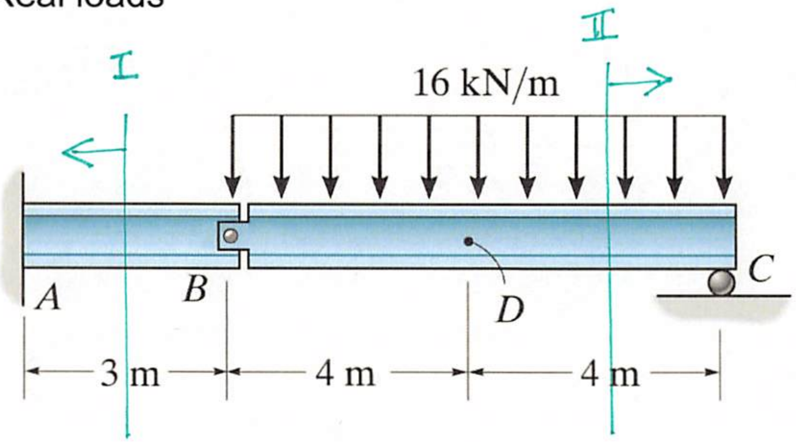
Problem 8b-7. Determine the slope at *C*. Use the principle of virtual work. *EI* is constant.

Real loads



SECTION BC

128 kN

4m 4m

V_B^+ V_C^+

$\circlearrowleft^{+} \sum M_B = 0 = -128\,kN(4m) - V_C(8m)$ $\quad \underline{V_C = -64\,kN}$

$+\uparrow \sum F_y = 0 = V_B - V_C - 128\,kN$ $\quad \underline{V_B = 64\,kN}$

SECTION AB

M_A^+ 3m V_A^+ V_B^+

$\circlearrowleft^{+} \sum M_A = 0 = -M_A - V_B(3m)$ $\quad \underline{M_A = -192\,kNm}$

$+\uparrow \sum F_y = 0 = V_A - V_B$ $\quad \underline{V_A = 64\,kN}$

(I) $\underline{0 \le x \le 3}$

192 kNm M_I 64 kN x

$\circlearrowleft^{+} \sum M_{CUT} = 0$

$= M_I + 192\,kNm - 64x$

$$\underline{M_I = [64x - 192]\,kNm} \checkmark$$

(II) $\underline{3 \le x \le 11}$

M_{II} d F 16 kN/m 64 kN x $11-x$

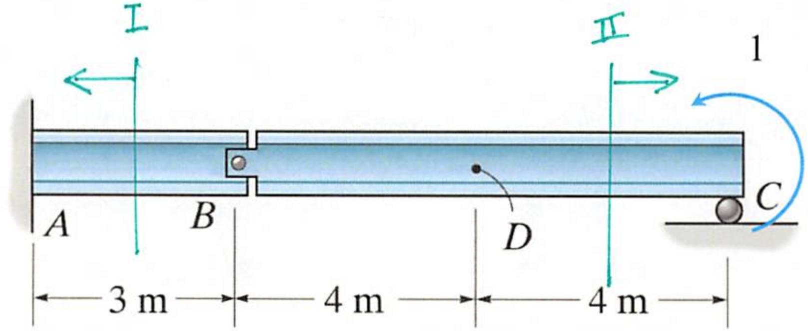
$F = 16(11-x)\,kN \qquad d = \frac{1}{2}(11-x)$

$\circlearrowleft^{+} \sum M_{CUT} = 0 = -M_{II} - Fd + 64(11-x)$ $\checkmark$

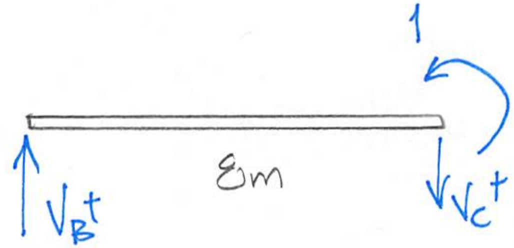
$$\underline{M_{II} = [-8(11-x)^2 + 64(11-x)]\,kNm}$$

Problem 8b-7. Determine the slope at *C*. Use the principle of virtual work. *EI* is constant.

Virtual moment



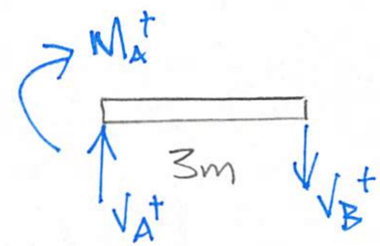
SECTION BC



$\circlearrowleft^{+} \sum M_C = 0 = 1 - V_B(8m)$ $\qquad$ $\underline{V_B = 1/8}$

$+\uparrow \sum F_y = 0 = V_B - V_C$ $\qquad$ $\underline{V_C = 1/8}$

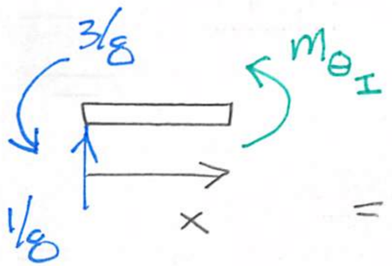
SECTION AB



$\circlearrowleft^{+} \sum M_A = 0 = -M_A - V_B(3m)$ $\qquad$ $\underline{M_A = -3/8}$

$+\uparrow \sum F_y = 0 = V_A - V_B$ $\qquad$ $\underline{V_A = 1/8}$

(I) $\underline{0 \le x \le 3}$

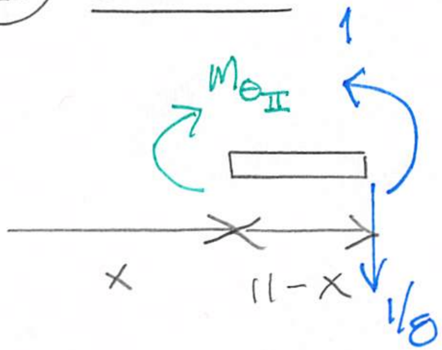


$\circlearrowleft^{+} \sum M_{CUT} = 0$

$= m_{\theta_I} + \frac{3}{8} - \frac{1}{8}x$

$\underline{m_{\theta_I} = \frac{x}{8} - \frac{3}{8}}$ ✓

(II) $\underline{3 \le x \le 11}$



$\circlearrowleft^{+} \sum M_{CUT} = 0$

$= -m_{\theta_{II}} - \frac{1}{8}(11-x) + 1$

$\underline{m_{\theta_{II}} = -\frac{1}{8}(11-x) + 1}$ ✓

$$\theta_C = \int_0^{11} \frac{M m_\theta}{EI} dx$$

$$= \frac{1}{EI}\left[\int_0^3 M_I m_{\theta_I} dx + \int_3^{11} M_{II} m_{\theta_{II}} dx\right]$$

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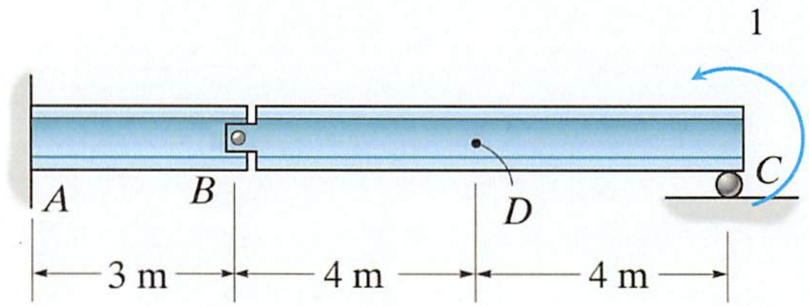
Virtual moment



$$\theta_C = \frac{1}{EI}\left[\int_0^3 (64x-192)\frac{1}{8}(x-3)\,dx + \int_3^{11}\left(-8(11-x)^2+64(11-x)\right)\frac{1}{8}(11-x)\,dx\right]$$

$$= \frac{1}{EI}\left[\int_0^3 (8x^2-48x+72)\,dx + \int_3^{11}(-x^3+17x^2-75x+99)\,dx\right]$$

$$= \frac{1}{EI}\left[72 + \frac{1024}{3}\right] = \frac{1240\,\mathrm{kNm}^2}{3EI}$$