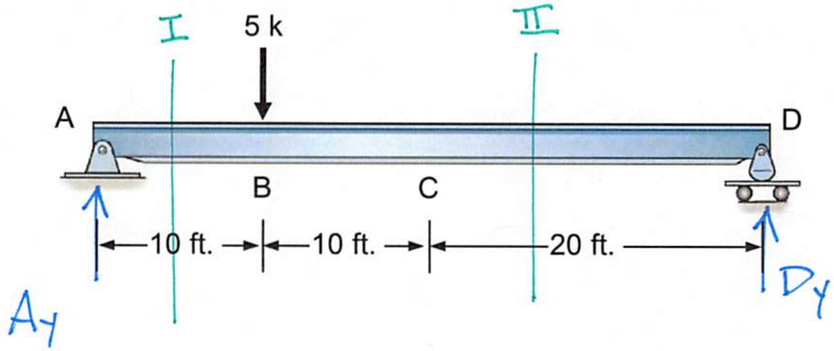
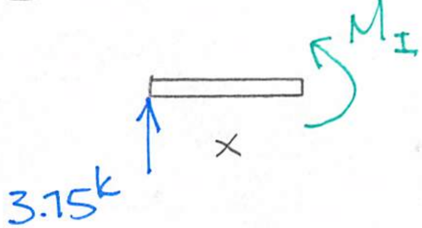
Example 8b-1: Determine the displacement at C. Assume $I = 240\ in^4$, $E = 29(10^3)\ ksi$.

Real loads



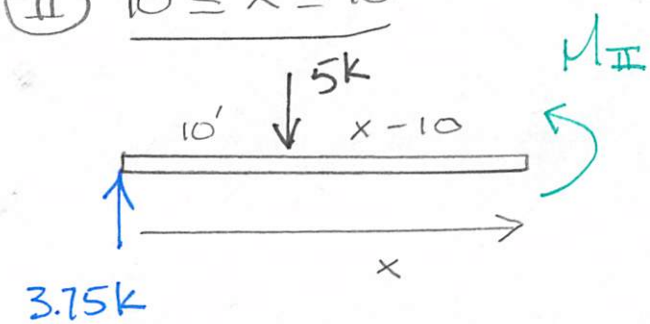
$$\circlearrowleft^{+} \sum M_D = 0 = 5k(30') - A_y(40') \qquad A_y = 3.75^k$$

(I) $0 \le x \le 10$



$$\circlearrowleft^{+} \sum M_{CUT} = 0 = M_I - 3.75x \qquad M_I = [3.75x]\ kft$$

(II) $10 \le x \le 40$

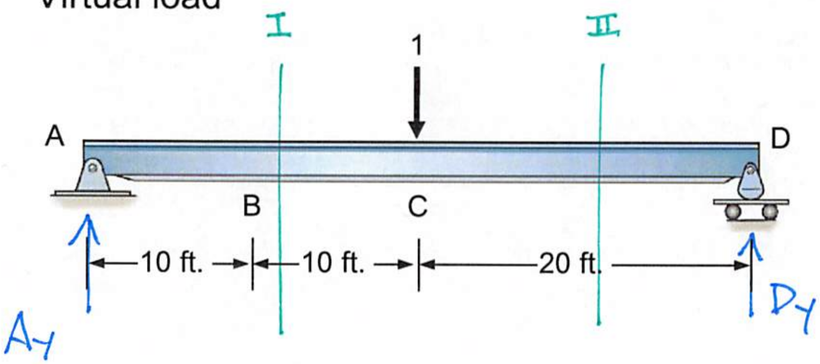


$$\circlearrowleft^{+} \sum M_{CUT} = 0 = M_{II} + 5^k(x-10) - 3.75x$$

$$M_{II} = [-1.25x + 50]\ kft$$

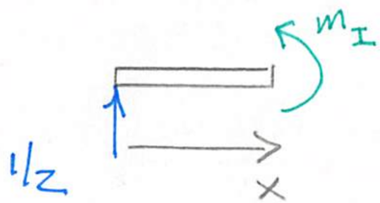
Example 8b-1: Determine the displacement at C. Assume $I = 240\ \text{in}^4$, $E = 29(10^3)\ ksi$.

Virtual load



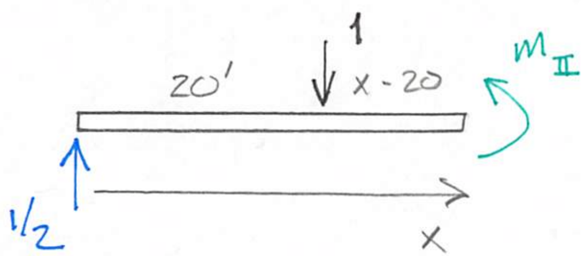
$$\circlearrowleft^{+} \sum M_D = 0 = 1(20') - A_y(40') \qquad \underline{A_y = 1/2}$$

I $\quad \underline{0 \le x \le 20}$



$$\circlearrowleft^{+} \sum M_{CUT} = 0 = m_I - \tfrac{1}{2}x \qquad \underline{\underline{m_I = \frac{x}{2}}} \checkmark$$

II $\quad \underline{20 \le x \le 40}$



$$\circlearrowleft^{+} \sum M_{CUT} = 0 = m_{II} + 1(x-20) - \tfrac{1}{2}(x)$$

$$\underline{\underline{m_{II} = -\frac{x}{2} + 20}} \checkmark$$

Example 8b-1: Determine the displacement at C. Assume I = 240 in^4, E = 29(10^3) *ksi*.

$$Y_C = \frac{1}{EI}\int_0^{40} Mm\,dx = \frac{1}{EI}\left[\int_0^{10} M_I m_I\,dx + \int_{10}^{20} M_{II} m_I\,dx + \int_{20}^{40} M_{II} m_{II}\,dx\right]$$

$$= \frac{1}{EI}\left[\int_0^{10}(3.75x)\left(\frac{x}{2}\right)dx + \int_{10}^{20}(50-1.25x)\left(\frac{x}{2}\right)dx + \int_{20}^{40}(50-1.25x)\left(20-\frac{x}{2}\right)dx\right]$$

$$= \frac{1}{EI}\left[\frac{5x^3}{8}\Big|_0^{10} - \frac{5x^2(x-60)}{24}\Big|_{10}^{20} + \frac{5x(x^2-120x+4800}{24}\Big|_{20}^{40}\right]$$

$$= \frac{13{,}750\,k\,ft^3}{EI}$$

$$= \frac{13{,}750\,k\,ft^3}{} \left|\frac{IN^2}{29000\,k}\right| \frac{}{240\,IN^4} \left|\frac{(12\,IN)^3}{ft^3}\right| = \underline{\underline{1.13\,IN}}$$