Analysis of Statically Determinate Trusses

In this section, we will discuss the determinacy, stability, and analysis of three forms of statically determinate trusses: **simple**, **compound**, and **complex**. If time allows, we will also consider space trusses.

Common Types Of Trusses

A **truss** is *a structure composed of slender members joined together at their end points.* Planar trusses lie in a single plane. Typically, the joint connections are formed by bolting or welding the end members together to a common plate, called a *gusset plate*.

- **Roof trusses** – in general, the roof load is transmitted to the truss by a series of *purlins*. The roof truss along with its supporting columns is termed a *bent*. The space between bents is called a *bay*.

- **Bridge trusses** – the load is transmitted by the *deck* to a series of *stringers* and then to a set of *floor beams*. The floor beams are supported by two parallel trusses. The supporting trusses are connected top and bottom by *lateral bracing*. Additional stability may be provided by *portal* and *sway* bracing.

- **Assumptions for design** – in order to design both the members and connections of a truss, the *force* in each member for a given loading must be determined. Two important assumptions are made in truss analysis

  1.. *Truss members are connected by smooth pins*. The stress produced in these elements is called the *primary stress*. The pin assumption is valid for bolted or welded connections if the members are concurrent. However, since the connection does provide some rigidity, the bending introduced in the members is called *secondary stress*. Secondary stress analysis is not commonly performed.

  2.. *All loading is applied at the joints of the truss*. Since the weight of each members is small compared to the member force, the member weight is often neglected. However, when the member weight is considered, it is applied at the end of each member.

  Because of these two assumptions, each truss member is a two–force member with either a compressive (C) or a tensile (T) axial force. In general, compression members are bigger to help with instability due to buckling.

Classification Of Coplanar Trusses

- **Simple truss** – the simplest framework that is rigid or stable is a *triangle*. Therefore, a simple truss is constructed starting with a basic triangular element and connecting two members to form additional elements. As each additional element of two members is placed on the a truss, the number of joints is increased by one.

- **Compound truss** – is formed by connecting two or more simple trusses together. This type of truss is often used for large spans. There are three ways in which simple trusses may be connected to form a compound truss:

  1. Trusses may be connected by a common joint and bar.
  2. Trusses may be joined by three bars.
  3. Trusses may be joined where bars of a large simple truss, called the *main truss,* have been substituted by simple trusses, called *secondary trusses*.

- **Complex truss** – is a truss that cannot be classified as being either simple or compound.

- **Determinacy** – since all the elements of a truss are two–force members, the moment equilibrium is automatically satisfied. Therefore there are two equations of equilibrium for each joint, j, in a truss. If r is the number of reactions and b is the number of bar members in the truss, determinacy is obtained by

$b + r = 2j$	**statically determinate**
$b + r > 2j$	**statically indeterminate**

  The degree of indeterminacy is equal to $(b + r) - 2j$

- **Stability** – if $b + r < 2j$, a truss will be *unstable*, which means the structure will collapse since there are not enough reactions to constrain all the joints. A truss may also be unstable if $b + r \geq 2j$. In this case, stability will be determined by inspection.

  1.. **External stability** – a structure (truss) is externally unstable if its reactions are concurrent or parallel.

  2. **Internal stability** – may be determined by inspection of the arrangement of truss members.

     In general, a *simple* truss will always be internally stable. If a truss is not constructed entirely of triangles, it may be unstable.

     The stability of a *compound* truss is determined by examining how the simple trusses are connected.

     The stability of a *complex* truss can often be difficult to determine by inspection. In general, the stability of any truss may be checked by performing a complete analysis of the structure. If a unique solution can be found for the set of equilibrium equations, then the truss is stable.

$b + r < 2j$	**unstable**
$b + r \geq 2j$	**unstable** if reactions are concurrent or parallel or a collapsible mechanism

The Method of Joints

If a truss is in equilibrium, then each of its joints must be in equilibrium. Therefore, the **method of joints** consists of satisfying the equilibrium equations: $\Sigma F_x = 0$ and $\Sigma F_y = 0$ for forces acting on each joint. Recall, that the line of action of a force acting on a joint is determined by the geometry of the truss member. The *line of action* is formed by connecting the two ends of each member with a straight line. Since direction of the force is known, the remaining unknown is the magnitude of the force.

- **Procedure for analysis** – the following is a procedure for analyzing a truss using the method of joints:

  1. First, if possible, determine the support reactions for the entire truss.

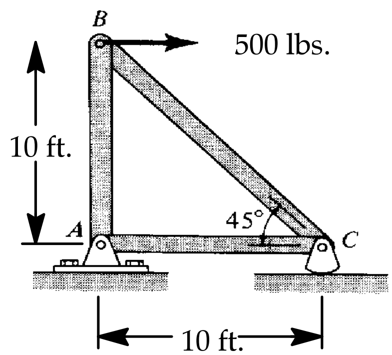
  2. Next, draw the free body diagram for each joint. In general, assume all the force member reactions are **tension** (this is not a rule, however, it is helpful in keeping track of tension and compression members).

  3. Write the equations of equilibrium for each joint, $\Sigma F_x = 0$ and $\Sigma F_y = 0$

  4a. If possible, begin solving the equilibrium equations at a joint where only two unknown reactions exist. Work your way from joint to joint, selecting the new joint using the criterion of two unknown reactions.

  4b. Solve the joint equations of equilibrium simultaneously, typically using a computer or an advanced calculator.

Example: Consider the following truss:



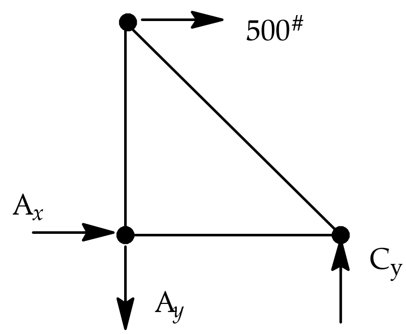
The force in each member is assumed to be in **tension** (T), therefore a negative value will correspond to a **compression** (C) force.

First, determine the support reactions for the entire truss.

$$\sum M_A = 0 = -500(10) + C_y(10) \quad C_y = 500 lbs.$$

$$\sum F_y = 0 = C_y - A_y \quad A_y = 500 lbs.$$

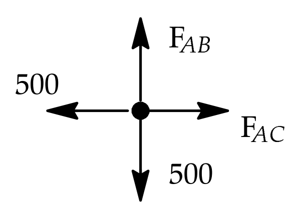
$$\sum F_x = 0 = A_x + 500 \quad A_x = -500 lbs.$$



The equations of equilibrium for joint ***A*** are

$$\sum F_x = 0 = F_{AC} - 500$$

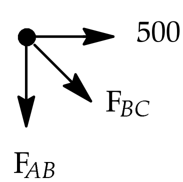
$$\sum F_y = 0 = F_{AB} - 500$$



The equations of equilibrium for joint ***B*** are

$$\sum F_x = 0 = F_{BC} \cos 45^o + 500$$

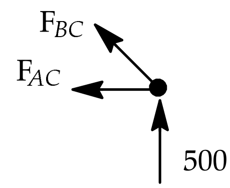
$$\sum F_y = 0 = -F_{AB} - F_{BC} \sin 45^o$$



The equations of equilibrium for joint ***C*** are

$$\sum F_x = 0 = -F_{BC}\cos 45^o - F_{AC}$$

$$\sum F_y = 0 = F_{BC}\sin 45^o + 500$$

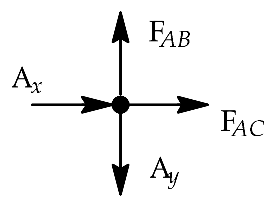


In this case, each set of joint equilibrium equations may be solved independently for two unknown member forces. These equations, including support reactions, may have be written in matrix form as a set of six simultaneous linear equations with six unknowns

The equations of equilibrium for joint ***A*** are

$$\sum F_x = 0 = F_{AC} + A_x$$

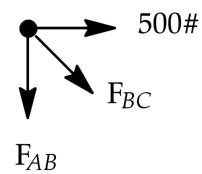
$$\sum F_y = 0 = F_{AB} + A_y$$



The equations of equilibrium for joint ***B*** are

$$\sum F_x = 0 = F_{BC}\cos 45^o + 500$$

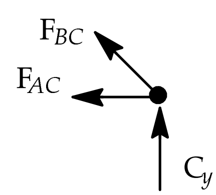
$$\sum F_y = 0 = -F_{AB} - F_{BC}\sin 45^o$$



The equations of equilibrium for joint ***C*** are

$$\sum F_x = 0 = -F_{BC}\cos 45^o - F_{AC}$$

$$\sum F_y = 0 = F_{BC}\sin 45^o + C_y$$



$$\begin{array}{cccccc} A_x & A_y & C_y & F_{AB} & F_{AC} & F_{BC} \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.707 \\ 0 & 0 & 0 & -1 & 0 & -0.707 \\ 0 & 0 & 0 & 0 & -1 & -0.707 \\ 0 & 0 & 1 & 0 & 0 & 0.707 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ C_y \\ F_{AB} \\ F_{AC} \\ F_{BC} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -500 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

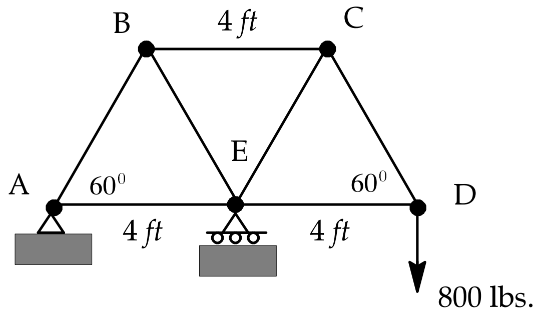
$A_x = -500\ lbs \qquad A_y = 500\ lbs \qquad C_y = 500\ lbs$

$F_{AB} = 500\ lbs \qquad F_{AC} = 500\ lbs \qquad F_{BC} = -707.21\ lbs$

Therefore, the member forces are

$F_{AB} = 500\ lbs\ (Tension) \qquad F_{AC} = 500\ lbs\ (Tension) \qquad F_{BC} = 707.21\ lbs\ (Compression)$

Example: Consider the following truss:



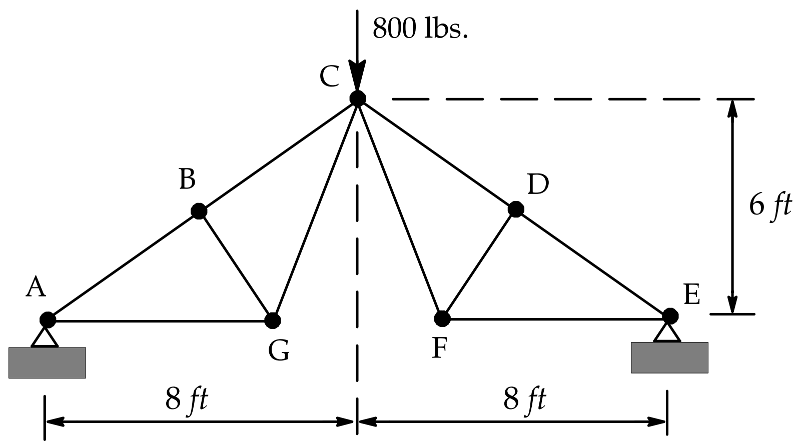
Determine the force in each member of the truss.

Zero–force Members

Truss analysis may be simplified by determining members with no loading or **zero–force.** These members may provide stability or be useful if the loading changes. Zero–force members may be determined by inspection of the joints.

- If only two members are connected at a joint and there is no external load at that joint, both members are zero–force members.
- The other common situation, is when three members connect at a joint with no loading and two of the members are colinear. In this case, the element off the colinear members is a zero–force member.

Example: Consider the following truss:



Determine the force in each member of the truss.

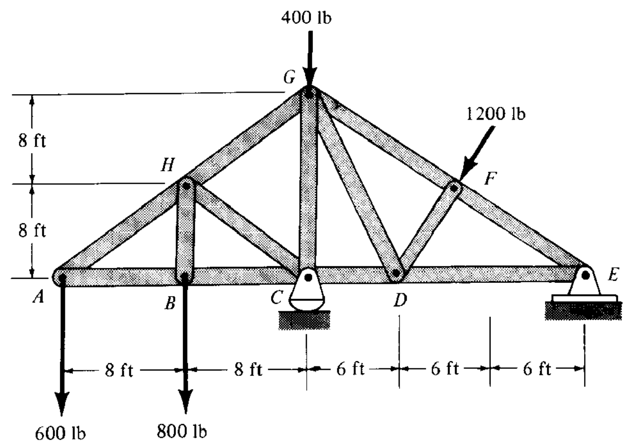
The Method of Sections

If the forces in only a few members of a truss are to be determined, the method of sections is generally the most appropriate analysis procedure. The method of sections consists of passing an *imaginary line* through the truss, cutting it into sections. Each imaginary section must be in equilibrium if the entire truss is in equilibrium. Since there are three independent equations of equilibrium, $\Sigma M_0 = 0$, $\Sigma F_x = 0$, and $\Sigma F_y = 0$ you should try to select a section where no more than three member forces are unknown.

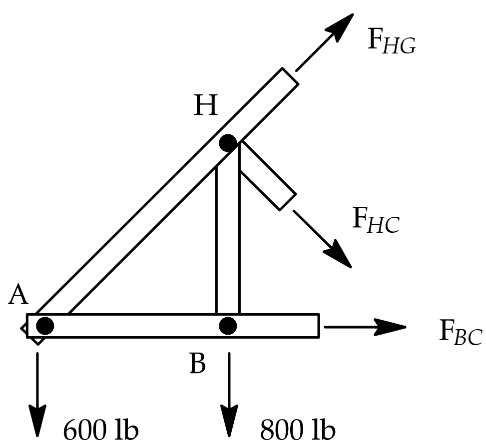
- **Procedure for analysis** – the following is a procedure for analyzing a truss using the method of sections:

  1. First, if necessary, determine the support reactions for the entire truss.
  2. Next, make a decision on how the truss should be "cut" into sections and draw the corresponding free-body diagrams.
  3. Try to apply the three equations of equilibrium such that simultaneous solution **is not** required. Moments should be summed about points that lie at the intersection of the lines of action of two unknown forces, so that the remaining force may be determined.

Example: Consider the following truss:



Find the forces in members HG, HC, and BC. Cut the truss into sections and apply the equations of equilibrium.



$$\sum M_H = 0 = F_{BC}(8) + 600(8) \quad F_{BC} = -600lb$$

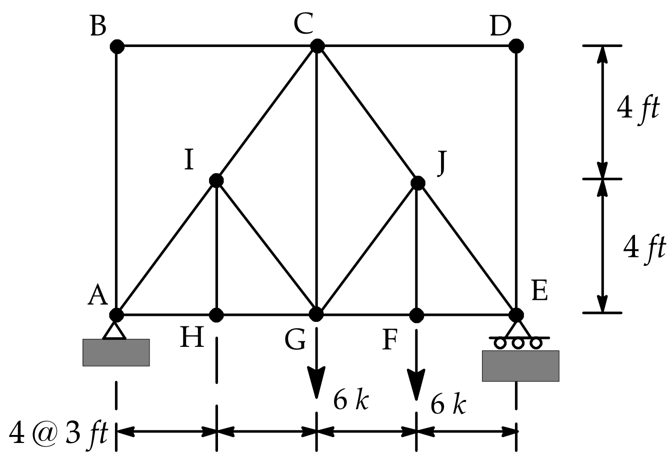
$$\sum M_A = 0 = -F_{HC}(11.31) - 800(8)$$

$$F_{HC} = -565.7lb$$

$$\sum M_C = 0 = -F_{HG}(11.31) + 800(8) + 600(16)$$

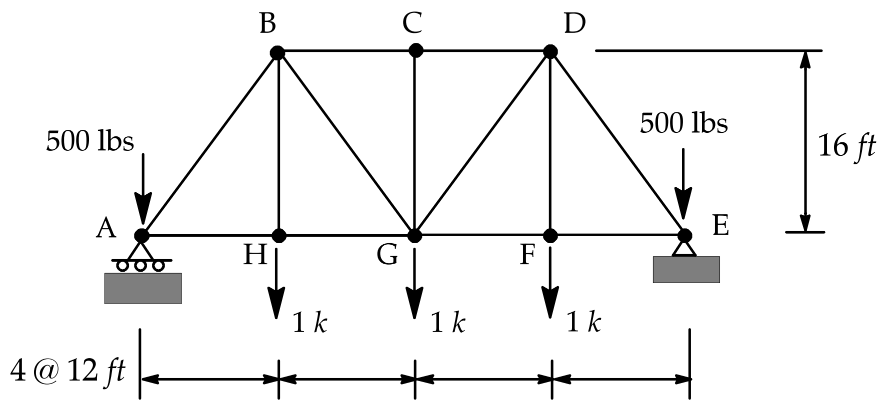
$$F_{HG} = 1414.2lb$$

Example: Consider the following truss:



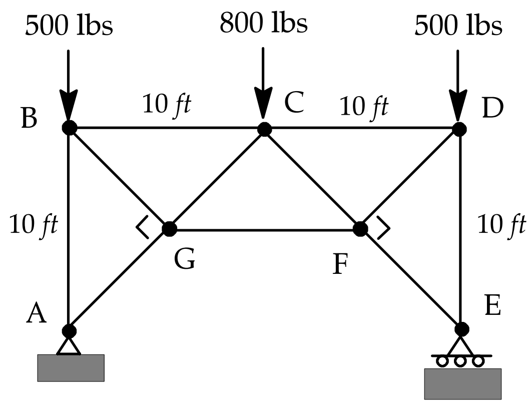
Determine the forces in member IC and CG of the truss.

Example: Consider the following truss:



Determine the forces in member BC, BG, HG, and CG of the truss.

Example: Consider the following truss:



Determine the forces in member BC, CG, and GF of the truss.

Compound Trusses

In general, a compound truss may be best analyzed by using both the method of sections and the method of joints.

- **Procedure for analysis** – first determine the type of compound truss being analyzed and follow the appropriate procedure:

  Type 1 – First, determine the external reactions. Next, use the method of sections to determine the force in the bar connecting the two simple trusses together. Next, analyze the simple trusses using the method of joints.

  Type 2 – First, determine the external reactions. Next, use the method of sections to determine the forces in the three bars connecting the two simple trusses together. Next, analyze the simple trusses using the method of joints.

  Type 3 – First, determine the external reactions. Next, replace the *secondary trusses* by an imaginary member in the *primary* structure (also apply the load on the secondary truss equally to the joints of the primary truss where the imaginary secondary system connects). Next, analyze the primary truss using the method of joints. Apply the idealized member force on the secondary system and solve using the method of joints.

Space Trusses

A *space truss* consists of members joined together to form a stable three–dimensional structure. The simplest element of a stable space truss is a *tetrahedron*.

- **Determinacy and stability** – in three-dimensions, there are three equations of equilibrium at each joint $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma F_z = 0$, therefore:

$b + r < 3j$	**unstable truss**
$b + r = 3j$	**statically determinate – check stability**
$b + r > 3j$	**statically indeterminate – check stability**

- **Procedure for analysis** – the methods and solution procedures discussed for simple and compound coplanar trusses may be applied to space trusses.