Reinforced Concrete Beam Example #2

Let's use the failure models to predict the ultimate strength-to-weight (SWR) of one of our reinforced concrete beams from lab

Consider a beam with the following characteristics:

- Concrete strength f'_c = 6,000 psi
- Steel strength f_y = 60,000 psi
- The tension reinforcement will be 2 # 4 rebars
- The shear reinforcement will be #3 rebars installed vertically at 4 in. spacing
- Use the minimum concrete cover of 1 inch and a bar spacing of 0.75 in.

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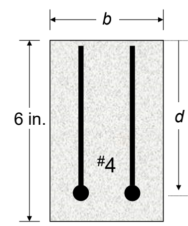
Reinforced Concrete Beam Example #2

Reinforcing bars are denoted by the bar number. The diameter and area of standard rebars are shown below.



Bar #	Diameter (in.)	As (in.^{2})
3	0.375	0.11
4	0.500	0.20
5	0.625	0.31
6	0.750	0.44
7	0.875	0.60
8	1.000	0.79
9	1.128	1.00
10	1.270	1.27
11	1.410	1.56

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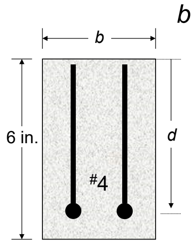
Reinforced Concrete Beam Example #2

Based on the choice of reinforcement we can compute an estimate of *b* and *d*

Minimum cover | #4 rebar diameter | Space between bars

$$b \geq 2(1.0\ in) + 2\ (0.5\ in) + (0.75\ in)$$

$$= 3.75\text{ in.}$$



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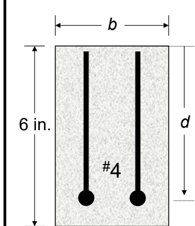
Reinforced Concrete Beam Example #2

If we allow a minimum cover under the rebars were can estimate *d*

Minimum cover | Half a #4 rebar diameter

$$d = 6 - 1.0 - \frac{0.5}{2}$$

$$d = 4.75\text{ in.}$$



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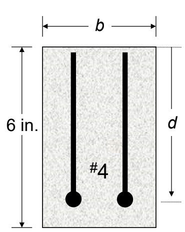
Reinforced Concrete Beam Example #2

The A_s for two #4 rebars is:

$$A_s = 2(0.20\text{ in.}^2) = 0.40\text{ in.}^2$$



$$a = \frac{A_s f_y}{0.85 f'_c\, b}$$

$$a = \frac{0.4\text{in.}^2(60ksi)}{0.85(6ksi)3.75\text{in.}} = 1.26\text{in.}$$

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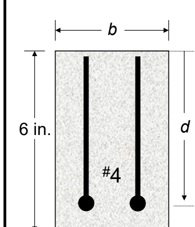
Reinforced Concrete Beam Example #2

We now have values for *a*, *d*, and A_s we can compute the moment capacity:

$$M = A_s f_y \left(d - \frac{a}{2}\right)$$



$$M = (0.4\text{in.}^2)60{,}000psi\left(4.75\text{in.} - \frac{1.26\text{in.}}{2}\right)$$

$$M = 98{,}880\text{ lb.}\cdot\text{in.}$$

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Reinforced Concrete Beam Example #2

Use the long formula to compute the moment capacity

$$M = A_s f_y\left(d - 0.59\frac{A_s f_y}{f'_c\, b}\right)$$

$$= 0.4\,\text{in.}^2(60ksi)\left(4.75\,\text{in.} - 0.59\frac{0.4\,\text{in.}^2(60ksi)}{6ksi(3.75\,\text{in.})}\right)$$

$$= 98.88\ \text{k}\cdot\text{in.} \Rightarrow \boxed{P = \frac{M}{4} = 24.72\ kips}$$

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Reinforced Concrete Beam Example #2

Let's check the shear model

$$P_{shear} = 2\left(\frac{A_v f_y d}{s} + 2\sqrt{f'_c}\,bd\right)$$

Area of a #3 rebars

$$= 2\left(\frac{2(0.11\text{in.}^2)(60{,}000\,psi)4.75\text{in.}}{4\,\text{in.}} + 2\sqrt{6{,}000\,psi}\,(3.75\text{in.})(4.75\text{in.})\right)$$

Shear reinforcement spacing

$$= 36{,}869\text{lb.} = \boxed{36.87\ kips}$$

Since $P_{tension} < P_{shear}$ therefore $P_{tension}$ controls

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Reinforced Concrete Beam Example #2

Let's check the reinforcement ratio

$$\rho = \frac{A_s}{bd}$$

$$\rho = 0.85\beta_1 \frac{c}{d}\frac{f'_c}{f_y}$$

ρ as function of *c/d*

To compute ρ, first we need to estimate β_1

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Reinforced Concrete Beam Example #2

The height of the stress box, ***a***, is defined as a percentage of the depth to the neural axis

$$f'_c \le 4000\ psi \Rightarrow \beta_1 = 0.85$$

$$f'_c \ge 4000\ psi$$

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) \ge 0.65$$

$$\beta_1 = 0.85 - 0.05\left(\frac{6{,}000 - 4{,}000}{1{,}000}\right) = \boxed{0.75}$$

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Reinforced Concrete Beam Example #2

Check the reinforcement ratio for the maximum steel allowed

$$\rho_{tension} = 0.85\beta_1 \frac{c}{d}\frac{f'_c}{f_y} = 0.85(0.75)0.375\frac{6\,ksi}{60\,ksi}$$

$$= 0.0239$$

$$\rho_{compression} = 0.85\beta_1 \frac{c}{d}\frac{f'_c}{f_y} = 0.85(0.75)0.600\frac{6\,ksi}{60\,ksi}$$

$$= 0.0383$$

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Reinforced Concrete Beam Example #2

Check the reinforcement ratio for the maximum steel allowed

$$\rho = \frac{A_s}{bd} = \frac{0.4\,\text{in.}^2}{3.75\,\text{in.}(4.75\,\text{in.})} = 0.0224$$

$\rho < \rho_{tension}$ or $0.0224 < 0.0239$

The beam should fail in ***tension***.

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Reinforced Concrete Beam Example #2

An estimate of the weight of the beam can be made as:

Size of concrete beam — Unit weight of concrete

$$W = \frac{(3.75\,\text{in.})(6\,\text{in.})(30\,\text{in.})}{1728\,{}^{\text{in.}^3}\!/_{\text{ft.}^3}}\left(\frac{145\,\text{lb.}}{\text{ft.}^3}\right)$$

Additional weight of rebars — Unit weight of steel

$$+\frac{(0.4\,\text{in.}^2)(30\,\text{in.})}{1728\,{}^{\text{in.}^3}\!/_{\text{ft.}^3}}\left(\frac{490\,\text{lb.} - 145\,\text{lb.}}{\text{ft.}^3}\right)$$

$$= 56.64\,\text{lb.} + 2.40\,\text{lb.} = \boxed{59.04\,\text{lb.}}$$

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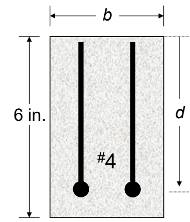
Reinforced Concrete Beam Example #2

In summary, this reinforced concrete beam will fail in tension



$$\Rightarrow P = 24.72 \;\; kips$$

$$SWR = \frac{24{,}720\,\text{lb.}}{59.04\,\text{lb.}} = \boxed{418}$$

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Reinforced Concrete Beam Example #2

The cost of steel may be estimated as follows:

$$\text{Cost of steel} = \frac{A_s L}{1{,}728\,{}^{\text{in.}^3}\!/_{\text{ft.}^3}}\left(490\frac{\text{lb.}}{\text{ft.}^3}\right)\left(\frac{\$530}{ton}\right)\left(\frac{ton}{2{,}000\,\text{lb.}}\right)$$

where $\boldsymbol{A_s}$ is the cross-sectional area of steel rebars, $\boldsymbol{L}$ is the length of the steel rebars, and 490 lb./ft.^{3} is the unit weight of steel.

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Reinforced Concrete Beam Example #2

For example, if two #4 rebar in placed in the beam the steel cost is estimated as:

Bar #	Diameter (in.)	As (in.^{2})
3	0.375	0.11
4	0.500	0.20
5	0.625	0.31
6	0.750	0.44
7	0.875	0.60
8	1.000	0.79
9	1.128	1.00
10	1.270	1.27
11	1.410	1.56

$$\text{Cost of steel} =$$

$$= \frac{2(0.2\,\text{in.}^2)(30\,\text{in.})}{1{,}728\,{}^{\text{in.}^3}\!/_{\text{ft.}^3}}\left(490\frac{\text{lb.}}{\text{ft.}^3}\right)\left(\frac{\$530}{ton}\right)\left(\frac{ton}{2{,}000\,\text{lb.}}\right)$$

$$= \$0.90$$

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Reinforced Concrete Beam Example #2

Consider the following mix for a yd.^{3} of concrete developed using the ACI mix design procedure.

Component	Amount (lb)
Water	315
Cement	768
Coarse aggregate	1,641
Fine aggregate	1,251

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Reinforced Concrete Beam Example #2

The cost of the concrete required for a 3.75 in. by 6 in. by 30 in. beam is estimated as:

$$\text{Cost of cement} = \frac{3.75\text{in.}(6\text{in.})30\text{in.}}{1{,}728\,{}^{\text{in.}^3}\!/_{\text{ft.}^3}}\left(\frac{768\,\text{lb.}}{27\,\text{ft.}^3}\right)\left(\frac{\$130}{ton}\right)\left(\frac{ton}{2{,}000\,\text{lb.}}\right)$$

$$= \$0.72$$

$$\text{Cost of coarse aggregate} = \frac{3.75\text{in.}(6\text{in.})30\text{in.}}{1{,}728\,{}^{\text{in.}^3}\!/_{\text{ft.}^3}}\left(\frac{1{,}641\,\text{lb.}}{27\,\text{ft.}^3}\right)\left(\frac{\$18}{ton}\right)\left(\frac{ton}{2{,}000\,\text{lb.}}\right)$$

$$= \$0.21$$

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Reinforced Concrete Beam Example #2

The cost of the concrete required for a 3.75 in. by 6 in. by 30 in. beam is estimated as:

$$\textit{Cost of fine aggregate} = \frac{3.75\,\text{in.}(6\,\text{in.})30\,\text{in.}}{1{,}728\,{}^{\text{in.}^3}\!/_{\text{ft.}^3}}\left(\frac{1{,}251\,\text{lb.}}{27\,\text{ft.}^3}\right)\left(\frac{\$10}{ton}\right)\left(\frac{ton}{2{,}000\,\text{lb.}}\right)$$

$$= \$0.09$$

The cost concrete is estimated as: $1.02

The cost reinforced concrete beam is estimated as: $1.92

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Reinforced Concrete Beam Example #2

The cost adjustment for the reinforced concrete beam is:

If cost < $1.50 then: *Cost Factor* = 1

If cost > $1.50 then:

$$\textit{Cost Factor} = \frac{\$1.50}{\textit{Cost}}$$

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Reinforced Concrete Beam Example #2

If the unadjusted ***SWR*** for a beam is 418 and the cost is $1.92, then the cost adjusted ***SWR*** is:

$$SWR_{Adjusted} = SWR \times \textit{Cost Factor}$$

$$SWR_{Adjusted} = 418 \times \frac{\$1.50}{\$1.92} = \boxed{327}$$

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Reinforced Concrete Beam Example #2

Questions?

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