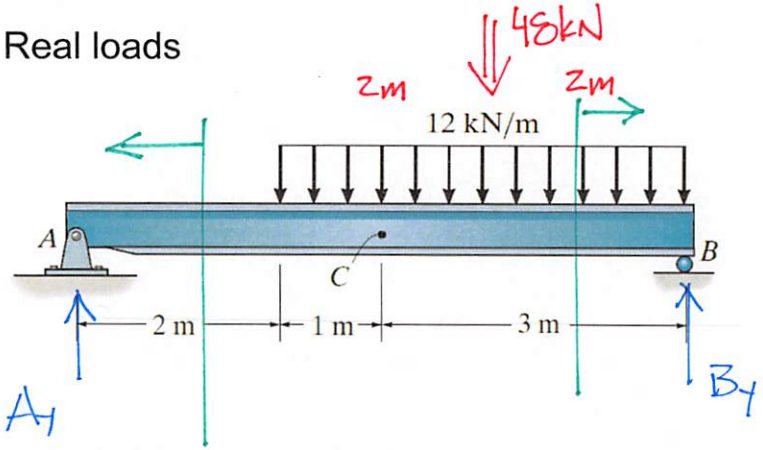
Problem 8b-5. Determine the displacement at $x = 2$ m. Use the principle of virtual work. *EI* is constant.

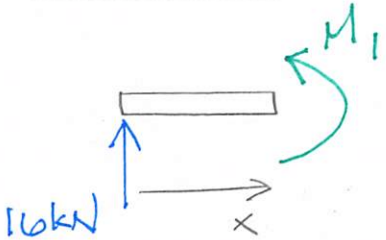
Real loads



$$\circlearrowleft + \sum M_B = 0 = 48\,kN(2m) - A_y(6m) \qquad A_y = 16\,kN$$

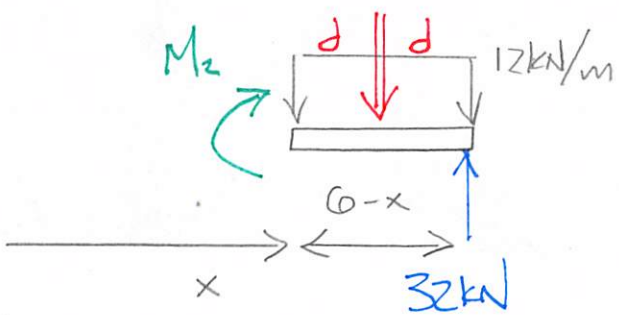
$$+\uparrow \sum F_y = 0 = A_y + B_y - 48\,kN \qquad B_y = 32\,kN$$

$0 \le x \le 2m$



$$\circlearrowleft + \sum M_{cut} = 0 = M_1 - 16\,kN(x) \qquad M_1 = [16x]\,kN\,m$$

$2 \le x \le 6m$



$$\circlearrowleft + \sum M_{cut} = 0 = -M_2 - 12(6-x)\tfrac{1}{2}(6-x) + 32(6-x)$$

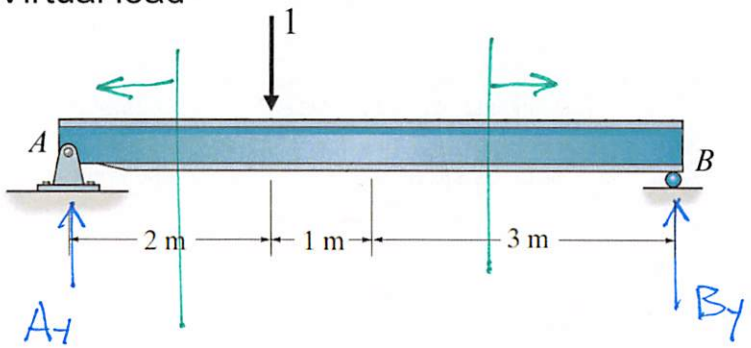
$$M_2 = \left[-6(6-x)^2 + 32(6-x)\right] kN\,m$$

$$M_1(x = 2m) = M_2(x = 2m) \checkmark$$

$d = \frac{1}{2}(6-x)$

Problem 8b-5. Determine the displacement at x = 2 m. Use the principle of virtual work. *EI* is constant.

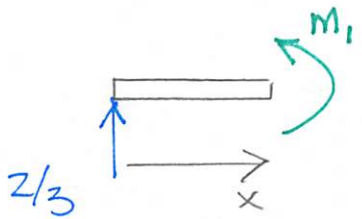
Virtual load



$$\circlearrowleft + \sum M_B = 0 = 1(4m) - A_y(6m) \qquad A_y = 2/3$$

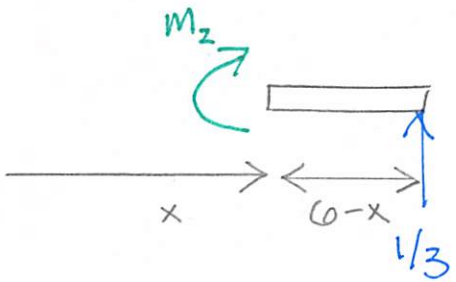
$$+\uparrow \sum F_y = 0 = A_y + B_y - 1 \qquad B_y = 1/3$$

$0 \le x \le 2m$



$$\circlearrowleft + \sum M_{cut} = 0 = m_1 - 2/3(x) \qquad m_1 = \frac{2x}{3}$$

$2 \le x \le 6m$



$$\circlearrowleft + \sum M_{cut} = 0 = -m_2 + \frac{1}{3}(6-x) \qquad m_2 = \frac{1}{3}(6-x)$$

$$m_1(x = 2m) = m_2(x = 2m) \quad \checkmark$$

Problem 8b-5. Determine the displacement at $x = 2$ m. Use the principle of virtual work. *EI* is constant.

$$Y(x=2m) = \int_0^6 \frac{Mm}{EI}dx = \frac{1}{EI}\left[\int_0^2 M_1 m_1 dx + \int_2^6 M_2 m_2 dx\right]$$

$$= \frac{1}{EI}\left[\frac{32x^2}{3}\Big|_0^2 + \left[\frac{x^4}{2} - \frac{76x^3}{9} + 44x^2 - 48x\right]_2^4\right]$$

$$= \frac{1}{EI}\left[\frac{256}{9} + \frac{896}{9}\right] = \underline{\underline{\frac{128\,kN\,m^3}{EI}}}$$