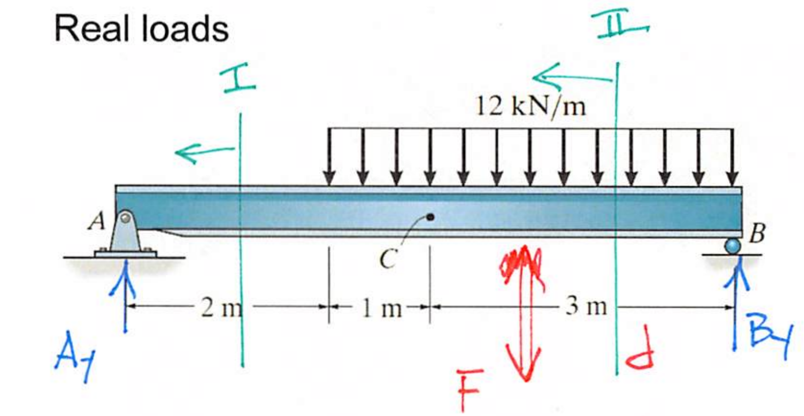
Problem 8b-5. Determine the displacement at x = 2 m. Use the principle of virtual work. EI is constant.

Real loads

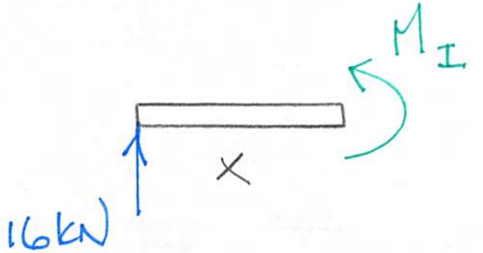


$$\overset{+}{\circlearrowleft} \sum M_B = 0 = Fd - A_y(6m) \qquad F = 12\,kN/m\,(4m)$$

$$d = \tfrac{1}{2}(4m) = 2m$$

$$\underline{A_y = 16\,kN}$$

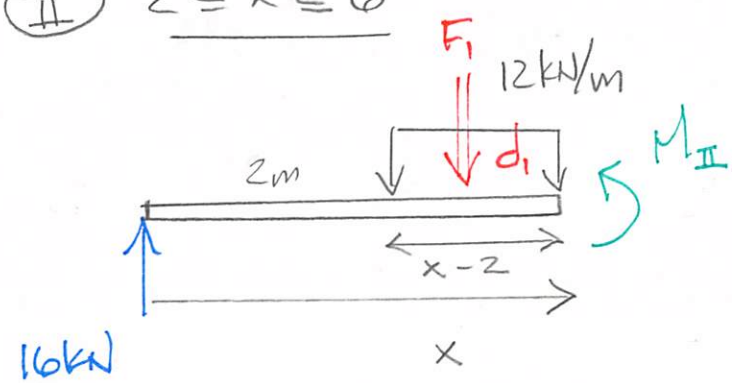
(I) $\underline{0 \le x \le 2}$



$$\overset{+}{\circlearrowleft} \sum M_{CUT} = 0 = M_I - 16kN\,x \qquad \underline{M_I = [16x]\,kNm}$$

$$M_I(x=0) = 0 \checkmark$$

(II) $\underline{2 \le x \le 6}$



$$F_1 = 12(x-2) \qquad d_1 = \tfrac{1}{2}(x-2)$$

$$\overset{+}{\circlearrowleft} \sum M_{CUT} = 0 = M_{II} + F_1 d_1 - 16x$$

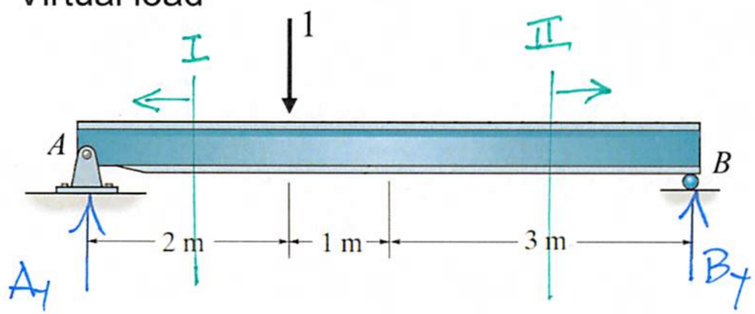
$$M_{II} = -F_1 d_1 + 16x$$

$$\underline{\underline{M_{II} = [-6(x-2)^2 + 16x]\,kNm}} \qquad M_{II}(x=6m) = 0 \checkmark$$

$$M_I(x=2) = M_{II}(x=2) \checkmark$$

Problem 8b-5. Determine the displacement at $x = 2$ m. Use the principle of virtual work. EI is constant.

Virtual load

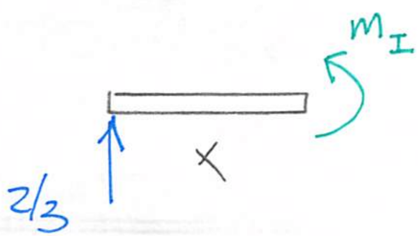


$\circlearrowleft^{+} \sum M_B = 0 = 1(4m) - A_y(6m)$ $\quad \underline{A_y = 2/3}$

$+\uparrow \sum F_y = 0 = A_y + B_y - 1$ $\quad \underline{B_y = 1/3}$

(I) $\underline{0 \le x \le 2}$

$\circlearrowleft^{+} \sum M_{cut} = 0 = M_I - \frac{2}{3}(x)$ $\quad \underline{M_I = \frac{2}{3}x}$ $\quad M_I(x=0) = 0$ ✓

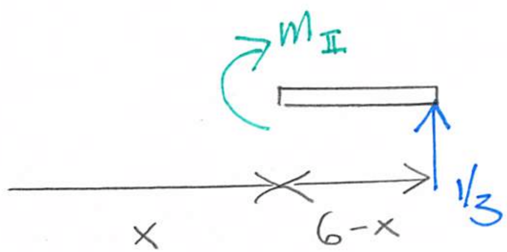


(II) $\underline{2 \le x \le 6}$

$\circlearrowleft^{+} \sum M_{cut} = 0 = -M_{II} + \frac{1}{3}(6-x)$ $\quad \underline{M_{II} = \frac{1}{3}(6-x)}$

✓

$M_{II}(x=6) = 0$



$M_I(x=2) = M_{II}(x=2)$ ✓

Problem 8b-5. Determine the displacement at x = 2 m. Use the principle of virtual work. EI is constant.

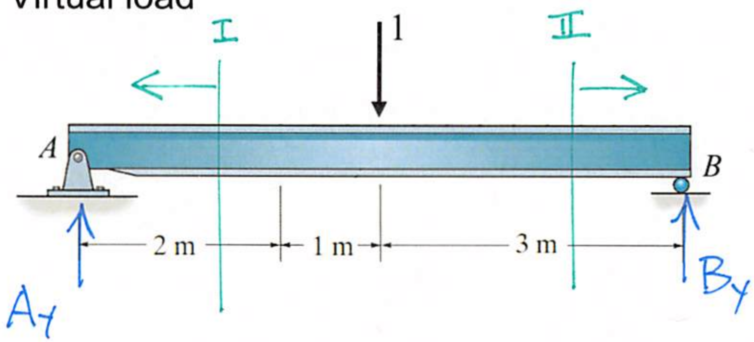
$$Y_{x=2m} = \int_0^6 \frac{Mm}{EI}dx = \frac{1}{EI}\left[\int_0^2 M_I m_I \, dx + \int_2^6 M_{II} m_{II} dx\right]$$

$$= \frac{1}{EI}\left[\frac{32x^3}{9}\Big|_0^2 + \left(\frac{x^4}{2} - \frac{76x^3}{9} + 44x^2 - 48x\right)_2^6\right]$$

$$= \frac{1}{EI}\left[\frac{256}{9} + \frac{896}{9}\right] = \frac{128 kNm^3}{EI}$$

Problem 8b-5. Determine the displacement at x = 3 m. Use the principle of virtual work. EI is constant.

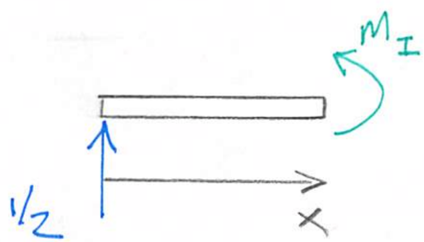
Virtual load



$$\circlearrowleft^{+} \sum M_B = 0 = 1(3m) - A_y(6m) \qquad \underline{A_y = 1/2}$$

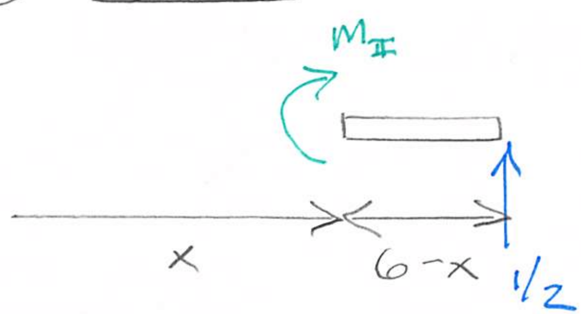
$$+\uparrow \sum F_y = 0 = A_y + B_y - 1 \qquad \underline{B_y = 1/2}$$

(I) $\underline{0 \le x \le 3}$



$$\circlearrowleft^{+} \sum M_{CUT} = 0 = M_I - \tfrac{1}{2}x \qquad \underline{M_I = \frac{x}{2}} \qquad M_I(x=0) = 0 \ \checkmark$$

(II) $\underline{3 \le x \le 6}$



$$\circlearrowleft^{+} \sum M_{CUT} = 0 = -M_{II} + \tfrac{1}{2}(6-x) \qquad \underline{M_{II} = \frac{1}{2}(6-x)}$$

$$M_{II}(x=6) = 0 \ \checkmark$$

$$M_I(x=3) = M_{II}(x=3) \ \checkmark$$

Problem 8b-5. Determine the displacement at x = 3 m. Use the principle of virtual work. EI is constant.

$$Y_{x=3m} = \int_0^6 \frac{Mm}{EI}dx = \frac{1}{EI}\left[\int_0^2 M_I m_I dx + \int_2^3 M_{II} m_I dx + \int_3^6 M_{II} m_{II} dx\right]$$

$$= \frac{1}{EI}\left[\frac{8x^3}{3}\Big|_0^2 + \left(-\frac{3x^4}{4} + \frac{20x^3}{3} - 6x^2\right)_2^3 + \left(\frac{3x^4}{4} - \frac{38x^3}{3} + 66x^2 - 72x\right)_3^6\right]$$

$$= \frac{1}{EI}\left[\frac{64}{3} + \frac{575}{12} + \frac{333}{4}\right] = \frac{305 kNm^3}{2EI}$$