Shear and Moment Diagrams

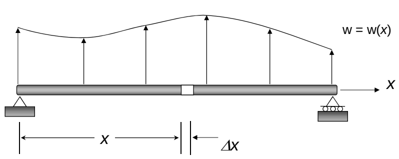
- If the variation of ***V*** and ***M*** are written as functions of position, *x*, and plotted, the resulting graphs are called the shear diagram and the moment diagram.
- Developing the shear and moment functions for complex beams can be quite tedious.

Shear and Moment Diagrams

- We will develop a simpler method for constructing shear and moment diagrams.
- We will derive the relationship between loading, shear force, and bending moment.

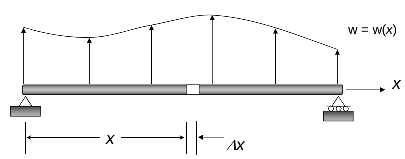
Shear and Moment Diagrams

- Consider the beam shown below subjected to an arbitrary loading.
- We will assume that *distributed loadings will be positive (+) if they act* ***upward.***



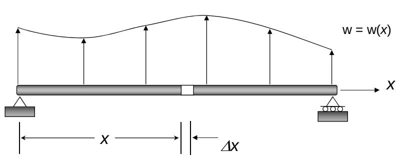
Shear and Moment Diagrams

- Let's draw a free body diagram of the small segment of length Δx and apply the equations of equilibrium.

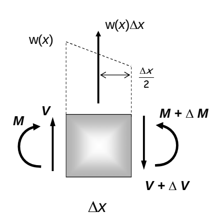


Shear and Moment Diagrams

- Since the segment is chosen at a point *x* where there is **no** concentrated forces or moments, the result of this analysis will **not** apply to points of concentrated loading



Shear and Moment Diagrams



$$\sum F_y = 0 = V + w(x)\Delta x - (V + \Delta V)$$

$$\boxed{\Delta V = w(x)\Delta x}$$

$$\sum M_0 = 0 = -M + (M + \Delta M) - V\Delta x - w(x)\Delta x\left(\frac{\Delta x}{2}\right)$$

$$\boxed{\Delta M = V\Delta x + \cancel{w(x)\frac{(\Delta x)^2}{2}}}$$

Shear and Moment Diagrams

- Dividing both sides of the ΔV and ΔM expressions by Δx and taking the limit as Δx tends to 0 gives:

$$\boxed{\frac{dV}{dx} = w(x)}$$

Slope of shear curve = Intensity of the loading

$$\boxed{\frac{dM}{dx} = V}$$

Slope of moment curve = Intensity of the shear

Shear and Moment Diagrams

- The slope of the shear diagram at a point is equal to the intensity of the distributed loading *w(x)* at that point

$$\boxed{\frac{dV}{dx} = w(x)}$$

Slope of shear curve = Intensity of the loading

$$\boxed{\frac{dM}{dx} = V}$$

Slope of moment curve = Intensity of the shear

Shear and Moment Diagrams

- The slope of the moment diagram at a point is equal to the intensity of the shear at that point.

$$\boxed{\frac{dV}{dx} = w(x)}$$

Slope of shear curve = Intensity of the loading

$$\boxed{\frac{dM}{dx} = V}$$

Slope of moment curve = Intensity of the shear

Shear and Moment Diagrams

- If we multiply both sides of each of the above expressions by *dx* and integrate:

$$\boxed{\Delta V = \int w(x)dx}$$

Change in shear = Area under the loading

$$\boxed{\Delta M = \int V(x)dx}$$

Change in moment = Area under the shear diagram

Shear and Moment Diagrams

- The change in shear between any two points is equal to the area under the loading curve between the points.

$$\boxed{\Delta V = \int w(x)dx}$$

Change in shear = Area under the loading

$$\boxed{\Delta M = \int V(x)dx}$$

Change in moment = Area under the shear diagram

Shear and Moment Diagrams

- The change in moment between any two points is equal to the area under the shear diagram between the points.

$$\boxed{\Delta V = \int w(x)dx}$$

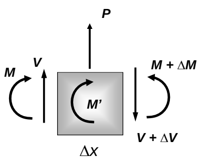
Change in shear = Area under the loading

$$\boxed{\Delta M = \int V(x)dx}$$

Change in moment = Area under the shear diagram

Shear and Moment Diagrams

- Let's consider the case where a concentrated force and/or a couple are applied to the segment.

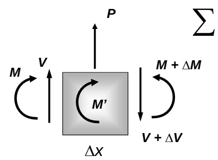


$$\sum F_y = 0 = V + P - (V + \Delta V)$$

$$\boxed{\Delta V = P}$$

Shear and Moment Diagrams

- Let's consider the case where a concentrated force and/or a couple are applied to the segment.



$$\sum M_0 = 0 = -M + (M + \Delta M) - V\Delta x - M'$$

$$\boxed{\Delta M = M'}$$

Shear and Moment Diagrams

- Therefore, when a force P acts downward on a beam, Δ**V** is negative so the "jump" in the shear diagrams is downward. Likewise, if P acts upward, the "jump" is upward.
- When a couple ***M*** ' acts clockwise, the resulting moment ΔM is positive, so the "jump" in the moment diagrams is up, and when the couple acts counterclockwise, the "jump" is downward.

Shear and Moment Diagrams

- **Procedure for analysis** - the following is a procedure for constructing the shear and moment diagrams for a beam .

1. Determine the support reactions for the structure.

Shear and Moment Diagrams

- **Procedure for analysis** - the following is a procedure for constructing the shear and moment diagrams for a beam .

2. To construct the shear diagram, first, establish the **V** and *x* axes and plot the value of the shear at each end of the beam.

Shear and Moment Diagrams

- **Procedure for analysis** - the following is a procedure for constructing the shear and moment diagrams for a beam .

Since the $d\mathbf{V}/dx = w$, the slope of the shear diagram at any point is equal to the intensity of the applied distributed loading.

Shear and Moment Diagrams

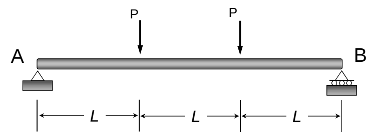
- **Procedure for analysis** - the following is a procedure for constructing the shear and moment diagrams for a beam .

  The *change in the shear force is equal to the area under the distributed loading*.

  If the distributed loading is a curve of degree *n*, the shear will be a curve of degree *n+1*.

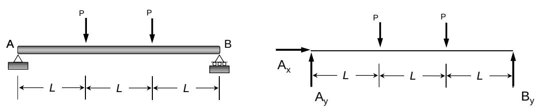
Shear and Moment Diagrams

- Draw the shear and moment diagrams for the following beam



Shear and Moment Diagrams

- Find the support reactions



$$\sum M_A = 0 = -P(L+2L) + B_y(3L) \qquad \boxed{B_y = P}$$

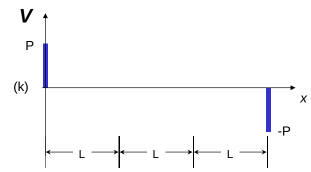
$$\sum F_y = 0 = A_y + B_y - 2P \qquad \boxed{A_y = P}$$

$$\sum F_x = 0 = A_x$$

Shear and Moment Diagrams

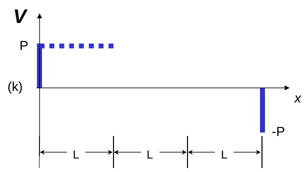
- Establish the ***V*** and *x* axes and plot the value of the shear at each end.

  In this case, the values are: at x = 0, ***V*** = P; and at x = 3L, ***V*** = -P.



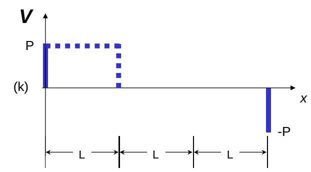
Shear and Moment Diagrams

- The slope of the shear diagram over the interval 0 < *x* < L is the equal to the loading. In this case *w(x)* = 0.



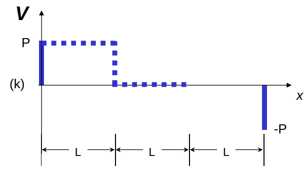
Shear and Moment Diagrams

- At a point *x* = L, a concentrated load P is applied. The shear diagram is discontinuous and “jumps” downward (recall Δ***V*** = -P).



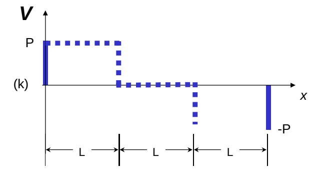
Shear and Moment Diagrams

- The slope of the shear diagram over the interval L < x < 2L is zero since, $w(x) = 0$.



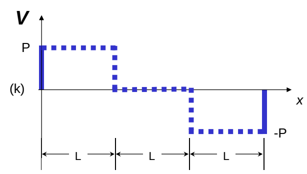
Shear and Moment Diagrams

- At 2L, P is applied and the shear diagram "jumps" downward (recall $\Delta \boldsymbol{V} = -P$).



Shear and Moment Diagrams

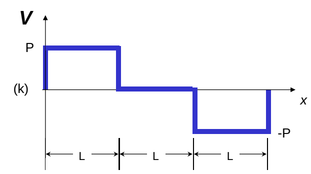
- The slope of the shear diagram over the interval 2L < x < 3L is zero since, $w(x) = 0$.



The resulting shear diagram matches the shear at the right end determined from the equilibrium equations.

Shear and Moment Diagrams

- The slope of the shear diagram over the interval 2L < x < 3L is zero since, $w(x) = 0$.

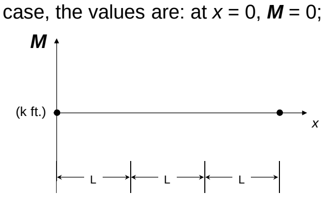


The resulting shear diagram matches the shear at the right end determined from the equilibrium equations.

Shear and Moment Diagrams

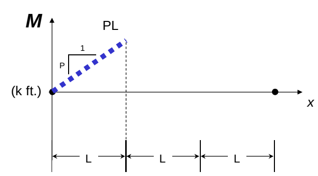
- Establish the ***M*** and x axes and plot the value of the moment at each end.

  In this case, the values are: at $x = 0$, $\boldsymbol{M} = 0$; and at $x = 3L$, $\boldsymbol{M} = 0$.



Shear and Moment Diagrams

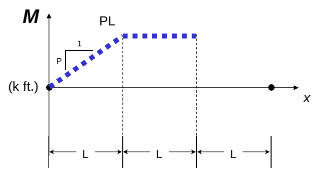
- The slope of the moment diagram over the interval 0 < x < L is the equal to value of the shear; in this case $\boldsymbol{V} = P$. This indicates a positive slope of constant value.



The change in moment is equal to the area under the shear diagram, in this case, $\Delta \boldsymbol{M} = PL$.

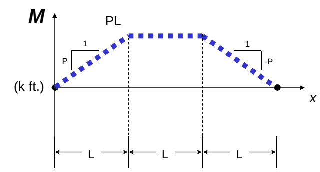
Shear and Moment Diagrams

- The slope of the moment diagram over the interval L < x < 2L is the equal to value of the shear; in this case $\boldsymbol{V}$ = 0.



Shear and Moment Diagrams

- The slope of the moment diagram over the interval 2L < x < 3L is the equal to value of the shear, $\boldsymbol{V}$ = -P.



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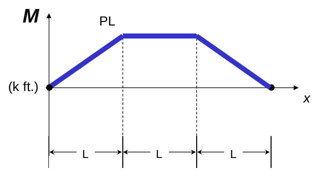
Shear and Moment Diagrams

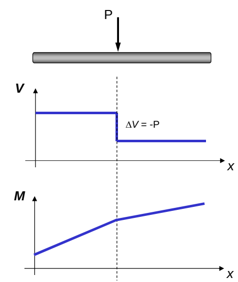
- The slope of the moment diagram over the interval 2L < x < 3L is the equal to value of the shear, $\boldsymbol{V}$ = -P.



The change in moment is equal to the area under the shear diagram, in this case, $\Delta \boldsymbol{M}$ = -PL.

Shear and Moment Diagrams

- The shape of the shear and moment diagrams for selected loadings



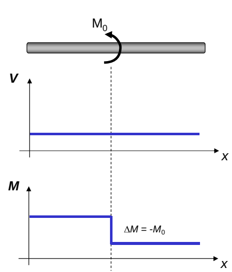
Loading $w(x)$

Shear Diagram $\frac{dV}{dx} = w$

Moment Diagram $\frac{dM}{dx} = V$

Shear and Moment Diagrams

- The shape of the shear and moment diagrams for selected loadings



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Shear and Moment Diagrams

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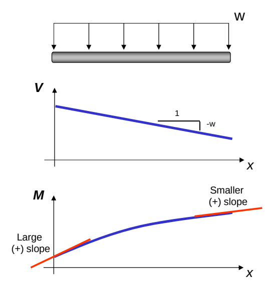

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Shear and Moment Diagrams

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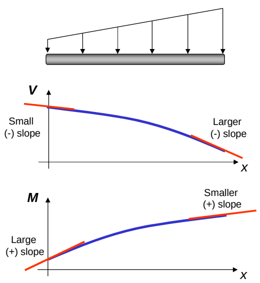

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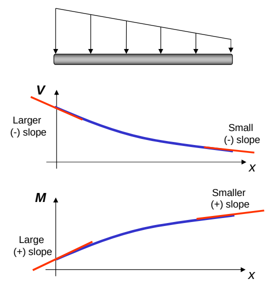


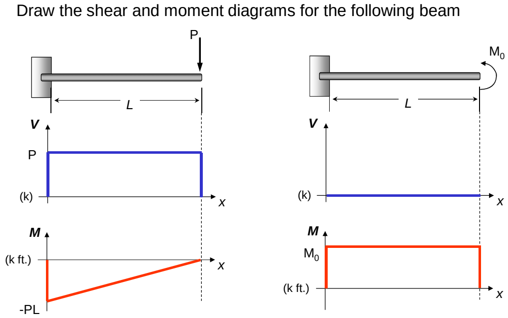
Loading $w(x)$

Shear Diagram $\frac{dV}{dx} = w$

Moment Diagram $\frac{dM}{dx} = V$

Shear and Moment Diagrams

- Draw the shear and moment diagrams for the following beam



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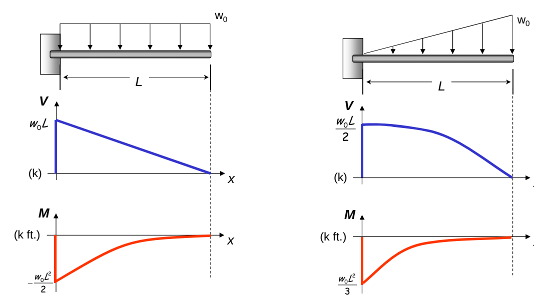

Shear and Moment Diagrams

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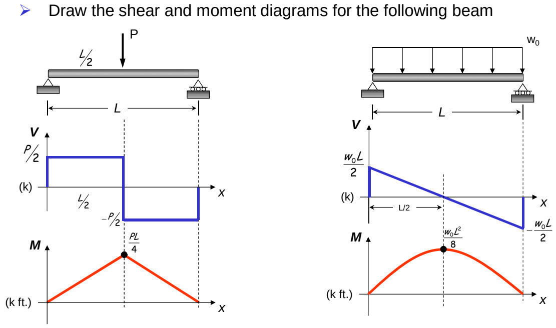

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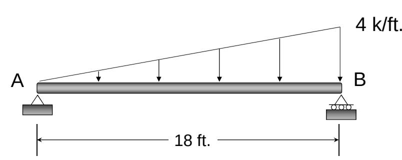

Shear and Moment Diagrams

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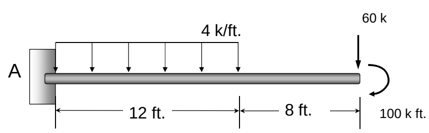

Shear and Moment Diagrams

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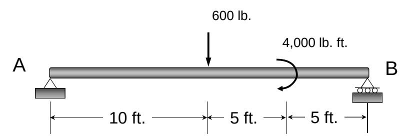

End of Internal Loads – Part 3

Any questions?