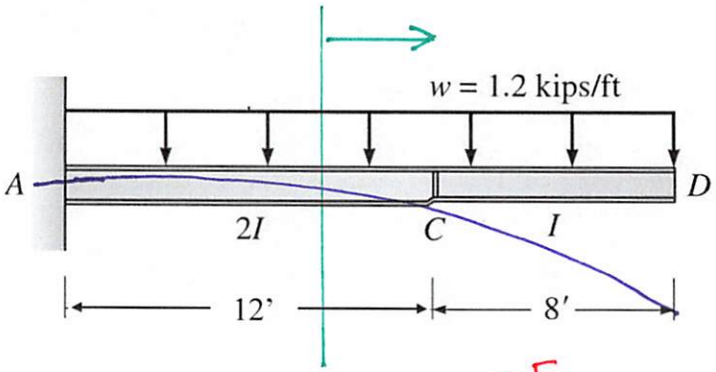
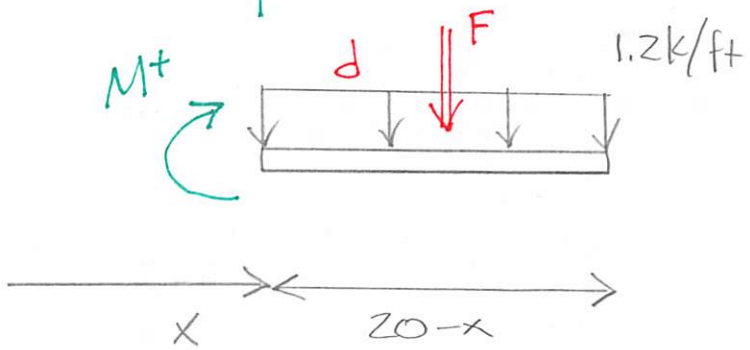
Example 8b-2: Determine the displacement at *D*. Assume $I = 400\ in^4$, $E = 29(10^3)\ ksi$.

Real loads





$\circlearrowleft + \sum M_{cut} = 0 = -M - Fd \qquad M = -Fd$

$M = -\left[1.2(20-x)\tfrac{1}{2}(20-x)\right] k\,ft$

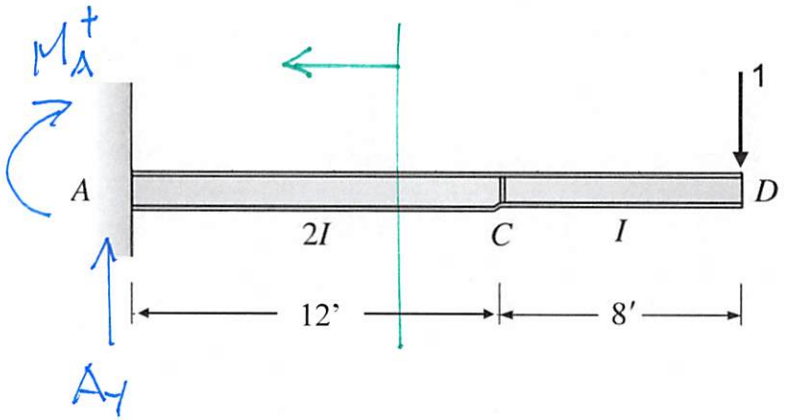
$M = -0.6(20-x)^2\ k\,ft$

$F = 1.2\,k/ft\,(20-x)\,ft = 1.2(20-x)^k$

$d = \tfrac{1}{2}(20-x)\,ft$

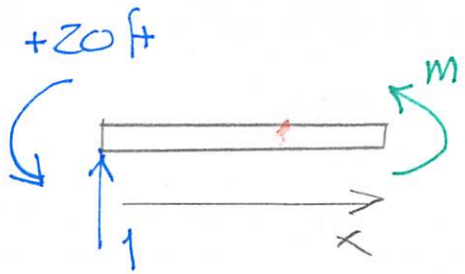
Example 8b-2: Determine the displacement at *D*. Assume $I = 400\ in^4$, $E = 29(10^3)\ ksi$.

Virtual load



$$\circlearrowleft^+ \Sigma M_A = 0 = -M_A - 1(20') \qquad M_A = -20\ ft\text{-}$$

$$+\uparrow \Sigma F_y = 0 = A_y - 1 \qquad A_y = 1$$



$$\circlearrowleft^+ \Sigma M_{cut} = 0 = m + 20\ ft - 1x \qquad m = (x - 20)\ ft$$

$$Y_D = \int_0^{20} \frac{Mm}{EI}\,dx = \frac{1}{EI}\left[\int_0^{12} \frac{Mm}{2}\,dx + \int_{12}^{20} Mm\,dx\right]$$

$$= \frac{1}{EI}\left[\int_0^{12} \frac{-0.6(20-x)^2}{2}(x-20)\,dx + \int_{12}^{20} -0.6(20-x)^2(x-20)\,dx\right]$$

$$= \frac{1}{EI}\left[-\frac{3(x-20)^4}{40}\Big|_0^{12} - \frac{3(x-20)^4}{20}\Big|_{12}^{20}\right] = \frac{61{,}536\ kft^3}{5EI}$$