A hole has been found in the nudist camp wall. The police are looking into it.
Objectives

- Understand the concept and representation of a distributed load
- Understand how to convert a distributed load into an equivalent point load for analysis purposes
Tools

- Basic Trigonometry
- Pythagorean Theorem
- Algebra
- Visualization
Distributed Loads

- We will do a lot more on this later in the semester but for now an introduction is in order to be able to work certain types of problems.
Distributed Loads

- Everything we have worked with so far represented point forces, forces that were concentrated at a single point.
- Very often we have loads that need to be reduced to points.
- These loads are distributed in space according to some loading variation and need to be reduced to point forces to help in our analysis.
Distributed loads are noted in many ways, your text chooses to denote a distributed load as a series of lines with arrows whose tails are connected with a straight line.

For example:

![Distributed Load Diagram](image)
Distributed Loads

- This represents a loading which has a uniform intensity of 15 lb/ft over the entire length of the loading.
- Notice the units of the intensity.
Distributed Loads

- The units are not units of force but _units of loading intensity_

- Units of force wouldn’t make any sense as we shall see when we convert this distributed load to an equivalent point load
Distributed Loads

- Now if we wanted to convert this to a point force, we need to find the area under the loading curve or loading diagram.
- In this case, the area of the rectangle containing all the arrows is the area under the loading curve.
If the 15lb/ft loading intensity extends over 10 feet, then we can calculate the equivalent force developed by the point load by calculating the area of the rectangle.
Distributed Loads

- The magnitude of the equivalent point force developed by this distributed load is equal to the base of the loading times the height.

\[10 \text{ ft} \times 15 \frac{\text{lb}}{\text{ft}}\]
Distributed Loads

- So the magnitude of the equivalent point force is 150 lb

\[10 \text{ ft} \times 15 \frac{\text{lb}}{\text{ft}} = 150 \text{lb}\]
Now we know the magnitude of the distributed load but not where the line of action is located.

\[ 10 \text{ ft} \times 15 \frac{\text{lb}}{\text{ft}} = 150 \text{ lb} \]
Distributed Loads

- The line of action of the distributed load is through the centroid of the loading diagram.
- You will see that again later in the semester.

\[ 10 \text{ ft} \times 15 \frac{lb}{\text{ft}} = 150 lb \]
Distributed Loads

- For now, I want you to remember the location for two types of loadings.
- For a rectangle, the centroid is located at the midpoint of the base.

\[
10 \text{ ft} \times 15 \frac{lb}{\text{ft}} = 150 lb
\]
Distributed Loads

- So if we replace the distributed load with a point load we have

\[
\begin{align*}
\text{5 ft} & \quad \text{15 lb/ft} \\
\text{10 ft} & \quad \text{150 lb}
\end{align*}
\]
Distributed Loads

- For a rectangular (uniform) distributed load
  - The magnitude of the equivalent point force is equal to the load intensity times the length over which the loading acts
  - The line of action of the equivalent point force is located at \( \frac{1}{2} \) the base over which the loading acts
  - The equivalent force has the same direction as the original distributed loading
Distributed Loads

- A second type of distributed loading that we will consider is a linearly increasing (or decreasing) loading.
- This is represented again as a series of arrows but the lengths of the arrows will increase or decrease in length across the loading.
Distributed Loads

- For example, if we have a distributed loading that starts at 0 lb/ft and over 20 feet increases to an intensity of 100 lb/ft
Distributed Loads

- Usually you won’t see the 0 lb/ft written out, it will just be assumed.
Distributed Loads

As with the uniform loading, the equivalent point force magnitude will be equal to the area under the loading diagram.
Distributed Loads

In this case, it is the area of a triangle, \( \frac{1}{2} \) base times height.

\[
\frac{20 \text{ ft}}{2} \times 100 \frac{\text{lb}}{\text{ft}} = 1000 \text{lb}
\]
Distributed Loads

- The location of the line of action for the equivalent point load is again at the centroid of the loading which in this case is \( \frac{2}{3} \) of the distance along the base.

\[
\frac{20 \text{ ft}}{2} \times 100 \frac{\text{lb}}{\text{ft}} = 1000 \text{ lb}
\]
Distributed Loads

If you will always measure from the smallest point on the loading diagram you won't get confused.

You may see the loading decreasing from right to left.

\[
\frac{20 \text{ ft}}{2} \times 100 \frac{\text{lb}}{\text{ft}} = 1000 \text{lb}
\]
Distributed Loads

- We locate the smallest intensity of the loading.
- In this case, it is the left edge of the loading diagram.

\[
\frac{20 \text{ ft}}{2} \times 100 \frac{\text{lb}}{\text{ft}} = 1000 \text{lb}
\]
Distributed Loads

- The location of the line of action is $2/3$ of the length of the base away from this point.

$$\frac{20 \text{ ft}}{2} \times 100 \frac{\text{lb}}{\text{ft}} = 1000 \text{lb}$$
So the equivalent point force to our linearly varying distributed load is

\[
\frac{20 \text{ ft}}{2} \times 100 \frac{\text{lb}}{\text{ft}} = 1000 \text{ lb}
\]
Distributed Loads

- At times, we combine the two shapes to form a more complex loading pattern.
Distributed Loads

- We break the pattern into easier form to analyze
Distributed Loads

- Taking care to transform the limits of the loadings correctly

Notice that the upper loading (the triangular loading) goes from 0 lb/ft at the left to 100 lb/ft at the right.
Distributed Loads

- Taking care to transform the limits of the loadings correctly

The rectangular loading has a uniform intensity of 100 lb/ft across its entire length.
Distributed Loads

- We can start with the top (blue) loading

The rectangular loading has a uniform intensity of 100 lb/ft across its entire length.
The magnitude is equal to the area of the rectangle and the location is 2/3 of the distance from the smallest loading.

The rectangular loading has a uniform intensity of 100 lb/ft across its entire length.
Now we can change the bottom (red) distributed load into a point load.

The rectangular loading has a uniform intensity of 100 lb/ft across its entire length.
Distributed Loads

Now we can change the bottom (red) distributed load into a point load.

The rectangular loading has a uniform intensity of 100 lb/ft across its entire length.
Distributed Loads

- While we can work with this, we often want to reduce the system to a single equivalent force.
Distributed Loads

- We can do this by selecting a point on the beam and looking at the net translational and rotational effects on the system.
Distributed Loads

- We can choose the left end of the beam as our reference point.
Distributed Loads

- The net translational effect on the system is just the sum of the forces. Since they are both in the same direction, we can just use scalars.

\[ F = 1000\text{lb} + 2000\text{lb} \]
\[ F = 3000\text{lb} \]
Distributed Loads

- Now we can calculate the net rotational effect about the left end of the beam by summing moments about that point.

\[ F = 1000\, lb + 2000\, lb \]
\[ F = 3000\, lb \]
Distributed Loads

- Now we can calculate the net rotational effect about the left end of the beam by summing moments about that point.

\[ F = \frac{40}{3} \text{ ft} \left( 1000 \text{ lb} \right) - \left( 10 \text{ ft} \right) \left( 2000 \text{ lb} \right) \]

\[ M = -33,333.33 \text{ ft} - \text{ lb} \]
Distributed Loads

- Now we know the translation effect and the rotational effect.
- We have to position to the force to get the same rotation.

\[ F = 1000\,\text{lb} + 2000\,\text{lb} \]
\[ F = 3000\,\text{lb} \]

\[ M = -\left(\frac{40}{3}\,\text{ft}\right)(1000\,\text{lb}) - (10\,\text{ft})(2000\,\text{lb}) \]
\[ M = -33,333.33\,\text{ft} - \text{lb} \]
Remembering that the magnitude of the moment is the perpendicular distance times the force we have.

\[
M = d \times F
\]

\[
33,333.33\text{ft} - \text{lb} = d \times 3000\text{lb}
\]

\[
d = \frac{33,333.33\text{ft} - \text{lb}}{3000\text{lb}} = 11.11\text{ft}
\]
Since the sense of the moment is CW about the left end of the system, we know that the distance is to the right.

\[
M = d \perp F \\
33,333.33 \text{ft} - \text{lb} = d \perp \times 3000 \text{lb} \\
d \perp = \frac{33,333.33 \text{ft} - \text{lb}}{3000 \text{lb}} = 11.11 \text{ft}
\]
Distributed Loads

So our equivalent single force system is

\[ M = d \times F \]

\[ 33,333.33\text{ft} - \text{lb} = d \times 3000\text{lb} \]

\[ d = \frac{33,333.33\text{ft} - \text{lb}}{3000\text{lb}} = 11.11\text{ft} \]

11.11 ft

3000 lb

20 ft