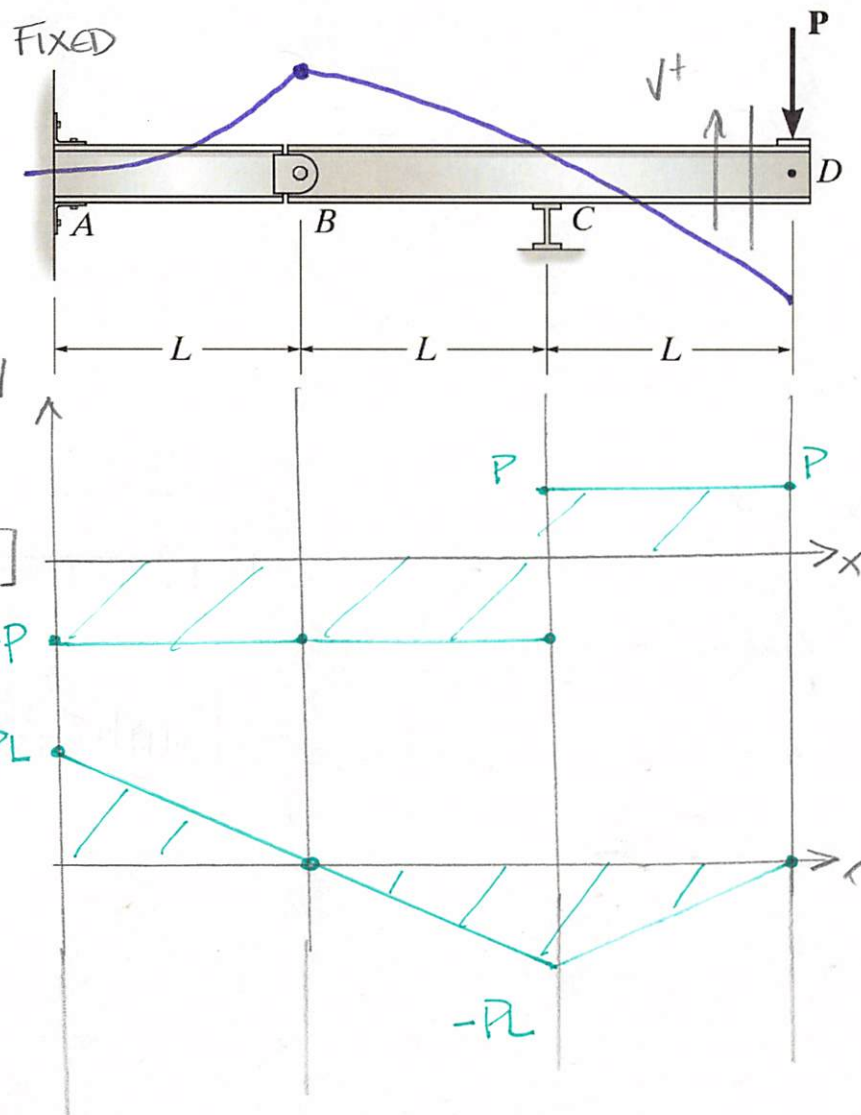


Example 7b-6: Use the conjugate beam method to determine the slope and displacement at point D. Assume that EI is constant.



FBD BCD

The FBD of segment BCD shows a horizontal beam of length $2L$. At B, there is an upward reaction force V_B^+ . At C, there is an upward reaction force C_1 . At D, there is a downward point load P . The equations for this segment are:

$$\sum M_B = 0 = C_1(L) - P(2L)$$

$$\underline{C_1 = 2P}$$

$$+\uparrow \sum F_y = 0 = V_B + C_1 - P \quad \underline{V_B = -P}$$

FBD AB

The FBD of segment AB shows a horizontal beam of length L . At A, there is a counter-clockwise moment M_A^+ and an upward reaction force V_A^+ . At B, there is a downward reaction force V_B^+ . The equations for this segment are:

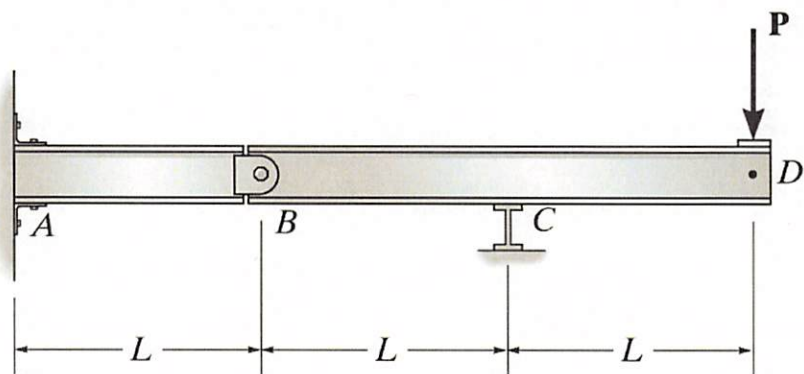
$$\sum M_A = 0 = -M_A - V_B(L)$$

$$\underline{M_A = PL}$$

$$+\uparrow \sum F_y = 0 = V_A - V_B$$

$$\underline{V_A = -P}$$

Example 7b-6: Use the conjugate beam method to determine the slope and displacement at point D. Assume that EI is constant.



FBD ABC



$$F_1 = \frac{1}{2}(L) \frac{PL}{EI} = \frac{PL^2}{2EI}$$

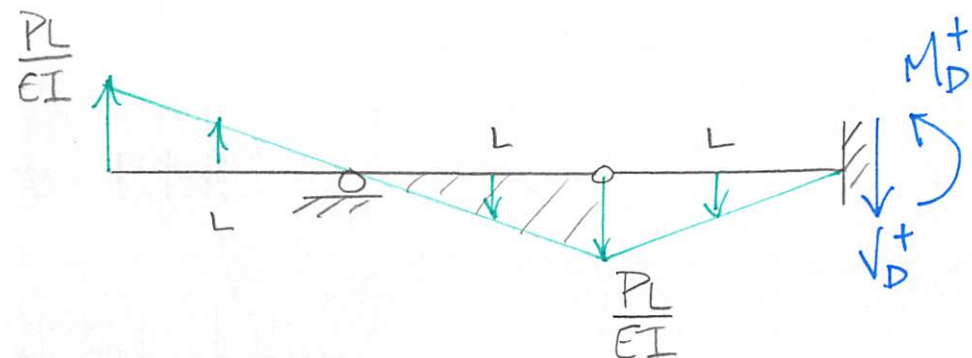
$$F_2 = F_1$$

$$d_1 = \frac{2}{3}L \quad d_2 = d_1$$

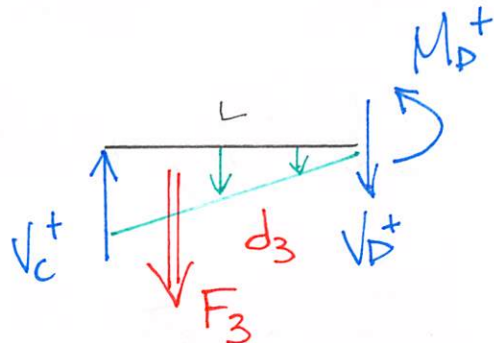
$$\sum M_B = 0 = -F_1 d_1 - F_2 d_2 - V_C L$$

$$V_C = \left[\frac{PL^2}{2EI} \left(\frac{2}{3}L \right) + \frac{PL^2}{2EI} \left(\frac{2}{3}L \right) \right] / L$$

$$V_C = -\frac{2PL^2}{3EI}$$



FBD CD



$$\sum M_D = 0 = M_D + F_3 d_3 - V_C (L)$$

$$M_D = -F_3 d_3 + V_C (L) = -\frac{PL^2}{2EI} \left(\frac{2L}{3} \right) - \frac{2PL^2}{3EI} (L)$$

$$= -\frac{PL^3}{EI} \Rightarrow \theta_C$$

$$+\uparrow \sum F_y = 0 = V_C - F_3 - V_D$$

$$V_D = -\frac{7PL^2}{6EI}$$