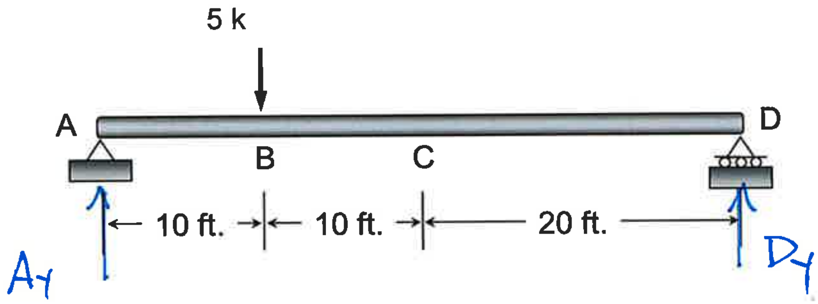
Example 8b-1: Determine the displacement at C. Assume $I = 240\ \text{in}^4$, $E = 29(10^3)$ *ksi*.

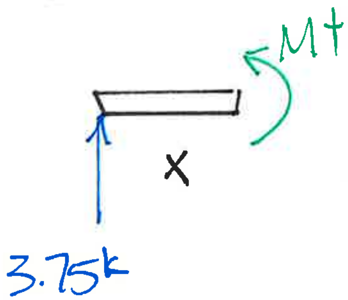
Real loads



$\circlearrowleft + \sum M_D = 0 = 5k(30') - A_y(40')$ $\quad \underline{A_y = 3.75^k}$

$+\uparrow \sum F_y = 0 = A_y + D_y - 5k$ $\quad \underline{D_y = 1.25^k}$

$\underline{0 \le x \le 10}$



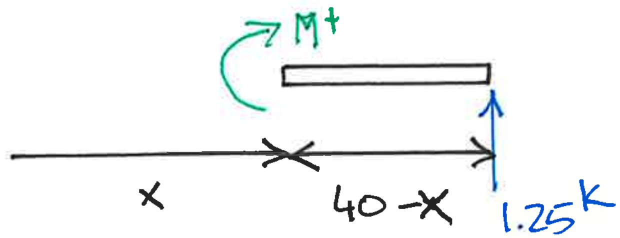
$\circlearrowleft + \sum M_{cut} = 0$

$= M - 3.75k(x)$

$$M_I = [3.75x]\ \text{kft}$$

$M_I(x=0) = 0$ ✓

$\underline{10 \le x \le 40}$



$\circlearrowleft + \sum M_{cut} = 0$

$= -M + 1.25k(40-x)$

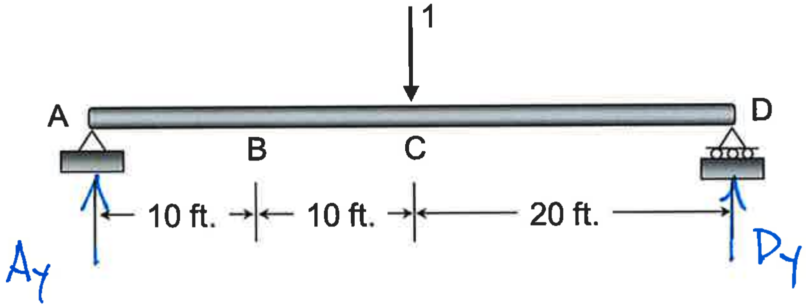
$$M_{II} = 1.25^k(40-x)$$

$M_{II}(x=40) = 0$ ✓

$M_I(10) = M_{II}(10) = 37.5\ \text{kft}$ ✓

Example 8b-1: Determine the displacement at C. Assume $I = 240\ \text{in}^4$, $E = 29(10^3)\ ksi$.

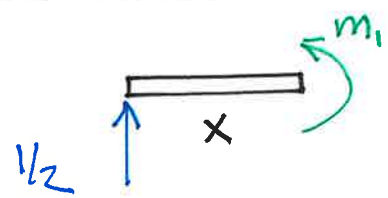
Virtual load



$$\circlearrowleft^{+} \sum M_D = 0 = 1(20') - A_y(40') \qquad \underline{A_y = 1/2}$$

$$+\uparrow \sum F_y = 0 = A_y + D_y - 1 \qquad \underline{D_y = 1/2}$$

$\underline{0 \le x \le 20}$

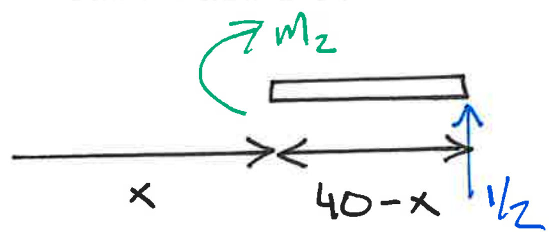


$$\circlearrowleft^{+} \sum M_{CUT} = 0$$

$$= m - \tfrac{1}{2}x \checkmark$$

$$\underline{m_1 = \frac{x}{2}}$$

$\underline{20 \le x \le 40}$



$$\circlearrowleft^{+} \sum M_{CUT} = 0$$

$$= -m_2 + \tfrac{1}{2}(40-x)$$

$$\underline{m_2 = \tfrac{1}{2}(40-x)} \checkmark$$

$$y_c = \int_0^{40} \frac{Mm}{EI}\,dx = \frac{1}{EI}\left[\int_0^{10} M_I\, m_1\, dx + \int_{10}^{20} M_{II}\, m_1\, dx + \int_{20}^{40} M_{III}\, m_2\, dx\right] *$$

$$= \frac{1}{EI}\left[\left.\frac{5x^3}{8}\right|_0^{10} + \left.-\frac{5x^2(x-60)}{24}\right|_{10}^{20} + \left.\frac{5x(x^2-120x+4800)}{24}\right|_{20}^{40}\right]$$

$$= \frac{1}{EI}\left[625 + \frac{6{,}875}{3} + \frac{5{,}000}{3}\right] = \frac{13{,}750\ \text{k ft}^3}{EI} = \underline{\underline{1.13\ \text{IN.}}}$$