Shear and Moment Diagrams

- If the variation of $V$ and $M$ are written as functions of position, $x$, and plotted, the resulting graphs are called the shear diagram and the moment diagram.
- Developing the shear and moment functions for complex beams can be quite tedious.

Shear and Moment Diagrams

- We will develop a simpler method for constructing shear and moment diagrams.
- We will derive the relationship between loading, shear force, and bending moment.

Shear and Moment Diagrams

- Consider the beam shown below subjected to an arbitrary loading.
- We will assume that distributed loadings will be positive (+) if they act upward.

Shear and Moment Diagrams

- Let's draw a free body diagram of the small segment of length $\Delta x$ and apply the equations of equilibrium.

Shear and Moment Diagrams

- Since the segment is chosen at a point $x$ where there is no concentrated forces or moments, the result of this analysis will not apply to points of concentrated loading.

\[
\Delta V = w(x)\Delta x
\]

\[
\sum F_y = 0 = V + w(x)\Delta x - (V + \Delta V)
\]

\[
\sum M_0 = 0 = -M + (M + \Delta M) - V\Delta x -w(x)\Delta x \left( \frac{\Delta x}{2} \right)
\]

\[
\Delta M = V\Delta x + w(x)\left( \frac{\Delta x^2}{2} \right)
\]
Shear and Moment Diagrams

- Dividing both sides of the $\Delta V$ and $\Delta M$ expressions by $\Delta x$ and taking the limit as $\Delta x$ tends to 0 gives:

$$\frac{dV}{dx} = w(x) \quad \frac{dM}{dx} = V$$

Slope of shear curve = Intensity of the loading
Slope of moment curve = Intensity of the shear

Shear and Moment Diagrams

- The slope of the shear diagram at a point is equal to the intensity of the distributed loading $w(x)$ at that point. 

$$\frac{dV}{dx} = w(x) \quad \frac{dM}{dx} = V$$

Slope of shear curve = Intensity of the loading
Slope of moment curve = Intensity of the shear

Shear and Moment Diagrams

- The slope of the moment diagram at a point is equal to the intensity of the shear at that point. 

$$\frac{dV}{dx} = w(x) \quad \frac{dM}{dx} = V$$

Slope of shear curve = Intensity of the loading
Slope of moment curve = Intensity of the shear

Shear and Moment Diagrams

- If we multiply both sides of each of the above expressions by $dx$ and integrate:

$$\Delta V = \int w(x)dx \quad \Delta M = \int V(x)dx$$

Change in shear = Area under the loading
Change in moment = Area under the shear diagram

Shear and Moment Diagrams

- The change in shear between any two points is equal to the area under the loading curve between the points.

$$\Delta V = \int w(x)dx \quad \Delta M = \int V(x)dx$$

Change in shear = Area under the loading
Change in moment = Area under the shear diagram

Shear and Moment Diagrams

- The change in moment between any two points is equal to the area under the shear diagram between the points.

$$\Delta V = \int w(x)dx \quad \Delta M = \int V(x)dx$$

Change in shear = Area under the loading
Change in moment = Area under the shear diagram
Shear and Moment Diagrams

- Let's consider the case where a concentrated force and/or a couple are applied to the segment.

\[ \sum F_y = 0 = V + P - (V + \Delta V) \]

\[ \Delta V = P \]

\[ \sum M_x = 0 = M + (M + \Delta M) - V \Delta x - M' \]

\[ \Delta M = M' \]

Shear and Moment Diagrams

- Therefore, when a force $P$ acts downward on a beam, $\Delta V$ is negative so the "jump" in the shear diagrams is downward. Likewise, if $P$ acts upward, the "jump" is upward.

- When a couple $M'$ acts clockwise, the resulting moment $\Delta M$ is positive, so the "jump" in the moment diagrams is up, and when the couple acts counterclockwise, the "jump" is downward.

Shear and Moment Diagrams

- **Procedure for analysis** - the following is a procedure for constructing the shear and moment diagrams for a beam.

1. Determine the support reactions for the structure.

2. To construct the shear diagram, first, establish the $V$ and $x$ axes and plot the value of the shear at each end of the beam.

Since the $dV/dx = w$, the slope of the shear diagram at any point is equal to the intensity of the applied distributed loading.
Procedure for analysis - the following is a procedure for constructing the shear and moment diagrams for a beam.

The change in the shear force is equal to the area under the distributed loading.

If the distributed loading is a curve of degree \( n \), the shear will be a curve of degree \( n+1 \).

Find the support reactions

\[
\begin{align*}
\sum M_A &= 0 = -P(L + 2L) + B_y(3L) \\
\sum F_y &= 0 = A_y + B_y - 2P \\
\sum F_x &= 0 = A_y
\end{align*}
\]

The slope of the shear diagram over the interval \( 0 < x < L \) is the equal to the loading. In this case \( w(x) = 0 \).

At a point \( x = L \), a concentrated load \( P \) is applied. The shear diagram is discontinuous and "jumps" downward (recall \( \Delta V = -P \)).
Shear and Moment Diagrams

- The slope of the shear diagram over the interval $L < x < 2L$ is zero since, $w(x) = 0$.

- At $2L$, $P$ is applied and the shear diagram "jumps" downward (recall $\Delta V = -P$).

- The slope of the shear diagram over the interval $2L < x < 3L$ is zero since, $w(x) = 0$.

- The resulting shear diagram matches the shear at the right end determined from the equilibrium equations.

- Establish the $M$ and $x$ axes and plot the value of the moment at each end.

- In this case, the values are: at $x = 0$, $M = 0$; and at $x = 3L$, $M = 0$.

- The slope of the moment diagram over the interval $0 < x < L$ is the equal to value of the shear; in this case $V = P$. This indicates a positive slope of constant value.

- The change in moment is equal to the area under the shear diagram, in this case, $\Delta M = PL$. 

CIVL 3121 Shear Force and Bending Moment Diagrams
Shear and Moment Diagrams

- The slope of the moment diagram over the interval \( L < x < 2L \) is equal to the value of the shear; in this case \( V = 0 \).

![Diagram showing moment and shear forces]

The change in moment is equal to the area under the shear diagram, in this case, \( \Delta M = -PL \).

Shear and Moment Diagrams

- The slope of the moment diagram over the interval \( 2L < x < 3L \) is equal to the value of the shear, \( V = -P \).

![Diagram showing moment and shear forces]

The change in moment is equal to the area under the shear diagram, in this case, \( \Delta M = -PL \).

Shear and Moment Diagrams

- The shape of the shear and moment diagrams for selected loadings:

  - \( \frac{dV}{dx} = w \)
  - \( \frac{dM}{dx} = V \)

![Diagram showing shear and moment forces]

Shear and Moment Diagrams

- The shape of the shear and moment diagrams for selected loadings:

  - \( \frac{dV}{dx} = w \)
  - \( \frac{dM}{dx} = V \)

![Diagram showing shear and moment forces]
Shear and Moment Diagrams

- The shape of the shear and moment diagrams for selected loadings

<table>
<thead>
<tr>
<th>Loading</th>
<th>$w(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large</td>
<td>$(+)$ slope</td>
</tr>
<tr>
<td>Smaller</td>
<td>$(+)$ slope</td>
</tr>
<tr>
<td>Larger</td>
<td>$(-)$ slope</td>
</tr>
<tr>
<td>Small</td>
<td>$(-)$ slope</td>
</tr>
</tbody>
</table>

$$\frac{dV}{dx} = w$$

$$\frac{dM}{dx} = V$$

Shear and Moment Diagrams

- Draw the shear and moment diagrams for the following beam

1. Beam with a uniform load $w_0$ over the length $L$.
2. Beam with a uniform load $w_0$ over a portion of the length $L$.
3. Beam with a uniformly distributed load of $4 \text{ k/ft.}$ over a length of $18 \text{ ft.}$.
Shear and Moment Diagrams

Draw the shear and moment diagrams for the following beam:

- 12 ft.
- 4 k/ft.
- 60 k
- 8 ft.
- 100 k ft.

Shear and Moment Diagrams

Draw the shear and moment diagrams for the following beam:

- 10 ft.
- 600 lb.
- 4,000 lb. ft.
- 5 ft.
- 5 ft.

End of Internal Loads – Part 3

Any questions?