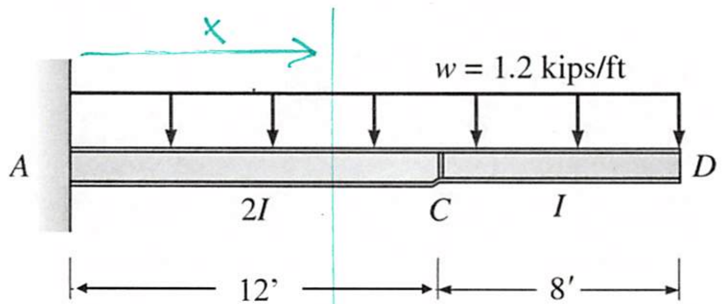
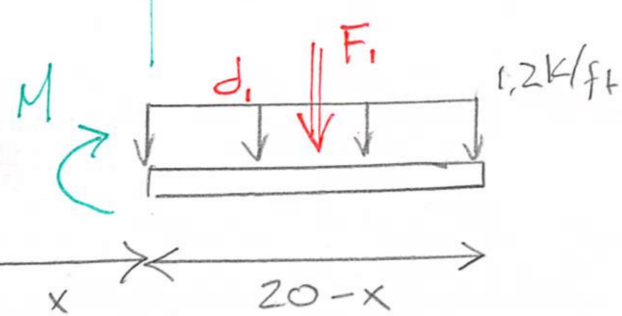
Example 8b-2: Determine the displacement at *D*. Assume $I = 400\ \text{in}^4$, $E = 29(10^3)$ *ksi*.

Real loads





$F_1 = 1.2(20-x)$ $\quad d_1 = \frac{1}{2}(20-x)$

$$\circlearrowleft + \sum M_{CUT} = 0 = -M - F_1 d_1$$

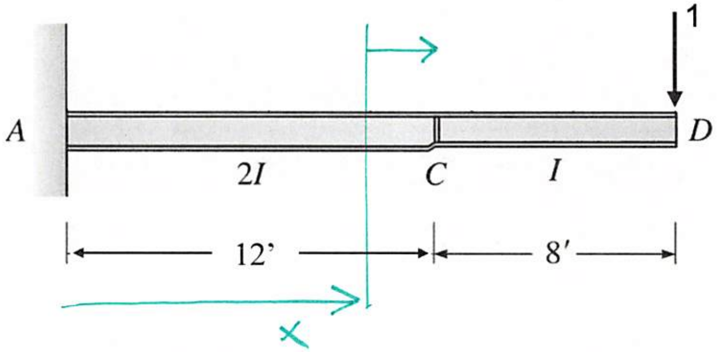
$$\underline{\underline{M = -F_1 d_1 = -0.6(20-x)^2}}$$

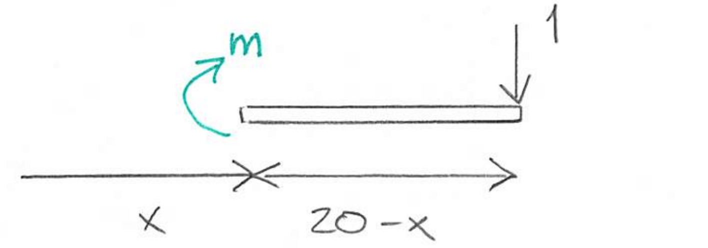
$M(x = 20') = 0$ ✓

$M(x = 0) = -240\,\text{k ft}$ ✓

Example 8b-2: Determine the displacement at *D*. Assume I = 400 in^4, E = 29(10^3) *ksi*.

Virtual load





$$\circlearrowleft^{+} \sum M_{CUT} = 0 = -m - 1(20-x)$$

$$\underline{m = -(20-x)}$$

$m(x=20') = 0$ ✓

$m(x=0) = -20$ ✓

$$Y_D = \int_0^{20} \frac{Mm}{EI}dx = \frac{1}{EI}\left[\int_0^{12}\frac{Mm}{2}dx + \int_{12}^{20} Mm\,dx\right]$$

$$= \frac{1}{EI}\left[\int_0^{12} \overset{M}{0.3(20-x)^2}\,\overset{m}{(20-x)}\,dx + \int_{12}^{20} \overset{M}{0.6(20-x)^2}\,\overset{m}{(20-x)}\,dx\right]$$

$$= \frac{1}{EI}\left[-\frac{3(x-20)^4}{40}\Big|_0^{12} - \frac{3(x-20)^4}{20}\Big|_{12}^{20}\right] = \frac{1}{EI}\left[\frac{58{,}464}{5} + \frac{3{,}072}{5}\right]k\,ft^3$$

$$= \frac{61{,}536\,k\,ft^3}{5\,EI} = \frac{61{,}536\,k\,ft^3}{5}\left|\frac{IN^2}{29{,}000\,k}\right|\frac{1}{400\,IN^4}\left|\frac{(12\,IN)^3}{ft^3}\right. = \underline{\underline{1.83\,IN}}$$