

Truss - Virtual Work

- Virtual work is a procedure for computing a single component of deflection at any point on a structure.
- To compute a component of deflection by the method of virtual work, the designer applies a force to structure at the point and in the direction of the desired displacement.
- The force is called the *dummy load* or the *virtual load*.
- The force system created by the virtual load is called the *Q-system*.

Truss - Virtual Work

- The force system created by the *actual loads* is the the *P-system*.
- As the structure deforms under the actual loads, *external virtual work* W_Q is done by the virtual loads as they move through real displacements.
- Due to the conservation of energy an equivalent quantity of *virtual strain energy* U_Q is stored in the structure.

$$W_Q = U_Q$$

Truss - Virtual Work

- Consider the method of virtual work applied to one-bar truss, as shown below.

deflection

W_p - real work done by P

elongation

U_p - real strain energy stored in AB due to P

$W_p = U_p$

Truss - Virtual Work

- Now consider the forces and displacements produced by the virtual load, as shown below.

deflection

W_D - real work done by Q

elongation

U_D - real strain energy stored in AB due to Q

$W_D = U_D$

Truss - Virtual Work

- Now consider the forces and displacements produced by the virtual load and actual loads acting together.

deflection

W_T - total work done by Q and P

elongation

U_T - total strain energy due to Q and P

$W_T = U_T$

$W_Q = U_Q$

Truss - Virtual Work

- By the principle of conservation of energy, it follows that $W_Q = U_Q$.

deflection

W_T - total work done by Q and P

elongation

U_T - total strain energy due to Q and P

$W_Q = Q\delta_p$

Truss - Virtual Work

- By the principle of conservation of energy, it follows that $W_Q = U_Q$.

$$U_Q = F_Q \Delta L_P$$

Truss - Virtual Work

- By the principle of conservation of energy, it follows that $W_Q = U_Q$.

$$Q \delta_p = F_Q \Delta L_P$$

Truss - Virtual Work

- By summing the energy expression for each member in a truss, we get:

$$\sum Q \delta_p = \sum F_Q \Delta L_P$$
- The bar elongation ΔL_P can be compute is terms of the actual load P and the properties of the section.

$$\sum Q \delta_p = \sum F_Q \Delta L_P = \sum F_Q \left(\frac{F_P L}{AE} \right) = \sum \left(\frac{n N L}{AE} \right)$$
- Where n is the force in each member of the truss due to the virtual load and N is the force in each member of the truss due to the real loads.

Truss - Virtual Work

- Typically, the virtual load Q is assumed to be 1 and dimensionless.
- Therefore, the virtual work expression for the a single displacement component in the direction of the applied virtual load Q is:

$$\delta_p = \sum \left(\frac{n N L}{AE} \right)$$

Truss - Virtual Work Example 1

- Example:** Compute the horizontal displacement at joint B in the truss shown below. Assume the $E = 30,000 \text{ kip/in}^2$, the area of bars AB and BC = 5 in^2 ; the area of all other bars = 4 in^2 .

Truss - Virtual Work Example 1

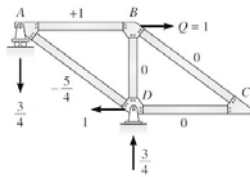
- Example:** Compute the horizontal displacement at joint B

- The value and direction of each real force is indicated on the truss.
- A (+) sign is tension and (-) sign is compression.

$$\text{Real Force System}$$

Truss - Virtual Work Example 1

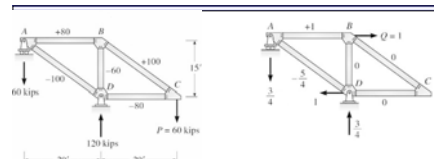
- Example: Compute the horizontal displacement at joint B



Virtual Force System

- The value and direction of each virtual force is indicated on the truss.
- A (+) sign is tension and (-) sign is compression.

Truss - Virtual Work Example 1



| Member | N (kips) | n | L (in) | A (in ²) | NnL/A (kip in/in ²) |
|--------|----------|-------|--------|----------------------|---------------------------------|
| AB | 80 | 1 | 240 | 4 | 4,800 |
| BC | 100 | 0 | 300 | 5 | 0 |
| CD | -80 | 0 | 240 | 4 | 0 |
| AD | -100 | -1.25 | 300 | 5 | 7,500 |
| BD | -60 | 0 | 180 | 4 | 0 |
| | | | | | 12,300 (kip/in) |

Truss - Virtual Work Example 1

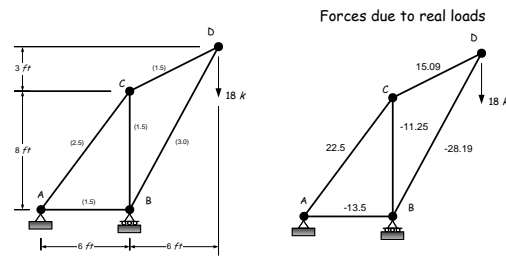
- Therefore the horizontal displacement at joint B may be computed as:

$$\delta_{Bx} = \frac{12,300 \text{ kip/in}}{30,000 \text{ kip/in}^2} = 0.41 \text{ in} \rightarrow$$

| Member | N (kips) | n | L (in) | A (in ²) | NnL/A (kip in/in ²) |
|--------|----------|-------|--------|----------------------|---------------------------------|
| AB | 80 | 1 | 240 | 4 | 4,800 |
| BC | 100 | 0 | 300 | 5 | 0 |
| CD | -80 | 0 | 240 | 4 | 0 |
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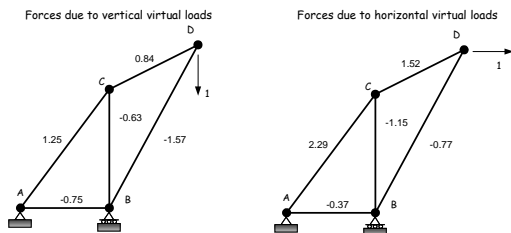
Truss - Virtual Work Example 2

- Example: Compute the magnitude and direction of the displacement at joint D in the truss shown below. Assume the E = 29,000 kip/in² and that the area of each bars is given in parenthesis (in²). See page 104 in the notes.



Truss - Virtual Work Example 2

- Example: Compute the magnitude and direction of the displacement at joint D in the truss shown below. Assume the E = 29,000 kip/in² and that the area of each bars is given in parenthesis (in²). See page 104 in the notes.



Truss - Virtual Work Example 2

- Example: Compute the magnitude and direction of the displacement at joint D in the truss shown below. Assume the E = 29,000 kip/in² and that the area of each bars is given in parenthesis (in²). See page 104 in the notes.

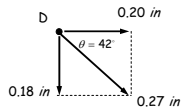
| Member | N (kips) | n _v | n _h | L (in) | A (in ²) | Nn _v L/A (kip/in) | Nn _h L/A (kip/in) |
|--------|----------|----------------|----------------|--------|----------------------|------------------------------|------------------------------|
| AB | -13.5 | -0.75 | -0.37 | 72 | 1.5 | 486.0 | 239.76 |
| AC | 22.5 | 1.25 | 2.29 | 120 | 2.5 | 1,350.0 | 2,473.20 |
| BC | -11.25 | -0.63 | -1.15 | 96 | 1.5 | 453.6 | 828.0 |
| BD | -28.19 | -1.57 | -0.77 | 150.35 | 3.0 | 2,218.08 | 1,087.92 |
| CD | 15.09 | 0.84 | 1.52 | 84.49 | 1.5 | 713.97 | 1,231.2 |
| | | | | | | 5,188.2 (kip/in) | 5,860.08 (kip/in) |

Truss - Virtual Work Example 2

- Therefore the total displacement a joint D may be computed as:

$$\delta_{D_x} = \frac{5,860.08 \text{ kip/in}}{29,000 \text{ kip/in}^2} = 0.20 \text{ in} \rightarrow$$

$$\delta_{D_y} = \frac{5,188.2 \text{ kip/in}}{29,000 \text{ kip/in}^2} = 0.18 \text{ in} \downarrow$$



$$\delta_D = \sqrt{(0.18 \text{ in})^2 + (0.20 \text{ in})^2} = 0.27 \text{ in}$$

$$\theta = \alpha \tan\left(\frac{0.18 \text{ in}}{0.20 \text{ in}}\right) = 42^\circ$$

End of Virtual Work - Trusses

Any questions?

