Deflections

- Question: What are **Structural Deflections?**
- Answer: The deformations or movements of a structure and its components, such as beams and trusses, from their original positions.
- It is as important for the designer to determine deflections and strains as it is to know the stresses caused by loads.

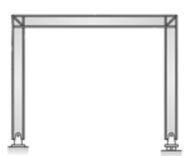
Deflections

- In general, a maximum deflection criteria for structures is frequently used.
- These limit states are mathematic expressed as:

$$\Delta_{live\,loads} = \frac{L}{360} \qquad \Delta_{total\,load} = \frac{L}{240}$$

Deflections

- In this section we will discuss how to calculate the deflections of elastic structures using both geometric and energy methods.
- A ***geometric method*** uses the strain of an elastic structure to determine the deflection.
- ***Energy methods*** are based on the principle of *conservation of energy*.



Deflections

Deflection Diagrams and the Elastic Curve

- The ability to determine the deflection of a structure is very important.
- Deflection is caused by many sources, such as, loads, temperature, construction error, and settlements.
- It is important to include the calculation of deflections into the design procedure to prevent structural damage to secondary structures (concrete or plaster walls or roofs) or to solve indeterminate problems.

Deflections

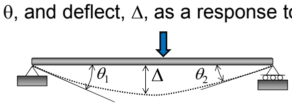
Deflection Diagrams and the Elastic Curve

- In this section, we will learn to compute the deflection of ***linear elastic*** structures.
- An elastic structure is one that returns to its original position after the load is removed.
- Deflections are most often caused by ***internal loadings*** such as bending moment and axial force.

Deflections

Deflection Diagrams and the Elastic Curve

- Usually, before the slope and deflection are calculated, it is important to sketch the shape of the structure when loaded.
- To do this, we need to know how different connections rotate, θ, and deflect, Δ, as a response to loading.

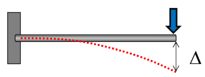


Pin or roller support $\Delta = 0$

Deflections

Deflection Diagrams and the Elastic Curve

- Usually, before the slope and deflection are calculated, it is important to sketch the shape of the structure when loaded.
- To do this, we need to know how different connections rotate, θ, and deflect, Δ, as a response to loading.

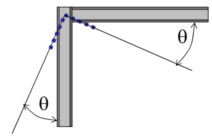


Fixed support $\Delta = 0$ and $\theta = 0$

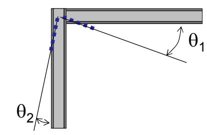
Deflections

Deflection Diagrams and the Elastic Curve

- Consider the rotation, θ, and deflection, Δ, of connections in frames



Fixed-connected joint – since the connection cannot rotate, the slope of the each member is the same

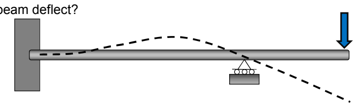


Pin-connected joint – the pinned joint may rotate which results in different slopes for each member same

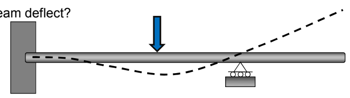
Deflections

Deflection Diagrams and the Elastic Curve

How will this beam deflect?



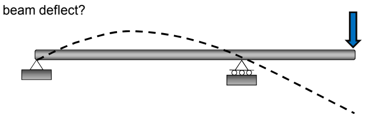
How will this beam deflect?



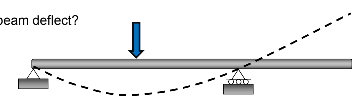
Deflections

Deflection Diagrams and the Elastic Curve

How will this beam deflect?

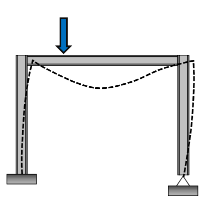


How will this beam deflect?

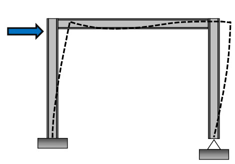


Deflections

Deflection Diagrams and the Elastic Curve



How will this frame deflect?

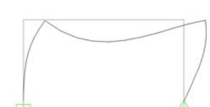


How will this frame deflect?

Deflections

Deflection Diagrams and the Elastic Curve

How will this frame deflect?

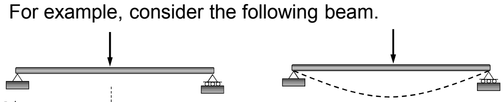


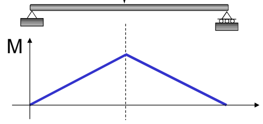
How will this frame deflect?

Deflections

Deflection Diagrams and the Elastic Curve

- If you have a difficult time drawing the deflected shape from the elastic response, try to construct the moment diagram and then use the sign of the moment to determine the curvature of the structure.
- For example, consider the following beam.



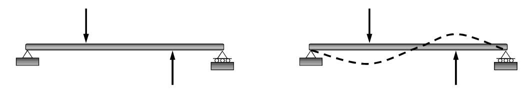


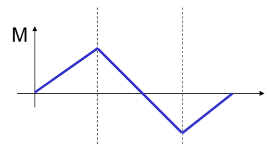
Since the moment is positive (+) the curvature of the beam is also positive or in this case ***concave upward***

Deflections

Deflection Diagrams and the Elastic Curve

For example, consider the following beam.



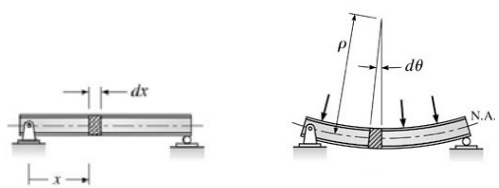


Where the moment is positive (+) the curvature of the beam is also positive or in this case ***concave upward***. Where the moment is negative (-), the curvature is ***concave downward***.

Deflections

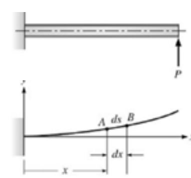
Elastic Beam Theory

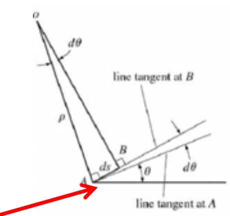
In this section, we will derive the relationship between the internal moment and the deflected shape. Consider a straight elastic beam deformed by a set of applied loads.



Deflections

Elastic Beam Theory





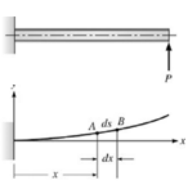
$$\tan\theta = \frac{dy}{dx}$$

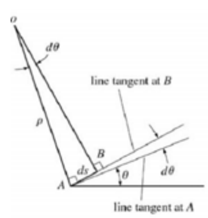
If the angles are small, then the tan θ ≈ θ, the slope can be written as:

$$\theta = \frac{dy}{dx}$$

Deflections

Elastic Beam Theory



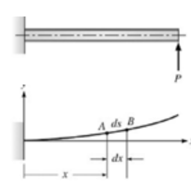


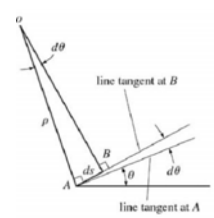
From geometry of the triangular segment AB we can write:

$$\rho\, d\theta = ds$$

Deflections

Elastic Beam Theory



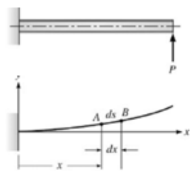


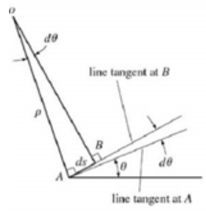
Dividing each side of the previous equation by ***ds*** and rearranging the terms gives:

$$\rho\, d\theta = ds \Rightarrow \frac{d\theta}{ds} = \frac{1}{\rho}$$

Deflections

Elastic Beam Theory



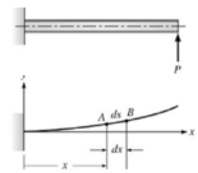


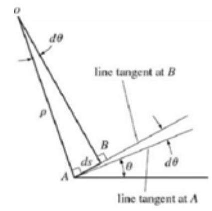
Since $d\theta/ds$ represents the change in slope per unit length of distance along the curve, this term is called the ***curvature***. Since slope are small in actual beams, $ds \approx dx$, therefore:

$$\frac{d\theta}{dx} = \frac{1}{\rho}$$

Deflections

Elastic Beam Theory



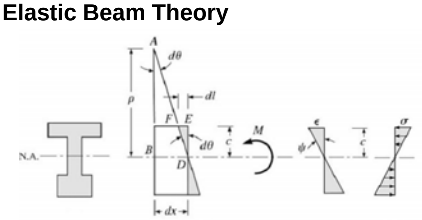


Differentiation both side of the first equation gives:

$$\theta = \frac{dy}{dx} \quad \Rightarrow \quad \frac{d\theta}{dx} = \frac{d^2y}{dx^2} \qquad \frac{d\theta}{dx} = \frac{1}{\rho}$$

Deflections

Elastic Beam Theory

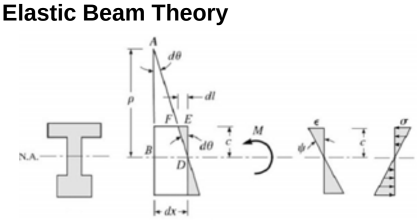


The change in length of the top fiber dl in terms of $d\theta$ and the distance c from the neutral axis is:

$$dl = d\theta\, c$$

Deflections

Elastic Beam Theory

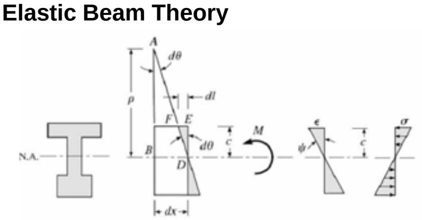


By definition the strain ε at the top fibers of the beam is:

$$\varepsilon = \frac{dl}{dx}$$

Deflections

Elastic Beam Theory



Combining the previous equations to solve for strain ε:

$$dl = d\theta\, c \qquad \varepsilon = \frac{dl}{dx} \quad \Rightarrow \quad \varepsilon = \frac{d\theta}{dx}c$$

Deflections

Elastic Beam Theory

Using the equation relating curvature and displacement we get:

$$\frac{d\theta}{dx} = \frac{d^2y}{dx^2} \quad \Rightarrow \quad \varepsilon = \frac{d\theta}{dx}c \quad \Rightarrow \quad \frac{d^2y}{dx^2} = \frac{\varepsilon}{c}$$

If the behavior is elastic, the flexural stress, *s*, can be related to the strain, *e*, at the top fibers by Hooke's Law, $\sigma = E\varepsilon$

$$\frac{d^2y}{dx^2} = \frac{\sigma}{Ec}$$

Deflections

Elastic Beam Theory

For elastic behavior the relationship between flexural stress at the top of the beam and the moment acting on the cross-section is:

$$\sigma = \frac{Mc}{I}$$

Substituting the value of *s* in the equation on the last page, gives the basic differential equation of the displacement of an elastic beam:

$$\boxed{\frac{d^2y}{dx^2} = \frac{M}{EI}} \qquad y = \int\int \frac{M}{EI}\,dx\,dx$$

Deflections

Elastic Beam Theory

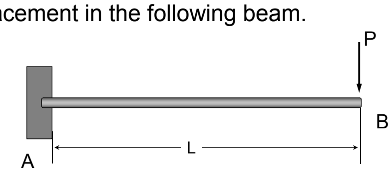
- How can we use the basic equation for elastic beams to solve for the displacement?

$$y = \int\int \frac{M}{EI}\,dx\,dx$$

- If we can write a continuous function for the moment over the beam we can integrate the function and find the displacement as a function of ***x***.
- This method is called ***direct integration***

Deflections

Example: Determine the equations for slope and displacement in the following beam.

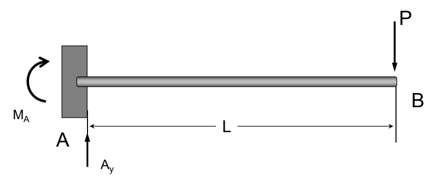


$$y = \int\int \frac{M}{EI}\,dx\,dx$$

First, find the reactions; and second, write the equation for moment as a function of *x*

Deflections

Find the reactions in the cantilever beam

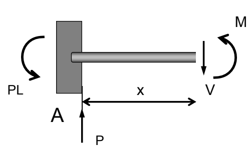


$$\circlearrowleft^{+} \sum M_A = 0 = -M_A - PL \quad \Rightarrow \quad M_A = -PL$$

$$^{+}\!\uparrow \sum F_y = 0 = A_y - P \quad \Rightarrow \quad A_y = P$$

Deflections

Write the equation for bending moment in the beam



$$\circlearrowleft^{+} \sum M_{cut} = 0 = M - Px + PL \quad \Rightarrow \quad M = P(x - L)$$

Deflections

Integrate the moment equation to determine the slope

$$M = P(x - L)$$

$$\theta = \int \frac{M}{EI}\,dx \quad = \int \frac{P(x-L)}{EI}\,dx$$

$$= \frac{P}{EI}\left(\frac{x^2}{2} - Lx\right) + C_1 \qquad \theta(x=0) = 0$$

$$\boxed{\theta = \frac{P}{EI}\left(\frac{x^2}{2} - Lx\right)} \qquad \theta(x=L) = -\frac{PL^2}{2EI}$$

Deflections

Integrate the slope equation to determine the displacement

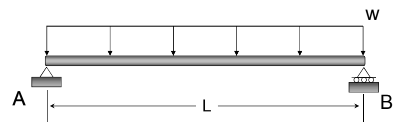
$$y = \int \theta \; dx \quad = \frac{P}{EI}\int\left(\frac{x^2}{2} - Lx\right) dx$$

$$= \frac{P}{EI}\left(\frac{x^3}{6} - \frac{Lx^2}{2}\right) + C_2 \qquad y(x=0)=0$$

$$\boxed{y = \frac{P}{EI}\left(\frac{x^3}{6} - \frac{Lx^2}{2}\right)} \qquad y(x=L) = -\frac{PL^3}{3EI}$$

Deflections

Example: Determine the equations for slope and displacement in the following beam.

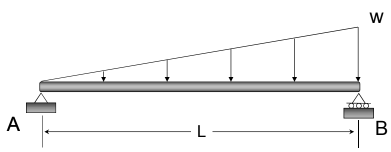


$$y = \iint \frac{M}{EI}\, dx\, dx$$

First, find the reactions; and second, write the equation for moment as a function of x

Deflections

Example: Determine the equations for slope and displacement in the following beam.

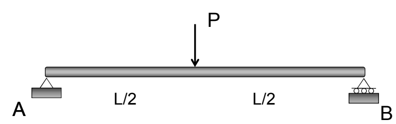


$$y = \iint \frac{M}{EI}\, dx\, dx$$

First, find the reactions; and second, write the equation for moment as a function of x

Deflections

Example: Determine the equations for slope and displacement in the following beam.



End of Defections – Part 1

Any questions?