CIVL 2131 - Statics

Resolution of a vector along an axes system
Cartesian Coordinates

Objectives

- Resolve a resultant vector into its components given any pair of axes
- View and Review the Cartesian Coordinate/axis system as a basis for vector operations

Madness takes its toll. Please have exact change.
Tools

- Law of Sines
- Law of Cosines
- Basic Trigonometry
- Pythagorean Theorem

Review

- Both Forces and Moments can be mathematically represented as vectors which allows us to have both a consistent representation and a convenient set of tools for manipulating the elements for analysis
Review

- Vectors are mathematical representations which have two components
- Magnitude and
- Direction

Resolving a Vector along Axes

- This time we will start with the resultant and try to find the components that were combined to get the resultant
- We will need to be given the directions of the axes
- Remember, they do not need to look like our typical x and y axes
Resolving a Vector along Axes

- If we are given two axes, a’ and b’

We would like to find the components of the vector F that lie along the a’ and b’ axes
Resolving a Vector along Axes

- The \(a'\) and \(b'\) axes in this case are not perpendicular (though they may appear so in my drawing).

When we added vectors, we chose one to remain stationary and one to move.
Resolving a Vector along Axes

- This time, we will keep one axis stationary and move the other axis so that it intersects with the head of the resultant.

Resolving a Vector along Axes

- Keeping the \( a' \) axis fixed, we move the \( b' \) axis until it intersects the head of the resultant.
Resolving a Vector along Axes

- Now, one of the components will go from the tail of the resultant to the intersection of the $a'$ axis and the shifted $b'$ axis.

- The other component will go from the head of the vector just constructed to the head of the resultant.
Resolving a Vector along Axes

Knowing some magnitudes and angles, you can then find the components using trig and geometry.

F2-5. The force $F = 450$ lb acts on the frame. Resolve the force into components acting along members AB and AC, and determine the magnitude of each component.
There is a special case when the axes are perpendicular

The coordinate system or axis system in this case is known as a Cartesian system

This is the case that we typically see

In the most common 2D case, we have the traditional x and y axis as we developed in the previous class

Combining component forces in this type of system allows for the use of the Pythagorean formula
For example, consider two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ whose directions are perpendicular to each other.

We can extend their lines of action to the point where they intersect.

Remember that force vectors are sliding vectors and so they can be moved anywhere along their line of action.

The line of action of a vector is the line that would be generated if we extended the vector infinitely in both directions.
Cartesian Coordinates

- So we can move (slide) the vectors until we reach a point where the tails of the vectors intersect

![Diagram of two vectors intersecting]

- We can set the origin of our coordinate system at this intersection

![Diagram of vectors with origin moved]
Cartesian Coordinates

- If we place a Cartesian (right angle) set of axis on the forces, we can see that they are also perpendicular.

![Diagram](image1)

- Again for convenience, we will label the horizontal axis as the x-axis, and the vertical axis as the y-axis.

![Diagram](image2)
Cartesian Coordinates

- We can move our new reference axis to coincide with the intersection of the two lines of action or the origin we specified.

- We are going to use the same method to add there two forces (vectors) together that we used before.
Cartesian Coordinates

- We move the $\mathbf{F}_1$ vector so that its tail lies on the head of the $\mathbf{F}_2$ vector

\[ \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 \]
In this case, since $\vec{F}_2$ lies along the x-axis, it is known as the x-component of $\vec{F}$.

\[ \vec{F} = \vec{F}_1 + \vec{F}_2 \]

In this case, since $\vec{F}_1$ lies along the y-axis, it is known as the y-component of $\vec{F}$.

\[ \vec{F} = \vec{F}_1 + \vec{F}_2 \]
Cartesian Coordinates

- We often rewrite this as

\[ \mathbf{F} = \mathbf{F}_x + \mathbf{F}_y \]

Notice that the resultant makes an angle \( \alpha \) that is CCW with the positive x-axis.
Cartesian Coordinates

- From the drawing, we have the following relationships

\[ \tan(\alpha) = \frac{F_y}{F_x} \]

\[ F_y = F \sin(\alpha) \]

\[ F_x = F \cos(\alpha) \]

- All of these are drawn from the basic trig relationships

\[ \tan(\alpha) = \frac{F_y}{F_x} \]

\[ F_y = F \sin(\alpha) \]

\[ F_x = F \cos(\alpha) \]
Knowing the magnitudes of $F_x$ and $F_y$, we can calculate the magnitude of the resultant $F$ using the Pythagorean theorem.

$$F = \sqrt{(F_x)^2 + (F_y)^2}$$

Unfortunatly, component vectors don’t always lie along the $+x$ and $+y$ axis, nor do resultants always sit so nicely in the $+x/+y$ plane.

We need to look at other cases.
Cartesian Coordinates

- For example, if we had a system like this one

Here the x component is in the negative x-direction and the y component is in the positive y-direction.
Cartesian Coordinates

- If we use our same method we can move either vector so that the tail of the moved vector is on the head of the stationary vector.

- Here the \( F_y \) vector is stationary and \( F_x \) vector is moved.
**Cartesian Coordinates**

- We can still use our trig relationships to calculate the components but we have to be careful of just what angle we are looking at.

![Diagram showing Cartesian Coordinates]

**Cartesian Coordinates**

- In this case, our angle is the CCW angle from the +y axis.
- This is no special angle, just what were given.

![Diagram showing Cartesian Coordinates with angle α]
Cartesian Coordinates

For example, if we had a system like this one

\[ \tan(\alpha) = \frac{F_x}{F_y} \]

\[ F_x = F \sin(\alpha) \]

\[ F_y = F \cos(\alpha) \]

Please remember that this way will only give you the magnitude of the components, not their direction.

\[ \tan(\alpha) = \frac{F_x}{F_y} \]

\[ F_x = F \sin(\alpha) \]

\[ F_y = F \cos(\alpha) \]
**Cartesian Coordinates**

- The direction (sign) of the components will be based on the directions of the +x and +y axes.

\[
\tan(\alpha) = \frac{F_x}{F_y}
\]

\[
F_x = F \sin(\alpha)
\]

\[
F_y = F \cos(\alpha)
\]

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**Cartesian Coordinates**

- REMEMBER, draw the picture before you try to use trig, even though the trig is always consistent, it does depend on where you draw the angle.
2-42 (not complete). Determine the components of the forces $F_A$ and $F_B$ along the $x$ and $y$ axes if $F_B = 600 \text{ N}$ and $\theta = 20^\circ$. 

$F_A = 700 \text{ N}$