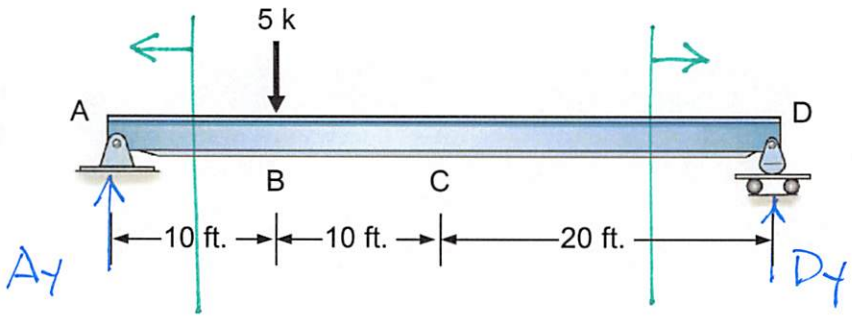
Example 8b-1: Determine the displacement at C. Assume $I = 240\ in^4$, $E = 29(10^3)\ ksi$.

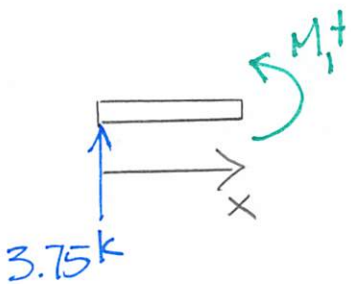
Real loads



$$\circlearrowleft +\sum M_D = 0 = 5^k(30\,ft) - A_y(40\,ft) \qquad A_y = 3.75^k$$

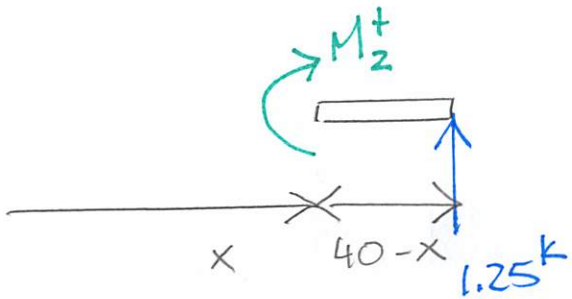
$$+\uparrow \sum F_y = 0 = A_y + D_y - 5^k \qquad D_y = 1.25^k$$

$0 \le x \le 10\,ft$



$$\circlearrowleft +\sum M_{CUT} = 0 = M_1 - 3.75x \qquad M_1(x) = (3.75x)\,k\,ft \;\checkmark$$

$10 \le x \le 40\,ft$

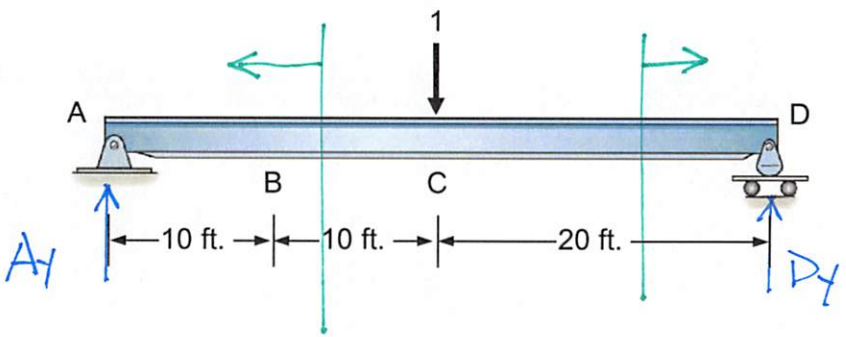


$$\circlearrowleft +\sum M_{CUT} = 0 = -M_2 + 1.25(40-x)$$

$$M_2(x) = [1.25(40-x)]\,k\,ft \;\checkmark$$

Example 8b-1: Determine the displacement at C. Assume $I = 240\ \text{in}^4$, $E = 29(10^3)\ ksi$.

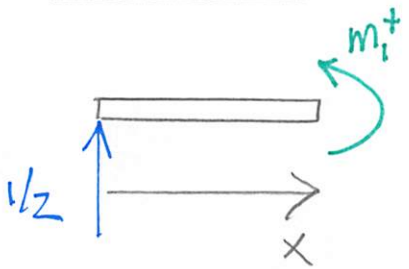
Virtual load



$\circlearrowleft + \Sigma M_D = 0 = 1(20\text{ ft}) - A_y(40\text{ ft})$ $\quad A_y = 1/2$

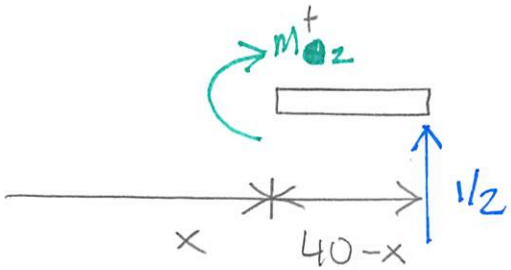
$+\uparrow \Sigma F_y = 0 = A_y + D_y - 1$ $\quad D_y = 1/2$

$0 \le x \le 20$



$\circlearrowleft + \Sigma M_{CUT} = 0 = m_1 - 1/2(x)$ $\quad m_1 = x/2$ ✓

$20 \le x \le 40\text{ ft}$



$\circlearrowleft + \Sigma M_{CUT} = 0 = -m_2 + 1/2(40-x)$ $\quad m_2 = \frac{1}{2}(40-x)$ ✓

Example 8b-1: Determine the displacement at C. Assume $I = 240\ \text{in}^4$, $E = 29(10^3)\ ksi$.

$$Y_C = \frac{1}{EI}\left[\int_0^{40} M\,m\,dx\right] = \frac{1}{EI}\left[\int_0^{10} M_1 m_1\,dx + \int_{10}^{20} M_2 m_1\,dx + \int_{20}^{40} M_2 m_2\,dx\right] \quad \text{ANSWER}$$

$$= \frac{1}{EI}\left[\int_0^{10} (3.75x)(x/2)\,dx + \int_{10}^{20} (1.25(40-x))(x/2)\,dx + \int_{20}^{40} (1.25(40-x))\frac{40-x}{2}\,dx\right]$$

$$= \frac{1}{EI}\left[\frac{5x^3}{8}\Big|_0^{10} - \frac{5x^2(x-60)}{24}\Big|_{10}^{20} + \frac{5x(x^2-120x+4{,}800)}{24}\Big|_{20}^{40}\right]$$

$$= \frac{13{,}750\,k\text{-}ft^3}{EI} = \frac{13{,}750\,k\text{-}ft^3}{29{,}000\,k/in^2 \cdot 240\,in^4 \cdot \left(\frac{12\,in}{ft}\right)^3} = 1.13\,in$$