Indeterminate Structures

Advantages and disadvantages of indeterminate structures:

- Advantages:
  - Smaller Stress
  - Greater Stiffness
  - Redundancies

- Disadvantages:
  - Stress due to support settlements
  - Stresses due to temperature changes and fabrication errors

Greater Stiffness

- Statically indeterminate structures, if properly designed, have the capacity for redistributing loads when certain structural elements become overstressed or collapse in cases of overloads due to earthquakes, tornadoes, impact, or other events.

Redundancies

Stress due to support settlements
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Stress due to temperature change and fabrication errors

Methods of Analysis

- When analyzing any indeterminate structure, it is necessary to satisfy equilibrium, compatibility, and force-displacement requirements for the structure.

  - **Equilibrium** is satisfied when the reactive forces hold the structure at rest.
  - **Compatibility** is satisfied when the various segments of the structure fit together without intentional breaks or overlaps.
  - **Force-Displacement** requirements depend upon the way the material responds (in our case linear-elastic).

Flexibility or Force Method

- The flexibility or force method is a procedure for analyzing linear elastic indeterminate structures.

  - This method was one of the first methods available for the analysis of statically indeterminate structures.

  - Since compatibility forms the basis for this method, it is also called the method of consistent deformations.

Flexibility or Force Method

- All indeterminate analysis methods require that the solution satisfy equilibrium and compatibility requirements.

  - By compatibility we mean that the structure must fit together - no gaps can exist - and the deflected shape must be consistent with the constraints imposed by the supports.

  - A key step in this method is the analysis of an indeterminate structure as a stable determinate structure with selected restraints (supports for example) removed.
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Flexibility or Force Method

- While this method can be applied to any type of structure (beams, trusses, frames, etc.) the computational effort increases exponentially with the degree of indeterminacy.
- Therefore this method is most applicable to structures with a low degree of indeterminacy.

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Selection of the redundant restraint

- To satisfy the original restraint at point B, we determine the deflection at B as a function of the redundant force $R_B$ and set that equation equal to zero.
- The additional compatibility equation combined with the equations of equilibrium will account for all the indeterminate reactions.

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Consider the following indeterminate beam

- One choice for the redundant in this problem is the reaction at point B.
- We know that the deflection at B is zero.

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Flexibility or Force Method

Selection of the redundant restraint

- The reaction at B is absolutely essential for the stability of the structure, it is termed redundant.

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Consider the following indeterminate beam

- $\Delta_B = 0$
- $\Delta_{BD} + \Delta_{BB} = 0$
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Flexibility or Force Method

- Consider the following indeterminate beam

  - The deflections at B can be evaluated with virtual work
  - Another method would be to use tables for deflections of structures and the method of superposition.

As a sign convention, we will assume that displacements are positive (+) when they are in the direction of the redundant. You are free to choice the direction of the redundant.

Returning to our original example indeterminate beam

\[
\Delta_{BO} + \Delta_{BB} = 0
\]

\[
-\frac{wL^4}{8EI} + \frac{X_2 L^2}{3EI} = 0 \quad X_2 = \frac{3wL}{8}
\]

Returning to our original example indeterminate beam

Compute the reactions and draw the shear and moment curves for the following beam
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Flexibility or Force Method

- Compute the reactions and draw the shear and moment curves for the following beam

![Beam Diagram]

End of Indeterminate Structures

Any questions?