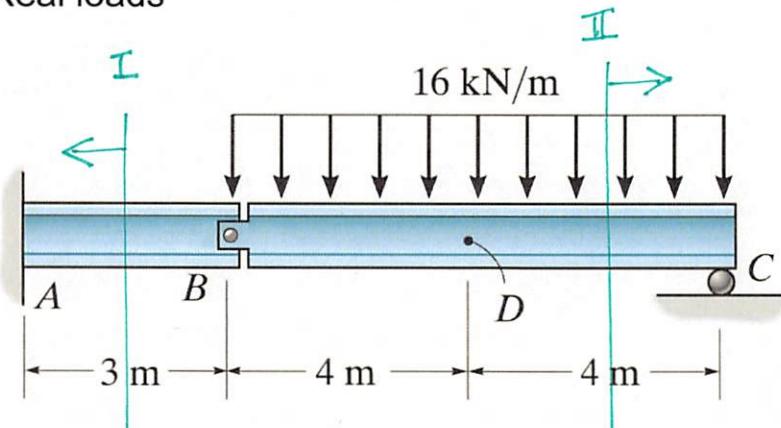
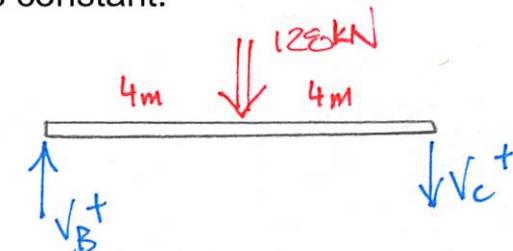


**Problem 8b-7.** Determine the slope at C. Use the principle of virtual work.  $EI$  is constant.

Real loads

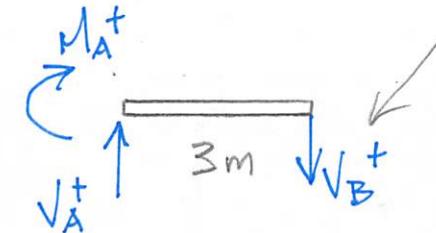


SECTION BC



$$\begin{aligned} \textcircled{1} \quad & \sum M_B = 0 = -128 \text{ kN}(4 \text{ m}) - V_c(8 \text{ m}) \quad V_c = -64 \text{ kN} \\ + \uparrow \sum F_y = 0 = & V_B - V_c - 128 \text{ kN} \quad V_B = 64 \text{ kN} \end{aligned}$$

SECTION AB



$$\begin{aligned} \textcircled{2} \quad & \sum M_A = 0 = -M_A - V_B(3 \text{ m}) \quad M_A = -192 \text{ kNm} \\ + \uparrow \sum F_y = 0 = & V_A - V_B \quad V_A = 64 \text{ kN} \end{aligned}$$

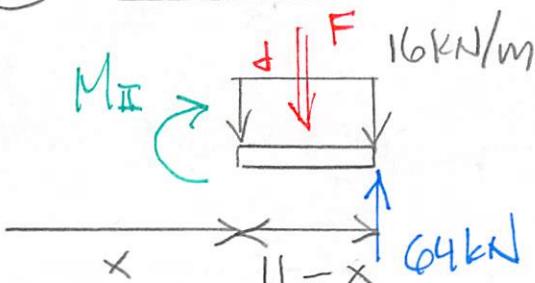
(I)

$$0 \leq x \leq 3$$

$$\begin{aligned} & \text{Virtual force } V_A = 64 \text{ kN} \\ & \text{Virtual moment } M_I = 64x - 192 \text{ kNm} \\ & \text{Equilibrium condition: } \sum M_{\text{cut}} = 0 \\ & \quad M_I + 192 \text{ kNm} - 64x = 0 \\ & \quad M_I = [64x - 192] \text{ kNm} \quad \checkmark \end{aligned}$$

(II)

$$3 \leq x \leq 11$$



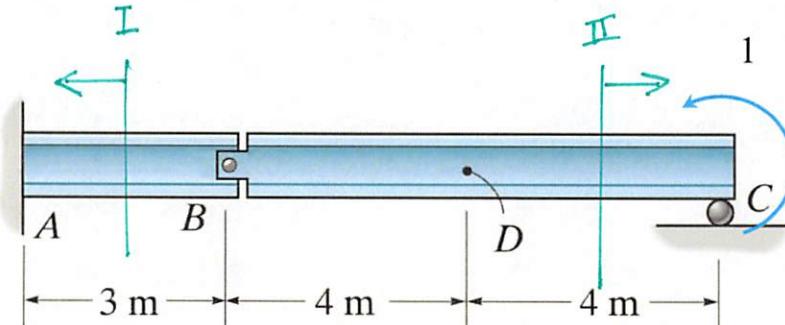
$$\sum M_{\text{cut}} = 0 = -M_{\text{II}} - Fd + 64(11-x)$$

$$M_{\text{II}} = [-8(11-x)^2 + 64(11-x)] \text{ kNm}$$

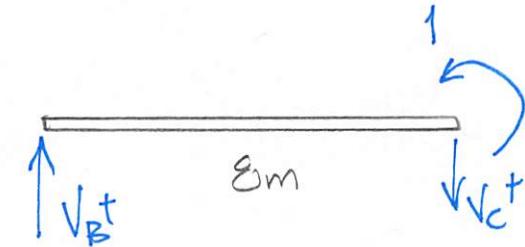
$$F = 16(11-x) \text{ kN} \quad d = \frac{1}{2}(11-x)$$

**Problem 8b-7.** Determine the slope at C. Use the principle of virtual work.  $EI$  is constant.

Virtual moment



SECTION BC



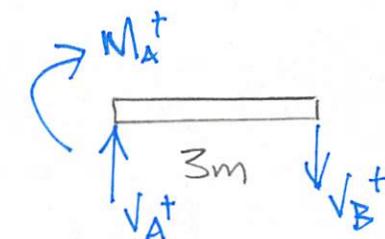
$$\begin{aligned}\textcircled{1} \sum M_C = 0 &= 1 - V_B(8m) \\ + \uparrow \sum F_y = 0 &= V_B - V_C\end{aligned}$$

$$\begin{aligned}\frac{V_B}{V_B} &= \frac{1}{8} \\ \frac{V_C}{V_C} &= \frac{1}{8}\end{aligned}$$

(I)  $0 \leq x \leq 3$

$$\begin{aligned}\text{Free body diagram: } &\text{Virtual moment } M_{\Theta_I} \text{ at } x, \text{ reaction } \frac{3}{8} \text{ at } x=0, \text{ reaction } \frac{1}{8} \text{ at } x=3. \\ \sum M_{\text{cut}} = 0 &= M_{\Theta_I} + \frac{3}{8} - \frac{1}{8}x \\ M_{\Theta_I} &= \frac{x}{8} - \frac{3}{8}\end{aligned}$$

SECTION AB



$$\begin{aligned}\textcircled{1} \sum M_A = 0 &= -M_A - V_B(3m) \\ + \uparrow \sum F_y = 0 &= V_A - V_B\end{aligned}$$

$$\begin{aligned}M_A &= -\frac{3}{8} \\ \frac{V_A}{V_A} &= \frac{1}{8}\end{aligned}$$

(II)  $3 \leq x \leq 11$

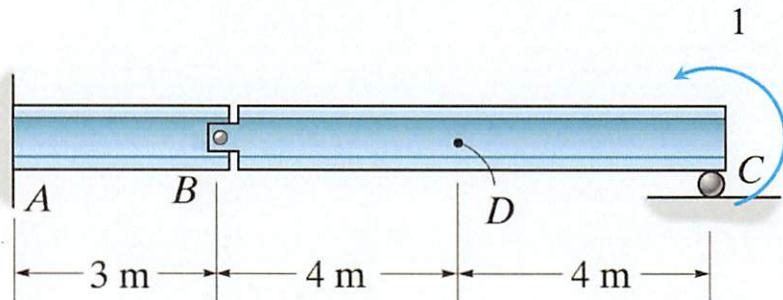
$$\begin{aligned}\text{Free body diagram: } &\text{Virtual moment } M_{\Theta_{II}} \text{ at } x, \text{ reaction } 1 \text{ at } x=3, \text{ reaction } \frac{1}{8} \text{ at } x=11. \\ \sum M_{\text{cut}} = 0 &= -M_{\Theta_{II}} - \frac{1}{8}(11-x) + 1 \\ M_{\Theta_{II}} &= -\frac{1}{8}(11-x) + 1\end{aligned}$$

$$\Theta_c = \int_0^1 \frac{M_I M_{\Theta_I}}{EI} dx$$

$$= \frac{1}{EI} \left[ \left( \int_0^3 M_I M_{\Theta_I} dx \right) + \left( \int_3^{11} M_{II} M_{\Theta_{II}} dx \right) \right]$$

**Problem 8b-7.** Determine the slope at C. Use the principle of virtual work.  $EI$  is constant.

Virtual moment



$$\begin{aligned}
 \Theta_C &= \frac{1}{EI} \left[ \int_0^3 (64x - 192) \frac{1}{8}(x-3) dx + \int_3^{11} (-8(11-x)^2 + 64(11-x)) \frac{1}{8}(11-x) dx \right] \\
 &= \frac{1}{EI} \left[ \int_0^3 (8x^2 - 48x + 72) dx + \int_3^{11} (-x^3 + 17x^2 - 75x + 99) dx \right] \\
 &= \frac{1}{EI} \left[ 72 + \frac{1024}{3} \right] = \underline{\underline{\frac{1240 \text{ kNm}^2}{3EI}}}
 \end{aligned}$$