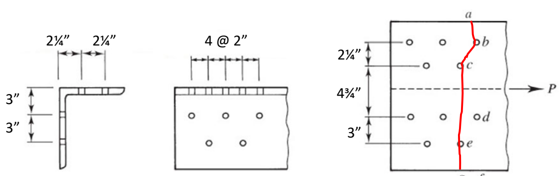
Chapter 3.4 – Staggered Fasteners

- **Example 3.7**: Because 1/10 of the load has been transferred from the member by the fastener at ***d***, this potential failure line must resist only 9/10 of the load.

$$A_n = A_n\left(\tfrac{10}{9}\right) = 5.71 in^2\left(\tfrac{10}{9}\right) = \boxed{6.34 in^2}$$



L8 x 6 x ½

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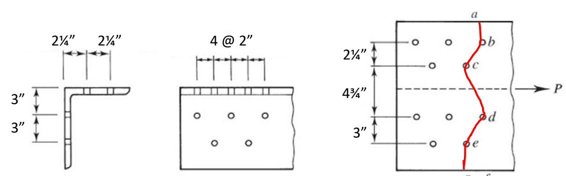
Chapter 3.4 – Staggered Fasteners

- **Example 3.7**: For line ***abcdeg***, the net area is:

$$A_n = A_g - \sum t_w (d \text{ or } d')$$



L8 x 6 x ½

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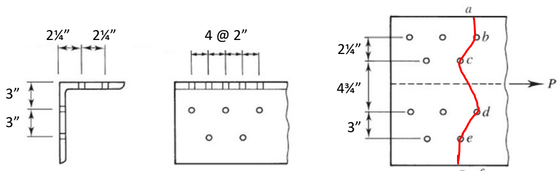
Chapter 3.4 – Staggered Fasteners

➢ **Example 3.7**: For line ***abcdeg***, the net area is:

$$A_n = 6.80\ in^2 - \left(\tfrac{1}{2}in\right)\left[\underset{b}{\left(\tfrac{7}{8}in\right)} + \underset{c}{\left(\left(\tfrac{7}{8}in\right) - \tfrac{(2in)^2}{4(2.25in)}\right)} + \underset{d}{\left(\left(\tfrac{7}{8}in\right) - \tfrac{(2in)^2}{4(4.75in)}\right)} + \underset{e}{\left(\left(\tfrac{7}{8}in\right) - \tfrac{(2in)^2}{4(3in)}\right)}\right] = \boxed{5.54\,in^2}$$



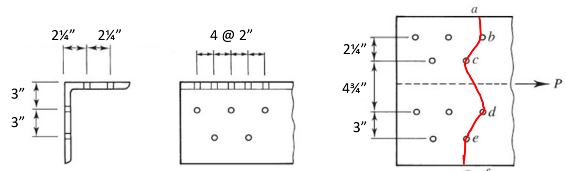
L8 x 6 x ½

Chapter 3.4 – Staggered Fasteners

➢ **Example 3.7**: Alternative approach for ***abcdeg***, the net area is:

$$A_n = 6.80\ in^2 - \left(\tfrac{1}{2}in\right)\left[\underbrace{4\ holes\left(\tfrac{7}{8}in\right)}_{\text{4 holes}} - \underbrace{\tfrac{(2in)^2}{4(2.25in)} - \tfrac{(2in)^2}{4(4.75in)} - \tfrac{(2in)^2}{4(3in)}}_{\text{3 staggered lengths}}\right]$$

$$= \boxed{5.54\,in^2}$$



L8 x 6 x ½

Chapter 3.4 – Staggered Fasteners

➢ **Example 3.7**: The last case controls: $A_n = \boxed{5.54\,in^2}$

Both legs of the angle are connected, so

$$A_e = A_n = 5.54\,in^2$$

The nominal strength based on rupture is

$$P_n = F_u A_e = (65ksi)5.54\,in^2 = \boxed{360.4k}$$

The nominal strength based on yielding is

$$P_n = F_y A_g = (50ksi)6.80\,in^2 = \boxed{340.0k}$$

Chapter 3.4 – Staggered Fasteners

➢ **Example 3.7**: The strength based on rupture using ***LRFD*** is

$$\phi_t P_n = (0.75)360.4k = \boxed{270.3k}$$

The design strength based on yielding using ***LRFD*** is:

$$\phi_t P_n = (0.90)340.0k = \boxed{306.0k}$$

The ***LRFD*** design strength is: $\boxed{270.3\ k}$

Chapter 3.4 – Staggered Fasteners

➢ **Example 3.7**: The limit state of rupture using ***ASD,*** the allowable stress is:

$$F_t = 0.5F_u = (0.5)65\ ksi = 32.5ksi$$

The allowable strength is:

$$F_t A_e = 32.5\ ksi\left(5.54\,in^2\right) = \boxed{180.2k}$$

The limit state of yielding using ***ASD,*** the allowable stress is:

$$F_t = 0.6F_y = (0.6)50\ ksi = 30.0ksi$$

$$F_t A_g = 30.0\ ksi\left(6.80\,in^2\right) = \boxed{204.0k}$$

The ***ASD*** allowable strength is: $\boxed{180.2\ k}$

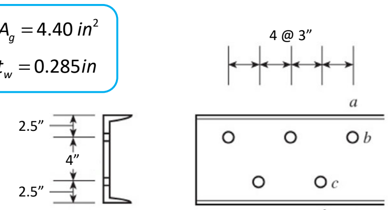
Chapter 3.4 – Staggered Fasteners

➢ **Example 3-8**: Determine the smallest net area for the American Standard Channel shown below. The holes are for ¾-inch diameter bolts.

For $d_{bolt} < 1\ in \quad d_{hole} = d_{bolt} + \frac{1}{8}in = \frac{3}{4}in + \frac{1}{8}in = \frac{7}{8}in$

$A_g = 4.40\ in^2$

$t_w = 0.285in$



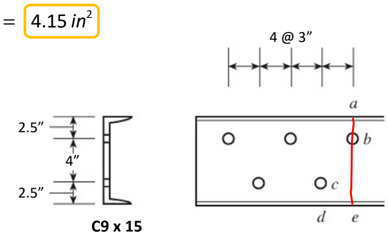
C9 x 15

Chapter 3.4 – Staggered Fasteners

- **Example 3-8**: Determine the smallest net area for the American Standard Channel shown below. The holes are for ¾-inch diameter bolts. For line ***abe***:

$$A_n = A_g - \sum t_w(d \text{ or } d') = 4.40\,in^2 - (0.285\,in)\left(\tfrac{7}{8}\,in\right)$$

$$= 4.15\,in^2$$



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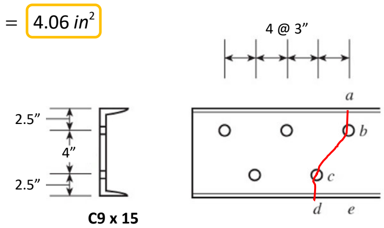
Chapter 3.4 – Staggered Fasteners

- **Example 3-8**: Determine the smallest net area for the American Standard Channel shown below. The holes are for ¾-inch diameter bolts. For line ***abcd***:

$$A_n = A_g - \sum t_w(d \text{ or } d')$$

$$= 4.40\,in^2 - \underbrace{2(0.285in)\left(\tfrac{7}{8}in\right)}_{\text{2 holes}} + \underbrace{\tfrac{(3in)^2}{4(4in)}(0.285in)}_{\text{1 staggered length}}$$

$$= 4.06\,in^2$$



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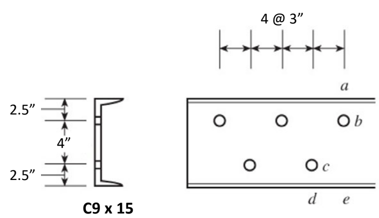
Chapter 3.4 – Staggered Fasteners

- **Example 3-8**: Determine the smallest net area for the American Standard Channel shown below. The holes are for ¾-inch diameter bolts. Smallest net area is:

$$A_n = 4.06\,in^2$$



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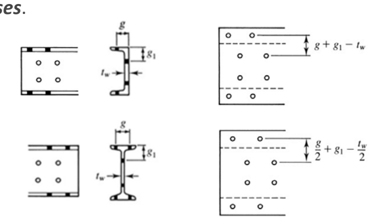
- When staggered holes are present in shapes other than ***angles***, and the holes are in different elements of the cross section, the shape can still be visualized as a ***plate***, even if it is an ***I-shape***.
- The ***AISC Specification*** furnishes no guidance for gage lines crossing a "fold" when the different elements have ***different thicknesses***.



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Chapter 3.4 – Staggered Fasteners

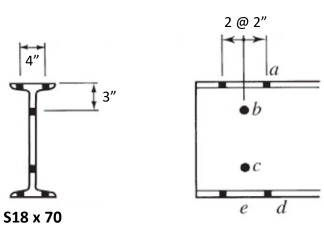
- **Example 3.9**: Find the available strength of the **S-shape** shown below. The holes are for ½-inch bolts. Use **A572, Grade 50** (F_y = 50 *ksi* and F_u = 65 *ksi*) steel.

For $d_{bolt} < 1\,in$ $\quad d_{hole} = d_{bolt} + \frac{1}{8}in = \frac{1}{2}in + \frac{1}{8}in. = \frac{5}{8}in$

$A_g = 20.5\,in^2$

$t_w = 0.711\,in$

$t_f = 0.691\,in$



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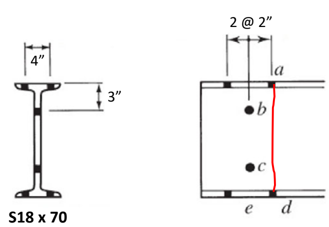
Chapter 3.4 – Staggered Fasteners

- **Example 3.9**: For line ***ad***:

$$A_n = A_g - \sum t_w(d \text{ or } d') = 20.5\,in^2 - (4\,holes)(\overset{t_f}{0.691\,in})\left(\tfrac{5}{8}in\right)$$

$$= 18.77\,in^2$$



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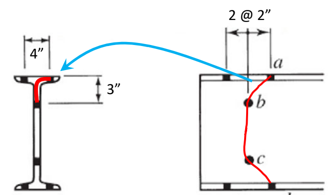
Chapter 3.4 – Staggered Fasteners

- **Example 3.9**: For line ***abcd,*** the gage distance for use in the $s^2/4g$ term is:

$$s = \frac{g}{2} + g_1 - \frac{t_w}{2} = \frac{4\,in}{2} + 3\,in - \frac{0.711\,in}{2} = 4.64\,in$$



S18 x 70

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Chapter 3.4 – Staggered Fasteners

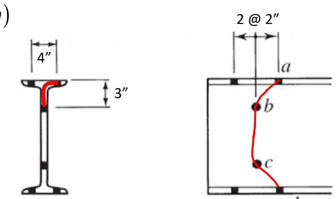
- **Example 3.9**: Starting at ***a*** and treating holes at ***b*** and ***d*** as stagged holes gives:

$$A_n = A_g - \sum t_w (d \text{ or } d') = 20.5\,in^2 - 2\overset{t_f}{(0.691in)}\left(\tfrac{5}{8}in\right)$$

$$-\overset{t_w}{(0.711in)}\left(\tfrac{5}{8}in - \tfrac{(2in)^2}{4(4.64in)}\right) - \overset{t_w}{(0.711in)}\left(\tfrac{5}{8}in\right) - \overset{t_f}{(0.691in)}\left(\tfrac{5}{8}in - \tfrac{(2in)^2}{4(4.64in)}\right)$$

$$-\overset{t_f}{(0.691in)}\left(\tfrac{5}{8}in\right)$$

$$= \boxed{18.33\text{ in}^2}$$



S18 x 70

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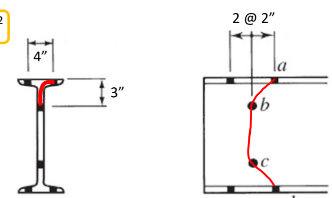
Chapter 3.4 – Staggered Fasteners

- **Example 3.9**: Starting at ***a*** and treating holes at ***b*** and ***d*** as stagged holes gives:

$$A_n = A_g - \sum t_w (d \text{ or } d') = 20.5\,in^2 - 4\ holes\overset{t_f}{(0.691in)}\left(\tfrac{5}{8}in\right)$$

$$-2\ holes\overset{t_w}{(0.711in)}\left(\tfrac{5}{8}in\right) + (0.711in)\overset{ab}{\left(\tfrac{(2in)^2}{4(4.64in)}\right)} + (0.691in)\overset{cd}{\left(\tfrac{(2in)^2}{4(4.64in)}\right)}$$

$$= \boxed{18.33\text{ in}^2}$$



S18 x 70

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Chapter 3.4 – Staggered Fasteners

- **Example 3.9**: The line ***abcd*** controls, and all elements of the cross-section are connected:

$$A_e = A_n = \boxed{18.33\,in^2}$$

The nominal strength based on rupture is

$$P_n = F_u A_e = (65ksi)18.33\,in^2 = 1{,}191.5k$$

The nominal strength based on yielding is

$$P_n = F_y A_g = (50ksi)20.5\,in^2 = 1{,}025.0k$$

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Chapter 3.4 – Staggered Fasteners

- **Example 3.9**: The design strength based on rupture using ***LRFD*** is:

$$\phi_t P_n = (0.75)1{,}191.5k = \boxed{893.6k}$$

The design strength based on yielding using ***LRFD*** is:

$$\phi_t P_n = (0.90)1{,}025.0k = \boxed{922.5k}$$

The ***LRFD*** design strength is: $\boxed{893.6\ k}$

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Chapter 3.4 – Staggered Fasteners

- **Example 3.9**: The limit state of rupture using ***ASD,*** the allowable stress is:

$$F_t = 0.5F_u = (0.5)65\ ksi = 32.5ksi$$

The allowable strength is:

$$F_t A_e = 32.5\ ksi(17.89\ in^2) = \boxed{581.4k}$$

The limit state of yielding using ***ASD,*** the allowable stress is:

$$F_t = 0.6F_y = (0.6)50\ ksi = 30ksi$$

$$F_t A_g = 30\ ksi(20.5\ in^2) = \boxed{615.0k}$$

The ***ASD*** allowable strength is: $\boxed{581.4\ k}$

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Let’s work on some problems.

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Chapter 3.4 – Staggered Fasteners

Any questions?

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