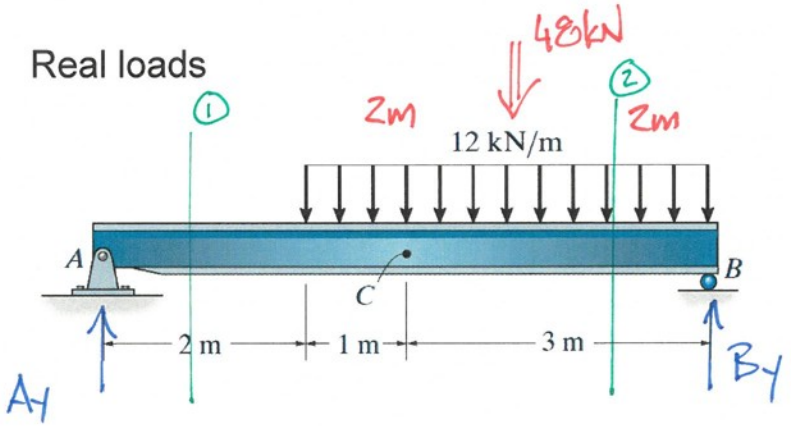
Problem 8b-5. Determine the displacement at $x = 2$ m. Use the principle of virtual work. *EI* is constant.

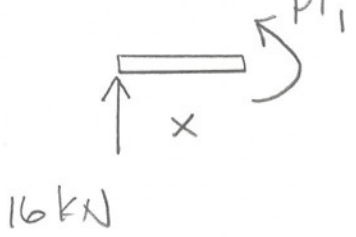
Real loads



$$\circlearrowleft + \sum M_B = 0 = 48\,\text{kN}(2\,\text{m}) - A_y(6\,\text{m}) \qquad A_y = 16\,\text{kN}$$

$$+\uparrow \sum F_y = 0 = A_y + B_y - 48\,\text{kN} \qquad B_y = 32\,\text{kN}$$

$0 \le x \le 2$

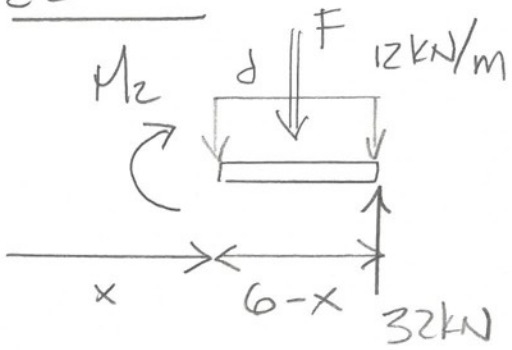


$$\circlearrowleft + \sum M_{CUT} = 0 = M_1 - 16x$$

$$M_1 = [16x]\,\text{kN m}$$

$$M_1(x=0) = 0 \;\checkmark$$

$2 \le x < 6$



$$\circlearrowleft + \sum M_{CUT} = 0 = -M_2 - 12(6-x)\frac{6-x}{2} + 32(6-x)$$

$$M_2 = \left[-6(6-x)^2 + 32(6-x)\right]\,\text{kN m}$$

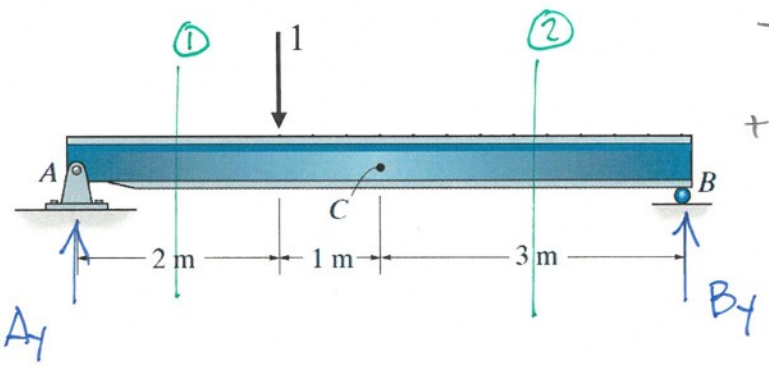
$$M_2(x=6) = 0 \;\checkmark$$

$$M_1(x=2) = M_2(x=2) \;\checkmark$$

$$F = 12(6-x)\,\text{kN} \qquad d = \frac{6-x}{2}$$

Problem 8b-5. Determine the displacement at x = 2 m. Use the principle of virtual work. *EI* is constant.

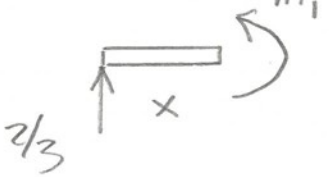
Virtual load



$$\circlearrowleft + \sum M_B = 0 = 1(4m) - A_y(6m) \qquad A_y = 2/3$$

$$+\uparrow \sum F_y = 0 = A_y + B_y - 1 \qquad B_y = 1/3$$

$0 \le x \le 2$

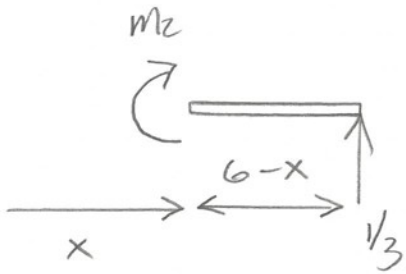


$$\circlearrowleft + \sum M_{cut} = 0$$

$$= m_1 - \tfrac{2}{3}x$$

$$m_1 = \frac{2x}{3}\,m$$

$2 \le x < 6$



$$\circlearrowleft + \sum M_{cut} = 0$$

$$= -m_2 + \tfrac{1}{3}(6-x)$$

$$m_2 = \tfrac{1}{3}(6-x)\,m$$

$$m_1(x=2) = m_2(x=2) \checkmark$$

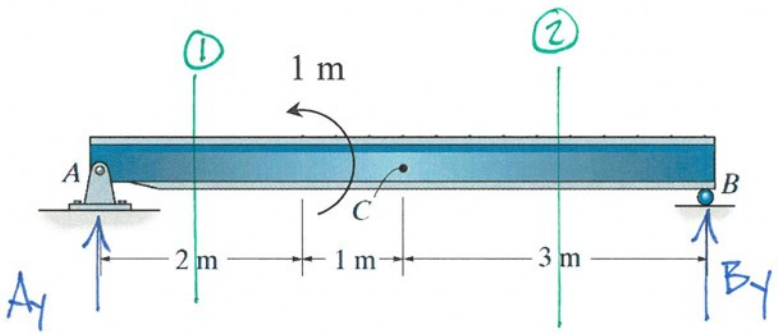
$$Y_{x=2} = \int_0^6 \frac{Mm}{EI}\,dx = \int_0^2 \frac{M_1 m_1}{EI}\,dx + \int_2^6 \frac{M_2 m_2}{EI}\,dx$$

$$= \frac{1}{EI}\left[\int_0^2 8x^2\,dx + \int_2^6 \left(3x^3 - 38x^2 + 132x - 72\right)dx\right]$$

$$= \frac{1}{EI}\left[\frac{64}{3} + \frac{448}{3}\right] = \frac{512\ \text{kN m}^2}{3EI}$$

Problem 8b-5. Determine the displacement at $x = 2$ m. Use the principle of virtual work. EI is constant.

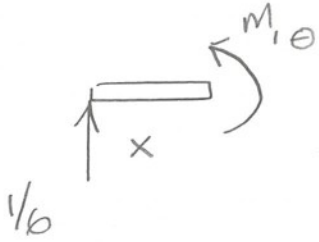
Virtual moment



$$\circlearrowleft +\sum M_B = 0 = 1 - A_y(6\text{m}) \qquad A_y = 1/6$$

$$+\uparrow \sum F_y = 0 = A_y + B_y \qquad B_y = -1/6$$

$0 \le x \le 2$

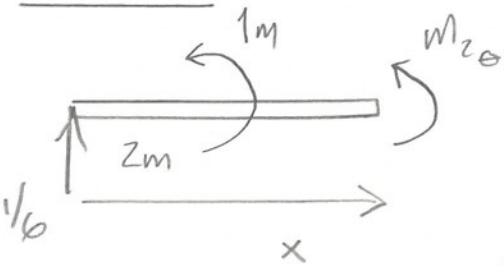


$$\circlearrowleft +\sum M_{CUT} = 0$$

$$= m_{1\theta} - 1/6\,x$$

$$m_{1\theta} = \frac{x}{6}$$

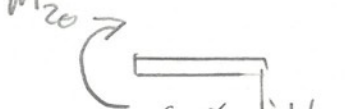
$2 \le x \le 6$



$$\circlearrowleft +\sum M_{CUT} = 0$$

$$= m_{2\theta} + 1 - 1/6\,x$$

$$m_{2\theta} = \frac{x}{6} - 1$$



$$\circlearrowleft \sum M_{CUT} = 0$$

$$-m_{2\theta} - 1/6(6-x)$$

$$\theta_{x=2} = \int_0^6 \frac{M m_\theta}{EI}\,dx = \int_0^2 \frac{M_1 m_{1\theta}}{EI}\,dx + \int_2^6 \frac{M_2 m_{2\theta}}{EI}\,dx$$

$$= \frac{1}{EI}\left[\int_0^2 \frac{8x^2}{6}\,dx + \int_2^6 \left(-x^3 + \frac{38x^2}{2} - 44x + 24\right)dx\right]$$

$$= \frac{1}{EI}\left[\frac{64}{9} - \frac{448}{9}\right] = -\frac{128\ \text{kN m}^2}{3EI}$$