Deflections

- > The previously discussed geometric methods are highly effective and straightforward for simple loadings.
- However, these methods are very tedious for complex loadings.
- In cases like this, an energy method is the preferred approach.
- Energy methods are based on the principle of the conservation of energy.

1



Typically, the effects of bending on deformation are much more significant than the effects of shear

2







Q (virtual load)







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Example: Determine the displacement and slope at point A on the beam ($I = 1,000 \text{ in}^4$ and $E = 29(10^3) \text{ ksi}$). Since the moment due to the virtual couple is discontinuous, we must break the integration into two parts.

$$\theta_{A} = \int_{0}^{10} \frac{m_{\theta}M}{EI} dx + \int_{10}^{10} \frac{m_{\theta}M}{EI} dx$$

Substituting the moment expression into the virtual work equation and integrating yields the following:

Deflections

$$\theta_{A} = \int_{0}^{10} \frac{x(15x - x^{2})}{15EI} dx + \int_{10}^{15} \frac{(x - 15)(15x - x^{2})}{15EI} dx$$



19



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