Let’s use the failure models to predict the ultimate strength-to-weight (SWR) of one of our reinforced concrete beams from lab.

Consider a beam with the following characteristics:

- Concrete strength $f'_c = 6,000$ psi
- Steel strength $f_y = 60,000$ psi
- The tension reinforcement will be 2 #4 rebars
- The shear reinforcement will be #3 rebars installed vertically at 3 in. spacing
- Use the minimum concrete cover of 1 in. and a bar spacing of 1 in.

Reinforced Concrete Beam Example #4

Based on the choice of reinforcement we can compute an estimate of $b$ and $d$:

- Minimum cover
- #4 rebar diameter
- Space between bars

$$ b \geq 2(1.0 \text{ in.}) + 2(0.5 \text{ in.}) + (1.0 \text{ in.}) = 4 \text{ in.} $$

Reinforcing bars are denoted by the bar number. The diameter and area of standard rebars are shown below.

<table>
<thead>
<tr>
<th>Bar #</th>
<th>Diameter (in.)</th>
<th>$A_s$ (in.$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.375</td>
<td>0.11</td>
</tr>
<tr>
<td>4</td>
<td>0.500</td>
<td>0.20</td>
</tr>
<tr>
<td>5</td>
<td>0.625</td>
<td>0.31</td>
</tr>
<tr>
<td>6</td>
<td>0.750</td>
<td>0.44</td>
</tr>
<tr>
<td>7</td>
<td>0.875</td>
<td>0.60</td>
</tr>
<tr>
<td>8</td>
<td>1.000</td>
<td>0.79</td>
</tr>
<tr>
<td>9</td>
<td>1.128</td>
<td>1.00</td>
</tr>
<tr>
<td>10</td>
<td>1.270</td>
<td>1.27</td>
</tr>
<tr>
<td>11</td>
<td>1.410</td>
<td>1.56</td>
</tr>
</tbody>
</table>

If we allow a minimum cover under the rebars we can estimate $d$:

$$ d = 6 - 1.0 - \frac{0.5}{2} = 4.75 \text{ in.} $$

The $A_s$ for two #4 rebars is:

$$ A_s = 2(0.2 \text{ in.$^2$}) = 0.4 \text{ in.$^2$} $$

$$ a = \frac{A_s f_y}{0.85 f'_c b} = \frac{0.4 \text{ in.$^2$}(60 \text{ksi})}{0.85(6 \text{ksi})4 \text{ in.}} = 1.18 \text{ in.} $$

We now have values for $a$, $d$, and $A_s$ we can compute the moment capacity:

$$ M = A_s f_y \left( d - \frac{a}{2} \right) $$

$$ M = (0.4 \text{ in.$^2$})(60,000 \text{ psi})(4.75 \text{ in.} - \frac{1.18 \text{ in.}}{2}) $$

$$ M = 99,840 \text{ lb} \cdot \text{in.} $$
Reinforced Concrete Beam Example #4

Use the long formula to compute the moment capacity

$$M = A_y f_y \left( d - 0.59 \frac{A_y f_y}{f_{y}'} \right)$$

$$= 0.4 \text{ in.}^2 (60 \text{ ksi}) \left( 4.75\text{ in.} - 0.59 \frac{0.4 \text{ in.}^2 (60 \text{ ksi})}{6 \text{ ksi} (4 \text{ in.})} \right)$$

$$= 99.84 \text{ k} \cdot \text{in.} \Rightarrow P = \frac{M}{2} = 24.96 \text{ kips}$$

Let’s check the shear model

$$P_{\text{shear}} = 2 \left( \frac{A_y f_y d}{s} + 2 d f_{c} b \right)$$

Area of a #3 rebar

$$= 2 \left( \frac{2 (0.11 \text{ in.}) (60,000 \text{ psi}) (4.75 \text{ in.})}{3 \text{ in.}} \right) + 2 \left( 5,000 \text{ psi} (4 \text{ in.}) (4.75 \text{ in.}) \right)$$

$$= 47,687 \text{ lb.} = 47.69 \text{ kips}$$

Since $P_{\text{tension}} < P_{\text{shear}}$, therefore $P_{\text{tension}}$ controls.

Let’s check the reinforcement ratio

$$\rho = \frac{A_y}{bd}$$

$$\rho = 0.85 \frac{c f_{c}'}{d f_{y}}$$

To compute $\rho$, first we need to estimate $\beta_i$

The height of the stress box, $a$, is defined as a percentage of the depth to the neutral axis

$$f_{c}' \leq 4000 \text{ psi} \Rightarrow \beta_i = 0.85$$

$$f_{c}' \geq 4000 \text{ psi}$$

$$\beta_i = 0.85 - 0.05 \left( \frac{f_{c}' - 4000}{1000} \right) \geq 0.65$$

$$\beta_i = 0.85 - 0.05 \left( \frac{6000 - 4000}{1000} \right) = 0.75$$

Check the reinforcement ratio for the maximum steel allowed

$$\rho_{\text{tension}} = 0.85 \beta_i \frac{c f_{c}'}{d f_{y}} = 0.85 (0.75) \frac{6 \text{ ksi}}{60 \text{ ksi}}$$

$$= 0.0239$$

$$\rho_{\text{compression}} = 0.85 \beta_i \frac{c f_{c}'}{d f_{y}} = 0.85 (0.75) \frac{6 \text{ ksi}}{60 \text{ ksi}}$$

$$= 0.0383$$

Check the reinforcement ratio for the maximum steel allowed

$$\rho = \frac{A_y}{bd} = \frac{0.4 \text{ in.}^2}{4 \text{ in.} (4.75 \text{ in.})} = 0.0211$$

$$\rho < \rho_{\text{tension}}$$ or $0.0211 < 0.0239$

The beam should fail in Tension.
An estimate of the weight of the beam can be made as:

\[
W = \left( \frac{(4\text{ in.})(6\text{ in.})(30\text{ in.})}{1728 \text{ in}^3/\text{ft}^3} \cdot \frac{145\text{ lb}}{\text{ft}^3} \right) + \left( \frac{(0.4\text{ in.}^2)(30\text{ in.})}{1728 \text{ in}^3/\text{ft}^3} \cdot \frac{490\text{ lb} - 145\text{ lb}}{\text{ft}^3} \right) \\
= 60.42\text{ lb} + 2.40\text{ lb} = 62.82\text{ lb}
\]

In summary, this reinforced concrete beam will fail in tension

\[
\Rightarrow P = 24,960 \text{ lb.} \\
\text{SWR} = \frac{24,960 \text{ lb.}}{62.82 \text{ lb.}} = 397
\]

The cost of steel may be estimated as follows:

Cost of steel = \[\frac{A_L}{1728 \text{ in}^3/\text{ft}^3} \cdot \left( \frac{490 \text{ lb}}{\text{ft}^3} \right) \cdot \left( \frac{\$500}{\text{ton}} \right) \cdot \left( \frac{2,000 \text{ lb.}}{\text{ton}} \right)\]

where \(A_L\) is the cross-sectional area of steel rebars, \(L\) is the length of the steel rebars, and 490 lb/ft\(^3\) is the unit weight of steel.

For example, if two #4 rebar in placed in the beam the steel cost is estimated as:

\[
\text{Cost of steel} = \left( \frac{2(0.2\text{ in.}^2)(30\text{ in.})}{1728 \text{ in}^3/\text{ft}^3} \cdot \frac{490 \text{ lb}}{\text{ft}^3} \right) \cdot \left( \frac{\$500}{\text{ton}} \right) \cdot \left( \frac{2,000 \text{ lb.}}{\text{ton}} \right) \\
= \$0.85
\]

Consider the following mix for a yd.\(^3\) of concrete developed using the ACI mix design procedure.

<table>
<thead>
<tr>
<th>Component</th>
<th>Amount (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>315</td>
</tr>
<tr>
<td>Cement</td>
<td>768</td>
</tr>
<tr>
<td>Coarse aggregate</td>
<td>1,658</td>
</tr>
<tr>
<td>Fine aggregate</td>
<td>1,242</td>
</tr>
</tbody>
</table>

The cost of the concrete required for a 4.5 in. by 6 in. by 30 in. beam is estimated as:

Cost of cement = \[\frac{(4\text{ in.})(6\text{ in.})(30\text{ in.})}{1728 \text{ in}^3/\text{ft}^3} \cdot \left( \frac{768 \text{ lb.}}{\text{ft}^3} \right) \cdot \left( \frac{\$123}{\text{ton}} \right) \cdot \left( \frac{2,000 \text{ lb.}}{\text{ton}} \right)\]

= \$0.72

Cost of coarse aggregate = \[\frac{(4\text{ in.})(6\text{ in.})(30\text{ in.})}{1728 \text{ in}^3/\text{ft}^3} \cdot \left( \frac{1,641 \text{ lb.}}{\text{ft}^3} \right) \cdot \left( \frac{\$18}{\text{ton}} \right) \cdot \left( \frac{2,000 \text{ lb.}}{\text{ton}} \right)\]

= \$0.23
The cost of the concrete required for a 4.5 in. by 6 in. by 30 in. beam is estimated as:

\[
\text{Cost of fine aggregate} = \frac{4 \text{ in.}(6\text{ in.})(30\text{ in.})}{1728 \text{ ft}^3} \left( \frac{1.251 \text{ lb.}}{27 \text{ ft}^3} \right) \left( \frac{\$10}{\text{ton}} \right) \left( \frac{\text{ton}}{2000 \text{ lb.}} \right) = 0.10
\]

The cost concrete is estimated as: $1.05

The cost reinforced concrete beam is estimated as: $1.90

The cost adjustment for the reinforced concrete beam is:

\[
\text{Cost Factor} = \frac{\$1.50}{\text{Cost}}
\]

If the unadjusted SWR for a beam is 397 and the cost is $1.90, then the cost adjusted SWR is:

\[
SWR_{\text{Adjusted}} = SWR \times \text{Cost Factor}
\]

\[
SWR_{\text{Adjusted}} = 397 \times \left( \frac{\$1.50}{\$1.90} \right) = 313
\]

Questions?