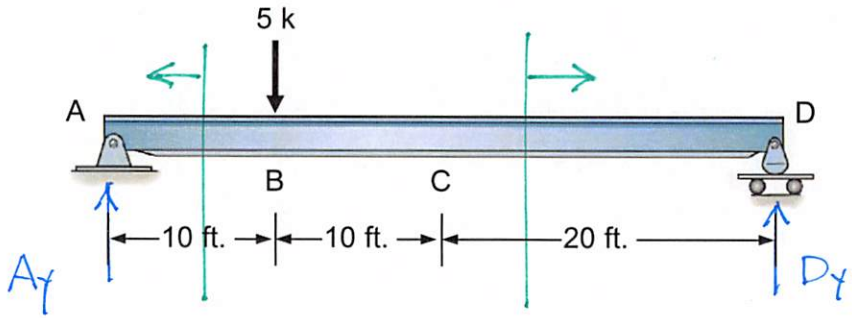
Example 8b-1: Determine the displacement at C. Assume $I = 240\ in^4$, $E = 29(10^3)\ ksi$.

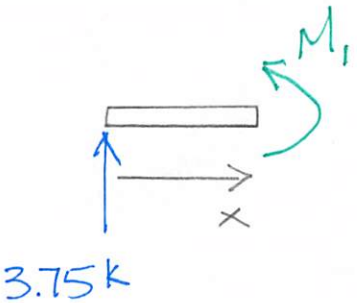
Real loads



$$\circlearrowleft + \sum M_D = 0 = 5^k(30\,ft) - A_y(40\,ft) \qquad A_y = 3.75^k$$

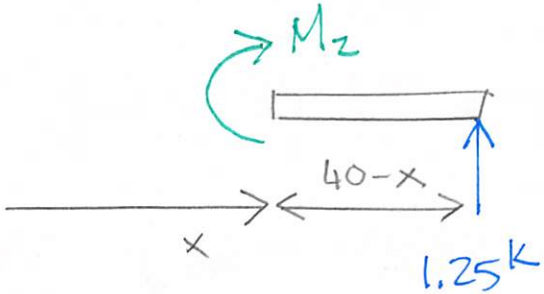
$$+\uparrow \sum F_y = 0 = A_y + D_y - 5^k \qquad D_y = 1.25^k$$

$0 \le x \le 10\,ft$



$$\circlearrowleft + \sum M_{cut} = 0 = M_1 - 3.75k(x) \qquad M_1 = [3.75x]\ k\,ft$$

$10 \le x \le 40\,ft$

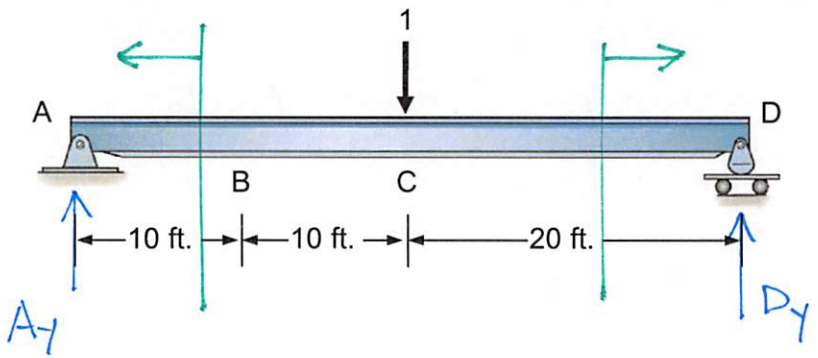


$$\circlearrowleft + \sum M_{cut} = 0 = -M_2 + 1.25^k(40-x) \qquad M_2 = [1.25(40-x)]\ k\,ft$$

$$M_1(x=10') = M_2(x=10') \checkmark$$

Example 8b-1: Determine the displacement at C. Assume $I = 240\ in^4$, $E = 29(10^3)\ ksi$.

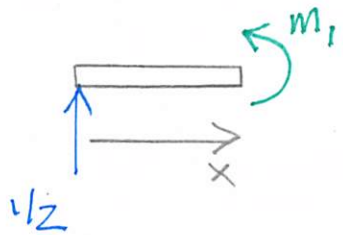
Virtual load



$$\circlearrowleft +\sum M_D = 0 = 1(20\,ft) - A_y(40\,ft) \qquad A_y = 1/2$$

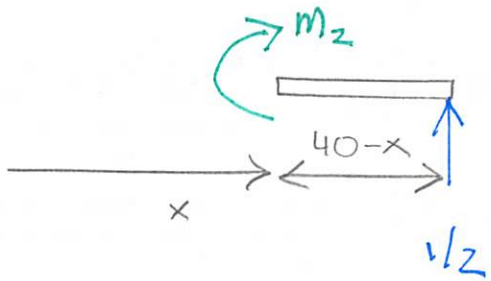
$$+\uparrow \sum F_y = 0 = A_y + D_y - 1 \qquad D_y = 1/2$$

$0 \le x \le 20\,ft$



$$\circlearrowleft +\sum M_{cut} = 0 = m_1 - 1/2(x) \qquad m_1 = [x/2]\ ft$$

$20 \le x \le 40\,ft$



$$\circlearrowleft +\sum M_{cut} = 0 = -m_2 + 1/2(40-x) \qquad m_2 = 1/2(40-x)\ ft$$

$$m_1(x=20\,ft) = m_2(x=20\,ft) \checkmark$$

Example 8b-1: Determine the displacement at C. Assume $I = 240\ \text{in}^4$, $E = 29(10^3)\ ksi$.

$$\Delta_C = \int_0^{40} \frac{Mm}{EI}\,dx = \frac{1}{EI}\left[\int_0^{10} M_1 m_1\,dx + \int_{10}^{20} M_2 m_1\,dx + \int_{20}^{40} M_2 m_2\,dx\right] \quad *$$

$$= \frac{1}{EI}\left[\int_0^{10} \overbrace{(3.75x)}^{M_1}\overbrace{(x/2)}^{m_1}dx + \int_{10}^{20} \overbrace{1.25(40-x)}^{M_2}\overbrace{(x/2)}^{m_1}dx + \int_{20}^{40} \overbrace{1.25(40-x)}^{M_2}\overbrace{\tfrac{1}{2}(40-x)}^{m_2}dx\right]$$

$$\underbrace{[k\text{-}ft]\ [ft]\ [ft]}_{k\text{-}ft^3}$$

$$= \frac{1}{EI}\left[\frac{5x^3}{8}\Big|_0^{10} - \frac{5x^2(x-60)}{24}\Big|_{10}^{20} + \frac{5x(x^2-120x+4{,}800)}{24}\Big|_{20}^{40}\right]$$

$$= \frac{1}{EI}\left[625 + \frac{6875}{3} + \frac{5000}{3}\right] = \frac{13{,}750\ k\text{-}ft^3}{3EI}$$

$$= \frac{13{,}750\ k\text{-}ft^3}{29{,}000\ k/in^2} \cdot \frac{1}{240\ in^4} \cdot \frac{1{,}728\ in^3}{ft^3} = 1.14\ in$$