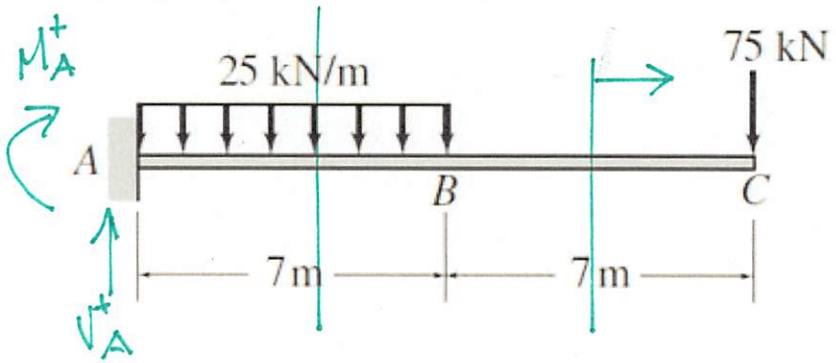
Example 8b-3: Determine the slope and displacement at *C*. Assume $I = 2{,}340\ (10^6)\ \text{mm}^4$ and 70 GPa.

Real loads



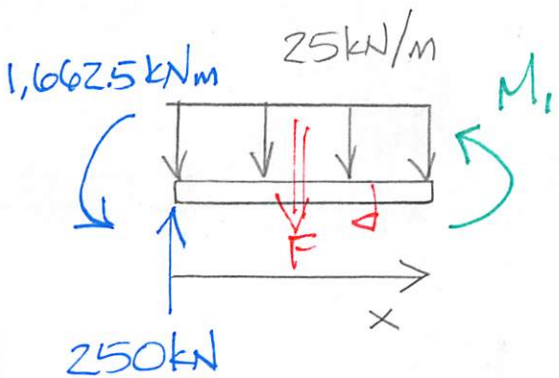
$$\overset{+}{\circlearrowleft}\Sigma M_A = 0 = -M_A - 25\,\text{kN/m}(7\,\text{m})(3.5\,\text{m}) - 75\,\text{kN}(14\,\text{m})$$

$$M_A = -1{,}662.5\,\text{kN m}$$

$$+\uparrow \Sigma F_y = 0 = V_A - 25\,\text{kN/m}(7\,\text{m}) - 75\,\text{kN}$$

$$V_A = 250\,\text{kN}$$

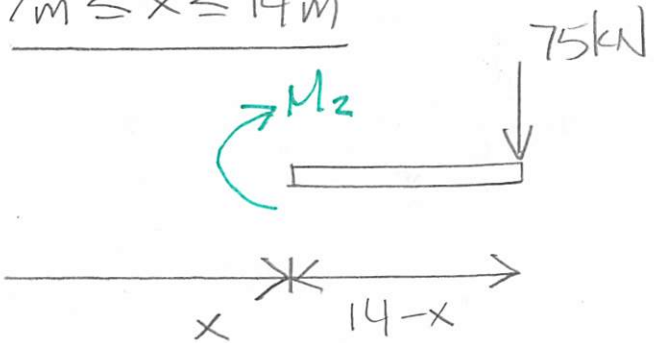
$0 \le x \le 7\,\text{m}$



$$\overset{+}{\circlearrowleft}\Sigma M_{CUT} = 0 = M_1 + 25\,\text{kN/m}(x)\frac{x}{2} + 1{,}662.5\,\text{kNm} - 250x$$

$$M_1(x) = \left[-12.5x^2 + 250x - 1{,}662.5\right]\text{kNm}$$ ✓

$7\,\text{m} \le x \le 14\,\text{m}$

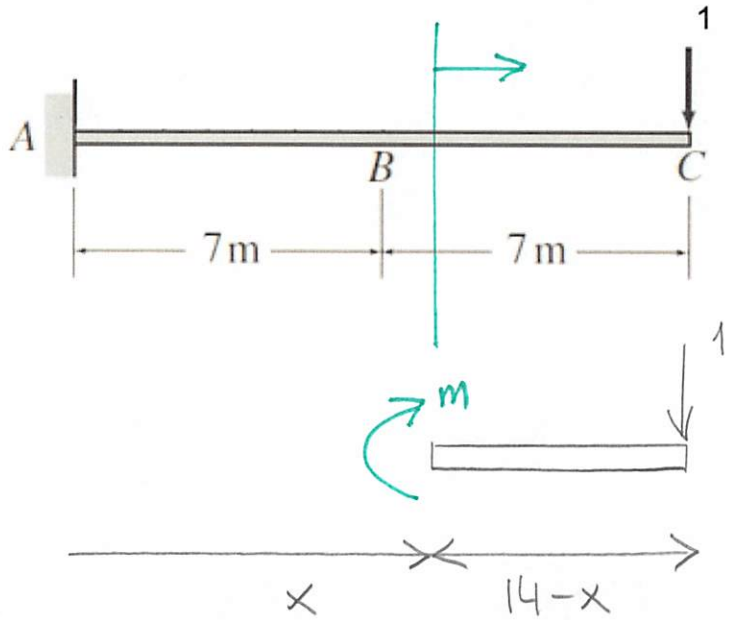


$$\overset{+}{\circlearrowleft}\Sigma M_{CUT} = 0 = -M_2 - 75(14 - x)$$

$$M_2(x) = \left[-75(14 - x)\right]\text{kN m}$$ ✓

Example 8b-3: Determine the slope and displacement at *C*. Assume I = 2,340 (10^6) mm^4 and 70 GPa.

Virtual load



$$\circlearrowleft + \Sigma M_{CUT} = 0 = -m - 1(14-x)$$

$$m = -(14-x)$$

$$Y_C = \frac{1}{EI}\int_0^{14} M m\,dx = \frac{1}{EI}\left[\int_0^{7} M_1 m\,dx + \int_7^{14} M_2 m\,dx\right]$$

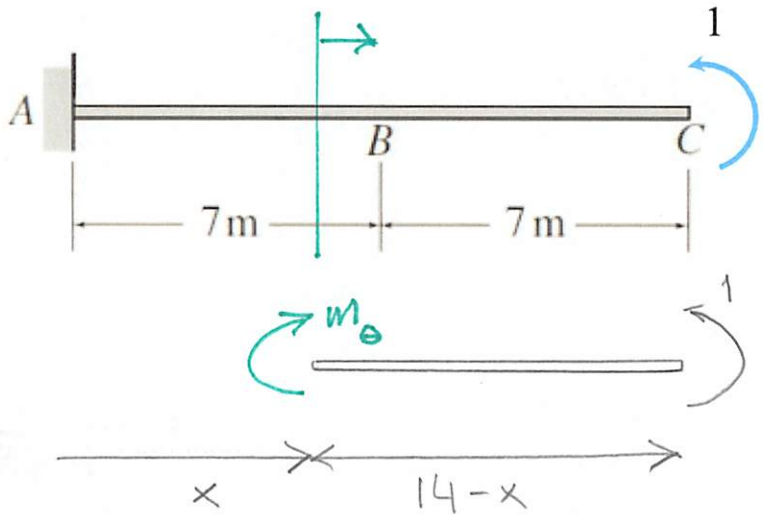
$$= \frac{1}{EI}\left[\left[-\frac{25x^4}{8} + \frac{425x^3}{3} + \frac{10,325x^2}{4} + 23,275x\right]_0^{7} + \left[25x^3 - 1,050x^2 + 14,700x\right]_7^{14}\right]$$

$$= \frac{86,107\,kNm^3}{EI} = \frac{86,107\,kNm^3}{70(10^6)\,kN/m^2 \cdot 2,340(10^6)\,mm^4 \cdot \left(\frac{10^3\,mm}{m}\right)^4}$$

$$= 0.5256\,m$$

Example 8b-3: Determine the slope and displacement at *C*. Assume $I = 2{,}340\ (10^6)\ \text{mm}^4$ and 70 GPa.

Virtual moment



$$\circlearrowleft + \Sigma M_{cut} = 0 = -m_\theta + 1 \qquad \underline{m_\theta = 1}$$

$$\theta_C = \frac{1}{EI}\int_0^{14} M m_\theta \, dx = \frac{1}{EI}\left[\int_0^{7} M_1 m_\theta \, dx + \int_7^{14} M_2 m_\theta \, dx\right]$$

$$= \frac{1}{EI}\left[-\frac{25x^3}{6} + 125x^2 - \frac{3{,}325x}{2}\Bigg|_0^7 + \frac{75x^2}{2} - 1{,}050x\Bigg|_7^{14}\right]$$

$$= -\frac{52{,}675\ \text{kN m}^2}{6EI}$$

$$= -\frac{52{,}675\ \text{kN m}^2}{6} \cdot \frac{\text{m}^2}{70(10^6)\ \text{kN}} \cdot \frac{1}{2{,}340(10^6)\ \text{mm}^4}\left(\frac{10^3\ \text{mm}}{\text{m}}\right)^4 = \underline{0.0536\ \text{RADIANS}}$$