Reinforced Concrete Beam Example #3

Let's use the failure models to predict the ultimate strength-to-weight (SWR) of one of our reinforced concrete beams from the lab.

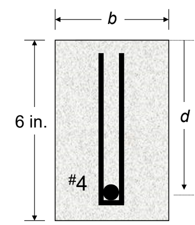
Consider a beam with the following characteristics:

- Concrete strength f'_c = 4,000 psi
- Steel strength f_y = 60,000 psi
- The tension reinforcement will be 1 #4 rebars
- The shear reinforcement will be #3 rebars, U-shaped, 3 in. spacing
- Use a minimum concrete cover of 1 in.
- Use the minimum width to accommodate the reinforcement.

1

Reinforced Concrete Beam Example #3

The bar number denotes reinforcing bars. The diameter and area of standard rebars are shown below.



Bar #	Diameter (in.)	As (in.^{2})
3	0.375	0.11
4	0.500	0.20
5	0.625	0.31
6	0.750	0.44
7	0.875	0.60
8	1.000	0.79
9	1.128	1.00
10	1.270	1.27
11	1.410	1.56

2

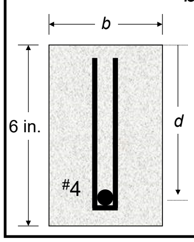
Reinforced Concrete Beam Example #3

Based on the choice of reinforcement we can compute an estimate of *b* and *d*

Minimum cover | #3 rebar in shear | #4 rebar in tension

$$b \geq 2(1\,in) + 2(0.375\,in) + 1\,(0.5\,in)$$

$$= \boxed{3.25\text{ in.}}$$



3

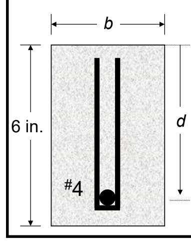
Reinforced Concrete Beam Example #3

If we allow a minimum cover under the rebars were can estimate *d*

Minimum cover | Half of #4 bar diameter | #3 rebar diameter

$$d = 6 - 1.0 - 0.375 - \frac{0.5}{2}$$

$$d = \boxed{4.375\text{ in.}}$$

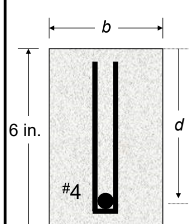


4

Reinforced Concrete Beam Example #3

We now have values for *b, d*, and A_s

$$\boxed{M = A_s f_y \left(d - 0.59 \frac{A_s f_y}{f'_c\, b} \right)}$$



The A_s for one #4 rebars is:

$$A_s = 0.2\text{ in.}^2$$

5

Reinforced Concrete Beam Example #3

Compute the moment capacity

$$M = A_s f_y \left(d - 0.59 \frac{A_s f_y}{f'_c\, b} \right)$$

$$= 0.2\text{in.}^2(60ksi)\left(4.375\text{in.} - 0.59 \frac{0.2\text{in.}^2(60ksi)}{4ksi(3.25\text{in.})} \right)$$

$$= 45.97 \;\; k \cdot in \Rightarrow \boxed{P_{tension} = \frac{M}{4} = 11.49\; kips}$$

6

Reinforced Concrete Beam Example #3

For proper anchorage, a minimum length of reinforcing, l_d is required:

$$l_d = \frac{f_y d_b}{24\sqrt{f'_c}\left(\frac{c}{d_b} - \frac{1}{2}\right)} = \frac{(60{,}000\,psi)(0.5\,in.)}{24\sqrt{4{,}000}\left(\frac{1.0\,in.}{0.5\,in.} - \frac{1}{2}\right)}$$

$= 13.2\,in.$ > 8 in. anchorage available at end of beam

Check for hooked bars. minimum L_{dh} is:

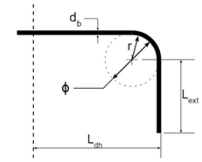
$$L_{dh} = \frac{1{,}200\,d_b}{\sqrt{f'_c}} = \frac{1{,}200(0.5\,in.)}{\sqrt{4{,}000}} = 9.48\,in.$$

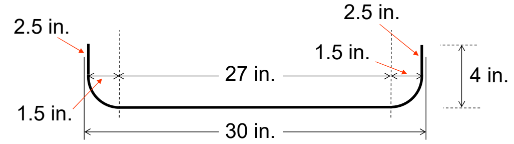
7

Reinforced Concrete Beam Example #3

For a hooked bar, the minimum length of reinforcing is:

ϕ_{inner} = 2.250 in.
r_{inner} = 1.125 in.
ϕ_{outer} = 3.000 in.
r_{outer} = 1.500 in.
L_{ext} = 2.500 in.





8

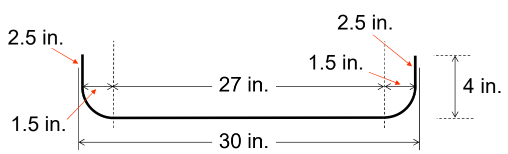
Reinforced Concrete Beam Example #3

For a hooked bar, the minimum length of reinforcing is:

$$L_{rebar} = 27\,\text{in.} + 2(2.5\,\text{in.}) + \frac{\pi}{2}(3\,\text{in.}) = 36.71\,\text{in.}$$

L_{ext}

Circumference of a hooks



9

Reinforced Concrete Beam Example #3

Let's check the shear model

$$P_{shear} = 2\left(\frac{A_v f_y d}{s} + 2\sqrt{f'_c}\,bd\right)$$

Area of a #3 rebars

$$= 2\left(\frac{2(0.11\,\text{in.}^2)(60{,}000\,psi)4.375\,\text{in.}}{3\,\text{in.}} + 2\sqrt{4{,}000\,psi}\,(3.25\,\text{in.})(4.375\,\text{in.})\right)$$

Shear reinforcement spacing

$$= 42{,}097\,\text{lb.} = \boxed{42.10\,kips}$$

Since $P_{tension} < P_{shear}$ therefore $P_{tension}$ controls

10

Reinforced Concrete Beam Example #3

Let's check the reinforcement ratio

$$\rho = \frac{A_s}{bd}$$

$$\rho = 0.85\beta_1 \frac{c}{d}\frac{f'_c}{f_y}$$

ρ as function of c/d

To compute ρ, first we need to estimate β_1

11

Reinforced Concrete Beam Example #3

The height of the stress box, ***a***, is defined as a percentage of the depth to the neural axis.

$$f'_c \le 4000\,psi \quad \Rightarrow \quad \beta_1 = 0.85$$

$$f'_c \ge 4000\,psi$$

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) \ge 0.65$$

$$\beta_1 = 0.85 - 0.05\left(\frac{4{,}000 - 4{,}000}{1{,}000}\right) = \boxed{0.85}$$

12

Reinforced Concrete Beam Example #3

Check the reinforcement ratio for the maximum steel allowed

$$\rho_{tension} = 0.85\beta_1 \frac{c}{d}\frac{f'_c}{f_y} = 0.85(0.85)0.375\frac{4\,ksi}{60\,ksi}$$

$$= 0.0181$$

$$\rho = \frac{A_s}{bd} = \frac{0.2\text{in.}^2}{3.25\text{in.}(4.375\text{in.})} = 0.0141$$

$$\rho < \rho_{tension}$$

Reinforced Concrete Beam Example #3

The minimum force controls

$$P_{tension} = 11.49\,kips$$

$$P_{shear} = 42.10\,kips$$

$$P = \boxed{11.49\,kips}$$

Reinforced Concrete Beam Example #3

An estimate of the weight of the beam can be made as:

Size of concrete beam; Unit weight of concrete

$$W = \frac{(3.25\text{in.})(6\text{in.})(30\text{in.})}{1728\,\text{in.}^3/\text{ft.}^3}\left(\frac{145\text{lb.}}{\text{ft.}^3}\right)$$

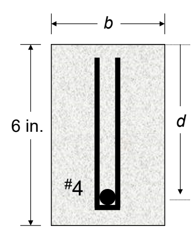
Additional weight of rebars; Unit weight of steel

$$+\frac{(0.2\text{in.}^2)(36.7\text{in.})}{1728\,\text{in.}^3/\text{ft.}^3}\left(\frac{490\text{lb.} - 145\text{lb.}}{\text{ft.}^3}\right)$$

$$= 49.09\text{lb.} + 1.47\text{lb.} = \boxed{50.46\text{lb.}}$$

Reinforced Concrete Beam Example #3

In summary, this reinforced concrete beam will fail in tension



$$\Rightarrow P = 14.58\,kips$$

$$SWR = \frac{11{,}491\,lb}{50.55\,lb} = \boxed{227}$$

Reinforced Concrete Beam Example #3

The cost of steel may be estimated as follows:

$$\text{Cost of steel} = \frac{A_s L}{1{,}728\,\text{in.}^3/\text{ft.}^3}\left(490\frac{\text{lb.}}{\text{ft.}^3}\right)\left(\frac{\$700}{ton}\right)\left(\frac{ton}{2{,}000\text{ lb.}}\right)$$

where $\boldsymbol{A_s}$ is the cross-sectional area of steel rebars, ***L*** is the length of the steel rebars, and 490 lb./ft.^{3} is the unit weight of steel.

Reinforced Concrete Beam Example #3

For example, if one #6 rebar is placed in the beam, the steel cost is estimated as follows:

Bar #	Diameter (in.)	As (in.^{2})
3	0.375	0.11
4	0.500	0.20
5	0.625	0.31
6	0.750	0.44
7	0.875	0.60
8	1.000	0.79
9	1.128	1.00
10	1.270	1.27
11	1.410	1.56

$$\text{Cost of steel} = \frac{(0.2\text{in.}^2)(36.7\text{in.})}{1{,}728\,\text{in.}^3/\text{ft.}^3}\left(490\frac{\text{lb.}}{\text{ft.}^3}\right)\left(\frac{\$700}{ton}\right)\left(\frac{ton}{2{,}000\text{ lb.}}\right)$$

$$= \$0.73$$

Reinforced Concrete Beam Example #3

Consider the following mix for a yd.^{3} of concrete developed using the ACI mix design procedure.

Component	Amount (lb)
Water	315
Cement	553
Coarse aggregate	1,641
Fine aggregate	1,431

19

Reinforced Concrete Beam Example #3

The cost of the concrete required for a 3 in. by 6 in. by 30 in. beam is estimated as:

$$\text{Cost of cement} = \frac{3.25\text{in.}(6\text{in.})30\text{in.}}{1{,}728\,{}^{\text{in.}^3}\!/_{\text{ft.}^3}}\left(\frac{553\text{ lb.}}{27\ ft.^3}\right)\left(\frac{\$150}{ton}\right)\left(\frac{ton}{2{,}000\text{ lb.}}\right)$$

$$= \$0.52$$

$$\text{Cost of coarse aggregate} = \frac{3.25\text{in.}(6\text{in.})30\text{in.}}{1{,}728\,{}^{\text{in.}^3}\!/_{\text{ft.}^3}}\left(\frac{1{,}641\text{ lb.}}{27\ ft.^3}\right)\left(\frac{\$18}{ton}\right)\left(\frac{ton}{2{,}000\text{ lb.}}\right)$$

$$= \$0.19$$

20

Reinforced Concrete Beam Example #3

The cost of the concrete required for a 3 in. by 6 in. by 30 in. beam is estimated as:

$$\text{Cost of fine aggregate} = \frac{3.25\text{in.}(6\text{in.})30\text{in.}}{1{,}728\,{}^{\text{in.}^3}\!/_{\text{ft.}^3}}\left(\frac{1{,}431\text{ lb.}}{27\ ft.^3}\right)\left(\frac{\$10}{ton}\right)\left(\frac{ton}{2{,}000\text{ lb.}}\right)$$

$$= \$0.09$$

The cost of concrete is estimated at $0.80

The reinforced concrete beam cost is estimated at $1.53

21

Reinforced Concrete Beam Example #3

The cost adjustment for the reinforced concrete beam is :

If cost < $2 then: *Cost Factor* = 1

If cost > $2 then:

$$\text{Cost Factor} = \frac{\$2}{Cost}$$

22

Reinforced Concrete Beam Example #3

If the unadjusted ***SWR*** for a beam is 227 and the cost is $1.53, then the cost-adjusted ***SWR*** is:

$$SWR_{Adjusted} = SWR \times \text{Cost Factor}$$

$$SWR_{Adjusted} = \boxed{277}$$

23

Reinforced Concrete Beam Example #3

Questions?

24