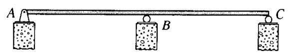
Indeterminate Structures

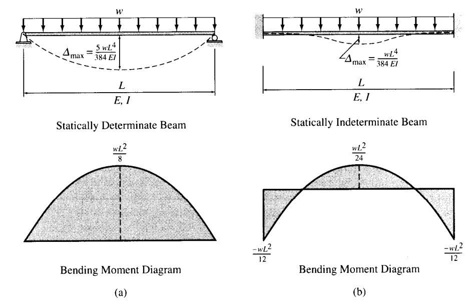
- Advantages and disadvantages of indeterminate structures:
  - Advantages:
    - Smaller Stress
    - Greater Stiffness
    - Redundancies
  - Disadvantages
    - Stress due to support settlements
    - Stresses due to temperature changes and fabrication errors



Statically Indeterminate Beam

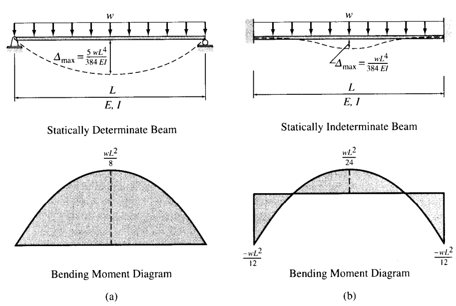
Indeterminate Structures

Smaller Stresses



Indeterminate Structures

Greater Stiffness



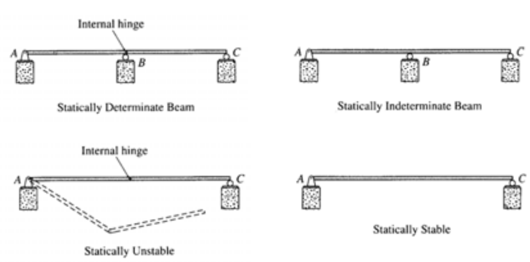
Indeterminate Structures

Redundancies

- Statically indeterminate structures, if properly designed, have the capacity for redistributing loads when certain structural elements become overstressed or collapse in cases of over loads due to earthquakes, tornadoes, impact, or other events

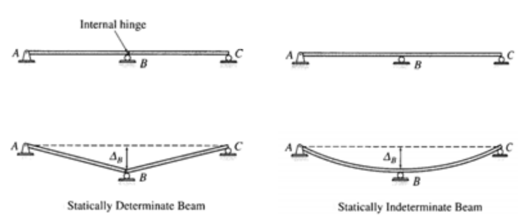
Indeterminate Structures

Redundancies



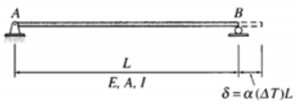
Indeterminate Structures

Stress due to support settlements

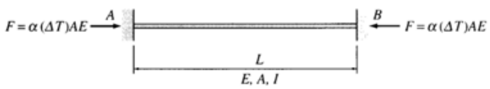


Indeterminate Structures

Stress due to temperature change and fabrication errors



Statically Determinate Beam



Statically Indeterminate Beam

Indeterminate Structures

Methods of Analysis

- When analyzing any indeterminate structure, it is necessary to satisfy ***equilibrium***, ***compatibility***, and ***force-displacement*** requirements for the structure.
- ***Equilibrium*** is satisfied when the reactive forces hold the structure at rest.
- ***Compatibility*** is satisfied when the various segments of the structure fit together without intentional breaks or overlaps.
- ***Force-Displacement*** requirements depend upon the way the material responds (in our case linear-elastic)

Indeterminate Structures

Flexibility or Force Method

- The ***flexibility or force method*** is a procedure for analyzing linear elastic indeterminate structures.
- This method was one of the first methods available for the analysis of statically indeterminate structures.
- Since compatibility forms the basis for this method, is has also called the ***method of consistent deformations.***

Indeterminate Structures

Flexibility or Force Method

- The method consists of writing equations that satisfy the compatibility requirements for the structure in order to determine redundant forces.
- Once these redundant forces are determined, the remaining reactive forces on the structure are determined by satisfying equilibrium requirements.

Indeterminate Structures

Flexibility or Force Method

- The ***flexibility or force method*** was originally developed by James Clerk Maxwell in 1874 and later refined by Otto Mohr and Heinrich Müller-Breslau.

James Clerk Maxwell

Christian Otto Mohr

Heinrich Müller-Breslau

Indeterminate Structures

Flexibility or Force Method

- All indeterminate analysis methods require that the solution satisfy equilibrium and compatibility requirements.
- By compatibility we mean that the structure must fit together – no gaps can exist – and the deflected shape must be consistent with the constraints imposed by the supports.
- A key step in this method is the analysis of an indeterminate structure as a stable determinate structure with selected restraints (supports for example) removed.

Indeterminate Structures

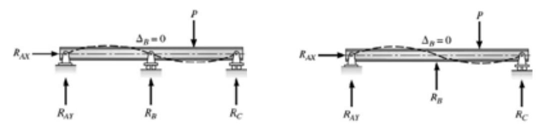
Flexibility or Force Method

- While this method can be applied to any type of structure (beams, trusses, frames, etc.) the computational effort increases exponentially with the degree of indeterminacy
- Therefore this method is most applicable to structures with a low degree of indeterminacy

Indeterminate Structures

Flexibility or Force Method

- Selection of the redundant restraint

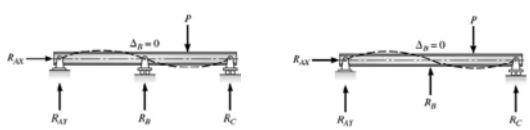


- The reaction at B is absolutely essential for the stability of the structure, it is termed ***redundant***

Indeterminate Structures

Flexibility or Force Method

- Selection of the redundant restraint

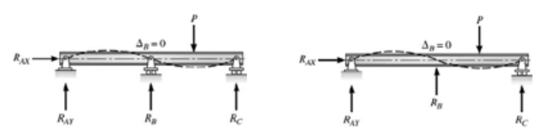


- To satisfy the original restraint at point B, we determine the deflection at B as a function of the redundant force R_B and set that equation equal to zero.

Indeterminate Structures

Flexibility or Force Method

- Selection of the redundant restraint

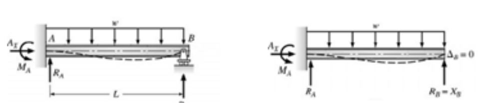


- The additional compatibility equation combined with the equations of equilibrium will account for all the indeterminate reactions.

Indeterminate Structures

Flexibility or Force Method

- Consider the following indeterminate beam

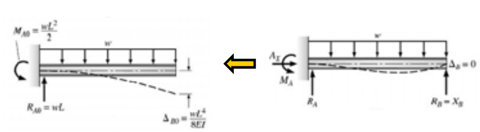


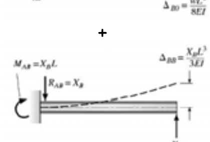
- One choice for the redundant in this problem is the reaction at point B.
- We know that the deflection at B is zero

Indeterminate Structures

Flexibility or Force Method

- Consider the following indeterminate beam





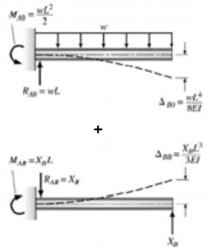
$$\Delta_B = 0$$

$$\Delta_{BO} + \Delta_{BB} = 0$$

Indeterminate Structures

Flexibility or Force Method

- Consider the following indeterminate beam

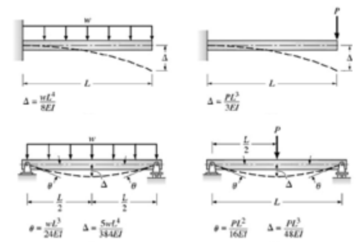


- The deflections at B can be evaluations with virtual work
- Another method would be to used tables for deflections of structures and the method of superposition.

Indeterminate Structures

Flexibility or Force Method

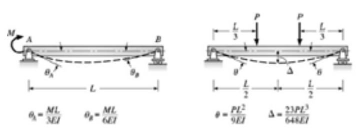
- A few tabulated values for deflections are given below:



Indeterminate Structures

Flexibility or Force Method

- A few tabulated values for deflections are given below:

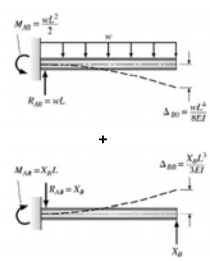


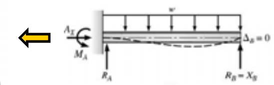
- As a sign convention, we will assume that displacements are positive (+) when they are in the direction of the redundant. You are free to choice the direction of the redundant.

Indeterminate Structures

Flexibility or Force Method

- Returning to our original example indeterminate beam





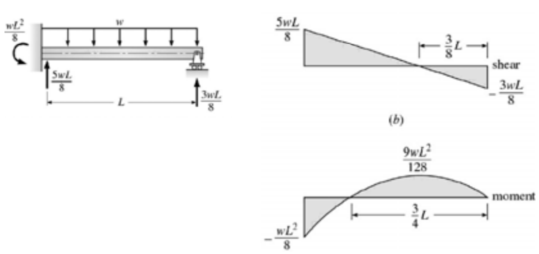
$$\Delta_{BO} + \Delta_{BB} = 0$$

$$-\frac{wL^4}{8EI} + \frac{X_B L^3}{3EI} = 0 \qquad X_B = \frac{3wL}{8}$$

Indeterminate Structures

Flexibility or Force Method

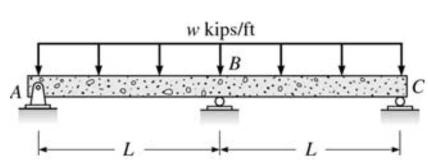
- Returning to our original example indeterminate beam



Indeterminate Structures

Flexibility or Force Method

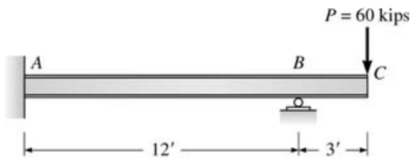
- Compute the reactions and draw the shear and moment curves for the following beam



Indeterminate Structures

Flexibility or Force Method

- Compute the reactions and draw the shear and moment curves for the following beam



End of Indeterminate Structures

Any questions?