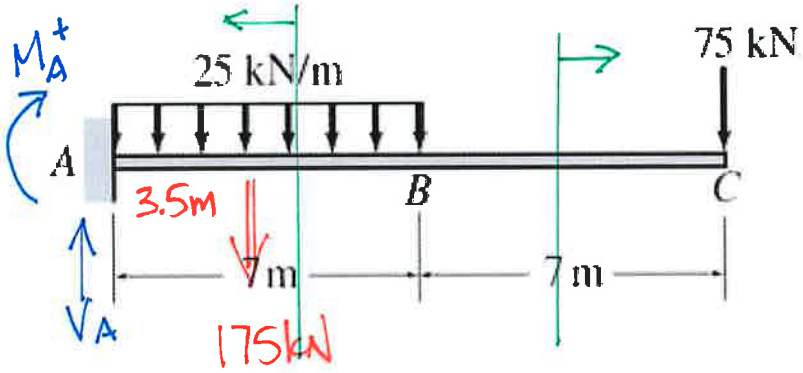
Example 8b-3: Determine the slope and displacement at *C*. Assume I = 2,340 (10^6) mm^4 and 70 GPa.

Real loads



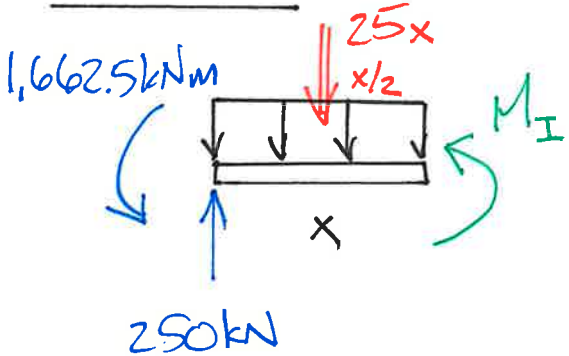
$$\circlearrowleft^+ \Sigma M_A = 0 = -M_A - 175\text{kN}(3.5\text{m}) - 75\text{kN}(14\text{m})$$

$$M_A = -1{,}662.5\ \text{kNm}$$

$$+\uparrow \Sigma F_y = 0 = V_A - 175\text{kN} - 75\text{kN}$$

$$V_A = 250\text{kN}$$

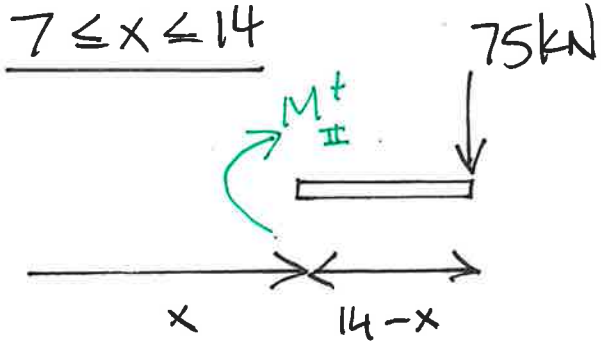
$0 \le x \le 7$



$$\circlearrowleft^+ \Sigma M_{CUT} = 0 = M_I + 25x\left(\frac{x}{2}\right) - 250x + 1{,}662.5\ \text{kNm}$$

$$M_I = \left[-\frac{25x^2}{2} + 250x - 1{,}662.5\right]\text{kNm}$$

$7 \le x \le 14$



$$\circlearrowleft^+ \Sigma M_{CUT} = 0 = -M_{II} - 75(14-x)$$

$$M_{II} = -75(14-x)\ \text{kNm}$$

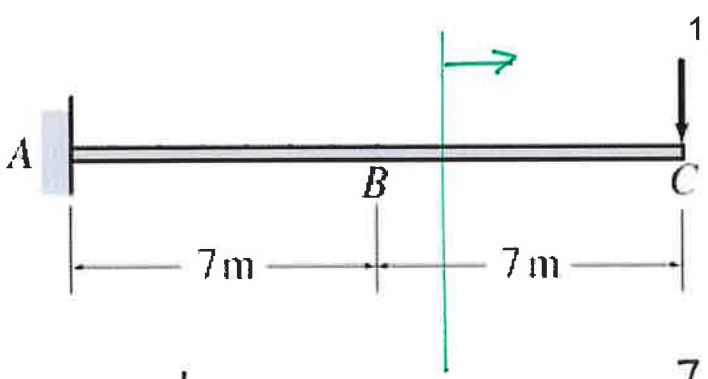
$M_I = (x=0) = -1{,}662.5\text{kNm}$ ✓

$M_{II}(x=14) = 0$ ✓

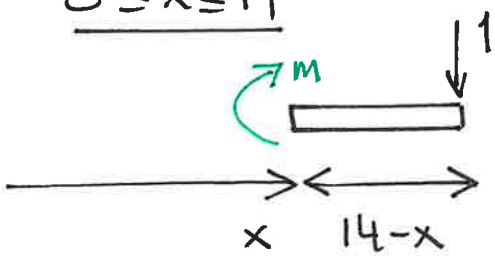
$M_I(7) = M_{II}(7)$ ✓

Example 8b-3: Determine the slope and displacement at *C*. Assume I = 2,340 (10^6) mm^4 and 70 GPa.

Virtual load



$0 \le x \le 14$



$$\curvearrowleft \Sigma M_{@1} = 0$$

$$= -m - 1(14-x)$$

$$m = x - 14$$

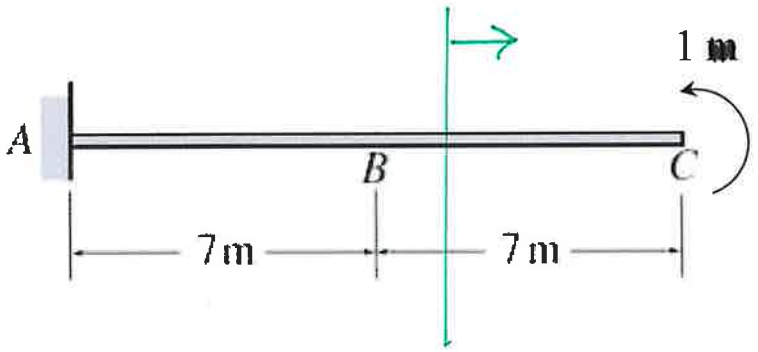
$$Y_c = \int_0^L \frac{Mm}{EI}dx = \frac{1}{EI}\left[\int_0^7 M_I m\,dx + \int_7^{14} M_{II} m\,dx\right] *$$

$$= \frac{1}{EI}\left[\int_0^7 \left(-\frac{25x^4}{8} + \frac{425x^3}{3} - \frac{10{,}325x^2}{4} + 23{,}275x\right) + \left(25x^3 - 1{,}050x^2 + 14{,}700x\right)_7^{14}\right]$$

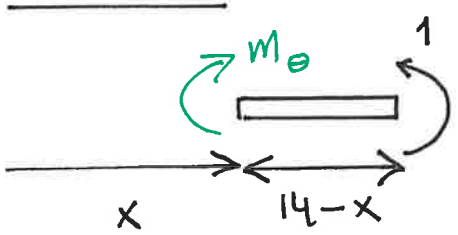
$$= \frac{1}{EI}\left[\frac{1{,}860{,}775}{24} + 8{,}575\right] = \frac{86{,}107\,kNm^3}{EI} = 0.526m$$

Example 8b-3: Determine the slope and displacement at *C*. Assume $I = 2{,}340\ (10^6)\ \text{mm}^4$ and 70 GPa.

Virtual moment



$0 \le x \le 14$



$$\circlearrowleft + \Sigma M_{OUT} = 0$$

$$= -M_\theta + 1$$

$$M_\theta = 1$$

$$\theta_C = \int_0^L \frac{M m_\theta}{EI}\,dx = \frac{1}{EI}\left[\int_0^7 M_I m_\theta\,dx + \int_7^{14} M_{II} m_\theta\,dx\right] \quad *$$

$$= \frac{1}{EI}\left[\left(-\frac{25x^3}{6} + 125x^2 - \frac{3{,}325x}{2}\right)\Bigg|_0^7 + \left(\frac{75x^2}{2} - 1{,}050x\right)\Bigg|_7^{14}\right] = -\frac{52{,}675\ \text{kNm}^2}{6EI}$$

$$= -0.0536\ \text{RADS}$$