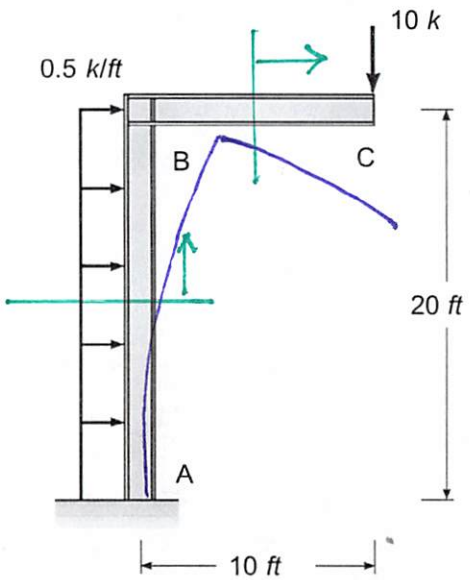
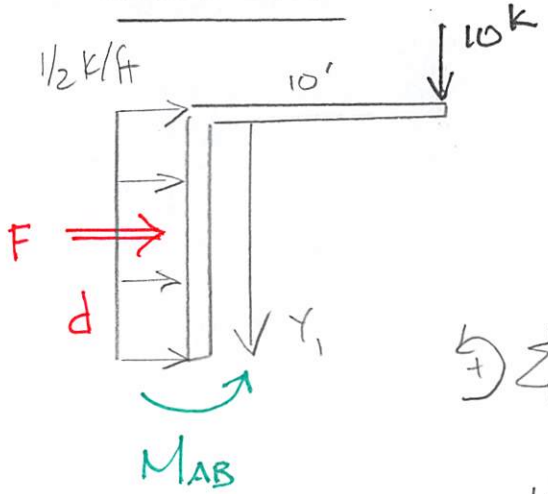
Example 8c-1: Compute the vertical deflection and rotation at point C on the frame shown. Include only the effects of the bending moment in your virtual work equations. Assume E = 29,000 *ksi* and I = 1,000 in^4.

Real loads



SECTION AB



$0 \le Y_1 \le 20\,ft$

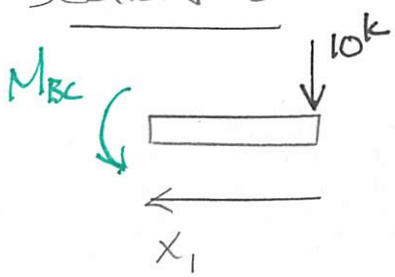
$F = \frac{1}{2}k/ft\,(Y_1) = \frac{Y_1}{2}\,k$

$d = \frac{1}{2}(Y_1)$

$\circlearrowleft + \sum M_{CUT} = 0 = M_{AB} - Fd - 10^k(10\,ft)$

$M_{AB} = \left[\frac{Y_1^2}{4} + 100\right] k\,ft$

SECTION BC



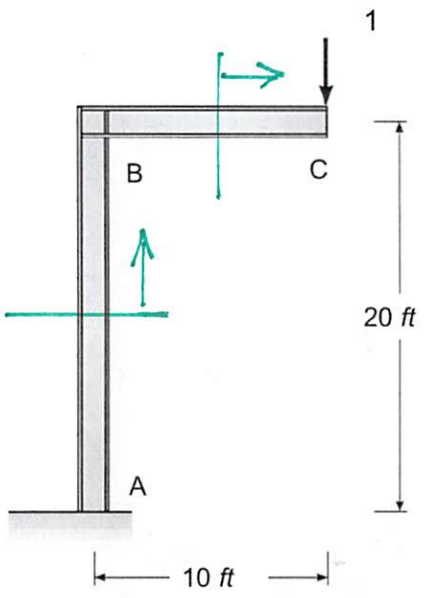
$0 \le X_1 \le 10\,ft$

$\circlearrowleft + \sum M_{CUT} = 0 = -M_{BC} - 10k(X_1)$

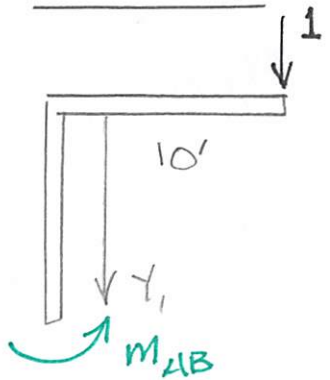
$M_{BC} = [10X_1]\,k\,ft$

Example 8c-1: Compute the vertical deflection and rotation at point C on the frame shown. Include only the effects of the bending moment in your virtual work equations. Assume E = 29,000 *ksi* and I = 1,000 in^4.

Virtual load



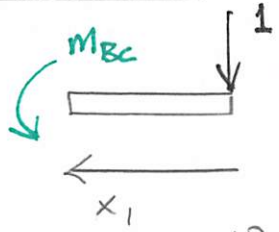
SECTION AB



$$\circlearrowleft + \Sigma M_{CUT} = 0 = M_{AB} - 1(10\,ft)$$

$$M_{AB} = 10\,ft$$

SECTION BC



$$\circlearrowleft + \Sigma M_{CUT} = 0 = M_{BC} - 1(x_1)$$

$$M_{BC} = x_1\,ft$$

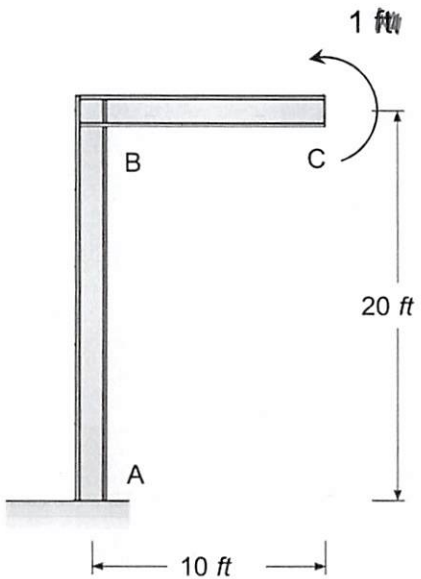
$$Y_C = \frac{1}{EI}\left[\int_0^{20}\left(\frac{Y_1^2}{4}+100\right)10\,dY_1 + \int_0^{10}(10x_1)x_1\,dx_1\right]$$

$[k\,ft]\quad [ft]\,[ft]$

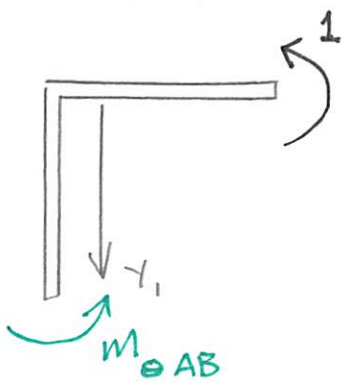
$$= \frac{1}{EI}\left[\frac{10Y_1^3}{12} + 1{,}000Y_1\right]_0^{20} + \frac{1}{EI}\left[\frac{10x_1^3}{3}\right]_0^{10} = \frac{30{,}000\,k\,ft^3}{EI} = 1.79\,in$$

Example 8c-1: Compute the vertical deflection and rotation at point C on the frame shown. Include only the effects of the bending moment in your virtual work equations. Assume E = 29,000 *ksi* and I = 1,000 in^4.

Virtual load



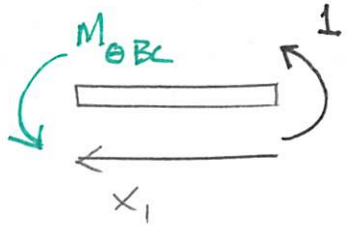
SECTION AB



$$\circlearrowleft + \sum M_{CUT} = 0 = M_{\theta AB} + 1$$

$$M_{\theta AB} = -1$$

SECTION BC



$$\circlearrowleft + \sum M_{CUT} = 0 = M_{\theta BC} + 1$$

$$M_{\theta BC} = -1$$

$$\theta_C = \frac{1}{EI}\left[\int_0^{20}\left(\frac{y_1^2}{4}+100\right)(-1)\,dy_1 + \int_0^{10}(10x_1)(-1)\,dx_1\right]$$

$[k\,ft]$ $[ft]$

$$= \frac{1}{EI}\left[-\frac{y_1^3}{12}-100y_1\right]_0^{20} + \frac{1}{EI}\left[-\frac{10x_1^2}{2}\right]_0^{10} = \frac{-9{,}500\,k\,ft^2}{3\,EI} = -0.0157\ RADS$$