Let’s use the failure models to predict the ultimate strength-to-weight (SWR) of one of our reinforced concrete beams from lab.

Consider a beam with the following characteristics:

- Concrete strength $f'_c = 4,000$ psi
- Steel strength $f_y = 60,000$ psi
- The tension reinforcement will be 1 #4 rebars
- The shear reinforcement will be #3 rebars, U-shaped, 3 in. spacing
- Use minimum cover of 0.75 in. and a width to accommodate the reinforcement

**Reinforced Concrete Beam Example #1**

Based on the choice of reinforcement we can compute an estimate of $b$ and $d$

\[
\begin{align*}
    b & \geq (0.5 \text{ in.}) + 2(0.75 \text{ in.}) + 2(0.375 \text{ in.}) \\
    & = 2.75 \text{ in.}
\end{align*}
\]

If we allow a minimum cover under the rebars were can estimate $d$

\[
\begin{align*}
    d & = 6 - \frac{0.5}{2} - 0.75 - 0.375 \\
    & = 4.625 \text{ in.}
\end{align*}
\]

We now have values for $b$, $d$, and $A_s$

\[
M = A_s f_y \left( d - 0.59 \frac{A_s f_y}{f'_c b} \right)
\]

The $A_s$ for one #4 rebars is:

\[
A_s = 0.20 \text{ in.}^2
\]

**Reinforced Concrete Beam Example #1**

Compute the moment capacity

\[
\begin{align*}
    M & = A_s f_y \left( d - 0.59 \frac{A_s f_y}{f'_c b} \right) \\
    & = 0.20 \text{ in.}^2 (60 \text{ ksi}) \left( 4.625 \text{ in.} - 0.59 \frac{0.20 \text{ in.}^2 (60 \text{ ksi})}{4 \text{ ksi (2.75 in.)}} \right) \\
    & = 47.78 \text{ k} \cdot \text{in.} \\
\end{align*}
\]

\[P_{\text{tension}} = \frac{M}{4} = 11.94 \text{ kips}\]
Let's check the shear model:

\[
P_{\text{shear}} = 2 \left( \frac{A_f f' c}{S} + 2 \frac{f' c d}{f_y} \right)
\]

Area of a #3 rebar:

\[
= 2 \left( \left[ \frac{0.11 \text{ in}^2}{60,000 \text{ psi}} \right] 4.625 \text{ in} \right) + 2 \left[ \frac{4,000 \text{ psi}}{2.75 \text{ in} \cdot 4.625 \text{ in}} \right]
\]

\[= 43,917 \text{ lb.} = 43.9 \text{kips} \]

Since \( P_{\text{tension}} < P_{\text{shear}} \), therefore \( P_{\text{tension}} \) controls.

Let's check the reinforcement ratio:

\[
\rho = \frac{A_s}{bd}
\]

\[
\rho = 0.85 \beta \frac{f'_c}{f_y}
\]

To compute \( \rho \), first we need to estimate \( \beta \).

The height of the stress box, \( a \), is defined as a percentage of the depth to the neutral axis:

\[
f'_c \leq 4000 \text{ psi} \quad \Rightarrow \quad \beta_1 = 0.85
\]

\[
f'_c \geq 4000 \text{ psi}
\]

\[
\beta_1 = 0.85 - 0.05 \left( \frac{f'_c - 4000}{1000} \right) \geq 0.65
\]

\[
\beta_1 = 0.85 - 0.05 \left( \frac{4000 - 4000}{1000} \right) = 0.85
\]

Check the reinforcement ratio for the maximum steel allowed:

\[
\rho = \frac{A_s}{bd} = \frac{0.20 \text{ in}^2}{2.75 \text{ in} \cdot 4.625 \text{ in}} = 0.0157
\]

\[
\rho_{\text{tension}} = 0.85 \beta \frac{f'_c}{f_y} = 0.85(0.85)0.375 \frac{4 \text{ ksi}}{60 \text{ ksi}} = 0.0181
\]

\[\rho < \rho_{\text{tension}}\]

An estimate of the weight of the beam can be made as:

\[
W = \left( \frac{2.75 \text{ in} \cdot 6 \text{ in} \cdot 30 \text{ in.}}{1728 \text{ in}^3/\text{ft}^3} \right) \left( 145 \text{ lb.} \right)
\]

\[
+ \left( \frac{0.20 \text{ in}^2 \cdot 30 \text{ in.}}{1728 \text{ in}^3/\text{ft}^3} \right) \left( 490 \text{ lb.} - 145 \text{ lb.} \right)
\]

\[= 41.54 \text{ lb.} + 1.20 \text{ lb.} = 42.74 \text{ lb.} \]
Reinforced Concrete Beam Example #1

In summary, this reinforced concrete beam will fail in tension

\[ P = 11.94 \text{ kips} \]

\[ SWR = \frac{11,907 \text{ lb}}{42.74 \text{ lb}} = 279 \]

This beam should fail in tension

Reinforced Concrete Beam Example #1

The cost of steel may be estimated as follows:

\[
\text{Cost of steel} = \frac{A_s L}{1728} \left( \frac{490 \text{ lb.}}{\text{ft}^2} \right) \left( \frac{700 \text{ ton}}{\text{ton}} \right) \left( \frac{2000 \text{ lb.}}{\text{ton}} \right)
\]

where \( A_s \) is the cross-sectional area of steel rebars, \( L \) is the length of the steel rebars, and 490 lb./ft. \(^2\) is the unit weight of steel.

Reinforced Concrete Beam Example #1

For example, if one #4 rebar in placed in the beam the steel cost is estimated as:

\[
\text{Cost of steel} = \frac{(0.2\text{ in.})(30\text{ in.})}{1728} \left( \frac{490 \text{ lb.}}{\text{ft}^2} \right) \left( \frac{700 \text{ ton}}{\text{ton}} \right) \left( \frac{2000 \text{ lb.}}{\text{ton}} \right)
\]

\[ = 0.60 \text{ lb.} \]

Reinforced Concrete Beam Example #1

Consider the following mix for a yd. \(^3\) of concrete developed using the ACI mix design procedure.

<table>
<thead>
<tr>
<th>Component</th>
<th>Amount (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>315</td>
</tr>
<tr>
<td>Cement</td>
<td>553</td>
</tr>
<tr>
<td>Coarse aggregate</td>
<td>1,641</td>
</tr>
<tr>
<td>Fine aggregate</td>
<td>1,431</td>
</tr>
</tbody>
</table>

Reinforced Concrete Beam Example #1

The cost of the concrete required for a 2.75 in. by 6 in. by 30 in. beam is estimated as:

\[
\text{Cost of cement} = \frac{2.75 \text{ in.}(6\text{ in.})(30\text{ in.})}{1728} \left( \frac{553 \text{ lb.}}{27 \text{ ft.}^2} \right) \left( \frac{116 \text{ ton}}{\text{ton}} \right) \left( \frac{2000 \text{ lb.}}{\text{ton}} \right)
\]

\[ = 0.34 \text{ lb.} \]

\[
\text{Cost of coarse aggregate} = \frac{2.75 \text{ in.}(6\text{ in.})(30\text{ in.})}{1728} \left( \frac{1,641 \text{ lb.}}{27 \text{ ft.}^2} \right) \left( \frac{18 \text{ ton}}{\text{ton}} \right) \left( \frac{2000 \text{ lb.}}{\text{ton}} \right)
\]

\[ = 0.16 \text{ lb.} \]

The cost concrete is estimated as: $0.58

The cost of the reinforced concrete beam is estimated as: $1.18
The cost adjustment for the reinforced concrete beam is:

If cost < $2.00 then: \( \text{Cost Factor} = 1 \)

If cost > $2.00 then:

\[
\text{Cost Factor} = \frac{\$2.00}{\text{Cost}}
\]

If the unadjusted SWR for a beam is 279 and the cost is $1.18, then the cost adjusted SWR is:

\[
\text{SWR}_{\text{Adjusted}} = \text{SWR} \times \text{Cost Factor}
\]

\[
\text{SWR}_{\text{Adjusted}} = 279 \times 1 = 279
\]