Analysis of Statically Determinate Structures

In general, the most common type of structure an engineer will analyze lies in a plane subject to a force system in the same plane, therefore we will concentrate on these structures.

Idealized Structures

In general, it is not possible to perform an exact analyze a of structure. Approximations for structure geometry, material parameters, and loading type and magnitude must be made.

- **Support connections** – Structural members may be joined in a variety of methods, the most common are pin and fixed joints

  A **pin** connection confines deflection and allows rotation

  A **fixed** connection confines deflection and rotation

  However, in reality, a pin connection has some resistance against rotation due to friction, therefore, a **torsional spring** connection may be more appropriate. If the stiffness k=0 the joint is a **pin**, if $k=\infty$, the joint is **fixed**.

  In general, co-planar supports can be idealized by noting how they *prevent* any degree of freedom or displacement of the member. In addition, the support will a force or moment on the structural member

- **Idealized Structures** – A complex structure may be idealized as a *line drawing* where orientation of members and type of connections are assumed.

  In many cases, loadings are transmitted to a structure under analysis by a secondary structure.

  In a *line drawing*, a pin support is represented by lines that do not touch and a fixed support by connecting lines

- **Tributary Loadings** – When frames or other structural members are analyzes, it is necessary to determine how walls, floors, or roofs transmit load to the element under consideration.

  A **one–way system** is typically a slab or plate structure supported along two opposite edges

  Examples, a slab of reinforced concrete with steel in one direction or a with steel in both directions with a span ratio $L_2/L_1 > 2$

  A **two–way system** is typically defined by a span ratio $L_2/L_1 < 2$ or if the all edges are supported

Principle Of Superposition

Basis for the theory of linear elastic structural analysis – *The total displacement or stress at a point in a structure subjected to several loadings can be determined by adding together the displacements or stresses caused by each load acting separately.*

There are two exceptions to these rule:

- If the material does not behave in a linear-elastic manner
- If the geometry of the structure changes significantly under loading (example, a column subjected to a bucking load)

Equations Of Equilibrium

From statics the equations of equilibrium are:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

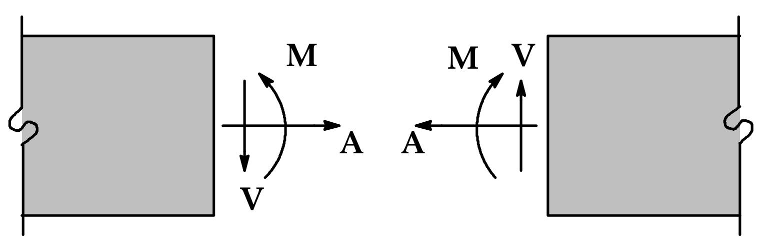
$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

However, since are dealing with co-planar structures the equations reduce to

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_z = 0$$

In order to apply these equations, we first must draw a **free-body diagram (FBD)** of the structure or its members. If the body is isolated from its supports, all forces and moments acting on the body are included.

If **internal loadings** are desired, the method of sections is used. A FBD of the cut section is used to isolate internal loadings. In general, internal loadings consist of an axial force **A**, a shear force **V**, and the bending moment **M**.



Internal Loadings

Determinacy And Stability

- **Determinacy** – provide both necessary and sufficient conditions for equilibrium.

  When all the forces in structure can be determined from the equations of equilibrium then the structure is considered *statically determinate*. If there are more unknowns than equations, the structure is *statically indeterminate*.

  For co–planar structures, there are three equations of equilibrium for each FBD, so that for n bodies and r reactions:

$r = 3n$	**statically determinate**
$r > 3n$	**statically indeterminate**

- **Stability** – Structures must be properly held or constrained by their supports

  Partial Constraints – a structure or one of its member with fewer reactive forces then equations of equilibrium

  Improper Constraints – the of reactions equals the number of equations of equilibrium, however, all the reactions are concurrent. In this case, the moment equations is satisfied and only two valid equations of equilibrium remain.

  Another case is when all the reactions are parallel

  In general, a structure is **geometrically unstable** if there are fewer reactive forces then equations of equilibrium.

$r < 3n$	**unstable**
$r \geq 3n$	**unstable** if reactions are concurrent or parallel or a collapsible mechanism

An unstable structure must be avoided in practice regardless of determinacy.

Application Of The Equations Of Equilibrium

- **Free-Body Diagram** – disassemble the structure and draw a free-body diagram of each member.
- **Equations of Equilibrium** – The total number of unknowns should be equal to the number of equilibrium equations