Reinforced Concrete Beam Example #1

Let's use the failure models to predict the ultimate strength-to-weight (SWR) of one of our reinforced concrete beams from lab

Consider a beam with the following characteristics:

- Concrete strength f'_c = 4,000 psi
- Steel strength f_y = 60,000 psi
- The tension reinforcement will be 1 #4 rebars
- The shear reinforcement will be #3 rebars, U-shaped, 3 in. spacing
- Use minimum cover of 0.75 in. and a width to accommodate the reinforcement

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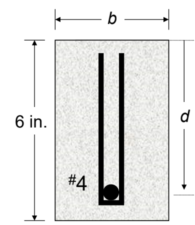
Reinforcing bars are denoted by the bar number. The diameter and area of standard rebars are shown below.



Bar #	Diameter (in.)	As (in.^{2})
3	0.375	0.11
4	0.500	0.20
5	0.625	0.31
6	0.750	0.44
7	0.875	0.60
8	1.000	0.79
9	1.128	1.00
10	1.270	1.27
11	1.410	1.56

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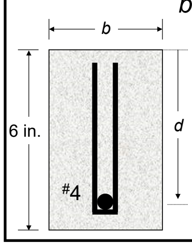
Based on the choice of reinforcement we can compute an estimate of *b* and *d*

Minimum cover | #3 rebar diameter | #4 rebar diameter

$$b \geq 2(0.75\ in) + 2(0.375\ in) + 1\,(0.5\ in)$$

$$= \boxed{2.75\text{ in.}}$$



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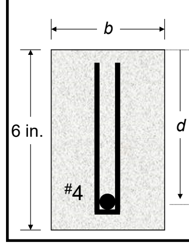
Reinforced Concrete Beam Example #1

If we allow a minimum cover under the rebars were can estimate *d*

Minimum cover | Half of #4 bar diameter | #3 rebar diameter

$$d = 6 - 0.75 - 0.375 - \frac{0.5}{2}$$

$$d = \boxed{4.625\text{ in.}}$$



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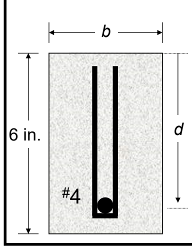
Reinforced Concrete Beam Example #1

We now have values for *b, d*, and A_s

$$\boxed{M = A_s f_y\left(d - 0.59\frac{A_s f_y}{f'_c\, b}\right)}$$



The A_s for one #4 rebars is:

$$A_s = 0.20\text{ in.}^2$$

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Reinforced Concrete Beam Example #1

Compute the moment capacity

$$M = A_s f_y\left(d - 0.59\frac{A_s f_y}{f'_c\, b}\right)$$

$$= 0.20\text{in.}^2(60ksi)\left(4.625\text{in.} - 0.59\frac{0.20\text{in.}^2(60ksi)}{4ksi(2.75\text{in.})}\right)$$

$$= 47.78\ k \cdot in \Rightarrow \boxed{P_{tension} = \frac{M}{4} = 11.94\ kips}$$

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Reinforced Concrete Beam Example #1

Let's check the shear model

$$P_{shear} = 2\left(\frac{A_v f_y d}{s} + 2\sqrt{f'_c}\,bd\right)$$

Area of a #3 rebars

$$= 2\left(\frac{2(0.11\text{in.}^2)(60{,}000\,psi)4.625\text{in.}}{3\text{in.}} + 2\sqrt{4{,}000\,psi}\,(2.75\text{in.})(4.625\text{in.})\right)$$

Shear reinforcement spacing

$$= 43{,}917\text{lb.} = \boxed{43.9\,kips}$$

Since $P_{tension} < P_{shear}$ therefore $P_{tension}$ controls

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Reinforced Concrete Beam Example #1

Let's check the reinforcement ratio

$$\rho = \frac{A_s}{bd}$$

ρ as function of *c/d*

$$\rho = 0.85\beta_1 \frac{c}{d}\frac{f'_c}{f_y}$$

To compute ρ, first we need to estimate β_1

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Reinforced Concrete Beam Example #1

The height of the stress box, ***a***, is defined as a percentage of the depth to the neural axis

$$f'_c \le 4000\ psi \quad \Rightarrow \quad \beta_1 = 0.85$$

$$f'_c \ge 4000\ psi$$

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) \ge 0.65$$

$$\beta_1 = 0.85 - 0.05\left(\frac{4{,}000 - 4{,}000}{1{,}000}\right) = \boxed{0.85}$$

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Reinforced Concrete Beam Example #1

Check the reinforcement ratio for the maximum steel allowed

$$\rho = \frac{A_s}{bd} = \frac{0.20\text{in.}^2}{2.75\text{in.}(4.625\text{in.})} = 0.0157$$

$$\rho_{tension} = 0.85\beta_1 \frac{c}{d}\frac{f'_c}{f_y} = 0.85(0.85)0.375\frac{4\,ksi}{60\,ksi}$$

$$= 0.0181$$

$$\rho < \rho_{tension}$$

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Reinforced Concrete Beam Example #1

The minimum force controls

$$P_{tension} = 11.94\ kips$$

$$P_{shear} = 43.92\,kips$$

$$P = \boxed{11.94\,kips}$$

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Reinforced Concrete Beam Example #1

An estimate of the weight of the beam can be made as:

Size of concrete beam

Unit weight of concrete

$$W = \frac{(2.75\text{in.})(6\text{in.})(30\text{in.})}{1728\,{}^{\text{in.}^3}\!/_{\text{ft.}^3}}\left(\frac{145\text{lb.}}{\text{ft.}^3}\right)$$

Additional weight of rebars

Unit weight of steel

$$+\frac{(0.20\text{in.}^2)(30\text{in.})}{1728\,{}^{\text{in.}^3}\!/_{\text{ft.}^3}}\left(\frac{490\text{lb.} - 145\text{lb.}}{\text{ft.}^3}\right)$$

$$= 41.54\text{lb.} + 1.20\text{lb.} = \boxed{42.74\text{lb.}}$$

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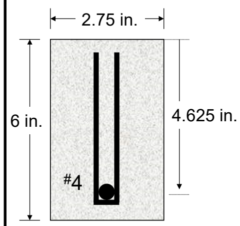
Reinforced Concrete Beam Example #1

In summary, this reinforced concrete beam will fail in tension



$$\Rightarrow P = 11.94\,kips$$

$$SWR = \frac{11{,}907\,lb}{42.74\,lb} = \boxed{279}$$

This beam should fail in tension

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Reinforced Concrete Beam Example #1

The cost of steel may be estimated as follows:

$$Cost\ of\ steel = \frac{A_s L}{1{,}728\,\text{in.}^3/\text{ft.}^3}\left(490\,\frac{\text{lb.}}{\text{ft.}^3}\right)\left(\frac{\$530}{ton}\right)\left(\frac{ton}{2{,}000\,\text{lb.}}\right)$$

where $\boldsymbol{A_s}$ is the cross-sectional area of steel rebars, $\boldsymbol{L}$ is the length of the steel rebars, and 490 lb./ft.^{3} is the unit weight of steel.

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Reinforced Concrete Beam Example #1

For example, if one #4 rebar in placed in the beam the steel cost is estimated as:

Bar #	Diameter (in.)	As (in.^{2})
3	0.375	0.11
4	0.500	0.20
5	0.625	0.31
6	0.750	0.44
7	0.875	0.60
8	1.000	0.79
9	1.128	1.00
10	1.270	1.27
11	1.410	1.56

$$Cost\ of\ steel =$$

$$= \frac{(0.2\,\text{in.}^2)(30\,\text{in.})}{1{,}728\,\text{in.}^3/\text{ft.}^3}\left(490\,\frac{\text{lb.}}{\text{ft.}^3}\right)\left(\frac{\$530}{ton}\right)\left(\frac{ton}{2{,}000\,\text{lb.}}\right)$$

$$= \$0.45$$

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Reinforced Concrete Beam Example #1

Consider the following mix for a yd.^{3} of concrete developed using the ACI mix design procedure.

Component	Amount (lb)
Water	315
Cement	553
Coarse aggregate	1,641
Fine aggregate	1,431

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Reinforced Concrete Beam Example #1

The cost of the concrete required for a 2.75 in. by 6 in. by 30 in. beam is estimated as:

$$Cost\ of\ cement = \frac{2.75\,\text{in.}(6\,\text{in.})30\,\text{in.}}{1{,}728\,\text{in.}^3/\text{ft.}^3}\left(\frac{553\,\text{lb.}}{27\,\text{ft.}^3}\right)\left(\frac{\$130}{ton}\right)\left(\frac{ton}{2{,}000\,\text{lb.}}\right)$$

$$= \$0.38$$

$$Cost\ of\ coarse\ aggregate = \frac{2.75\,\text{in.}(6\,\text{in.})30\,\text{in.}}{1{,}728\,\text{in.}^3/\text{ft.}^3}\left(\frac{1{,}641\,\text{lb.}}{27\,\text{ft.}^3}\right)\left(\frac{\$18}{ton}\right)\left(\frac{ton}{2{,}000\,\text{lb.}}\right)$$

$$= \$0.16$$

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Reinforced Concrete Beam Example #1

The cost of the concrete required for a 2.75 in. by 6 in. by 30 in. beam is estimated as:

$$Cost\ of\ fine\ aggregate = \frac{2.75\,\text{in.}(6\,\text{in.})30\,\text{in.}}{1{,}728\,\text{in.}^3/\text{ft.}^3}\left(\frac{1{,}431\,\text{lb.}}{27\,\text{ft.}^3}\right)\left(\frac{\$10}{ton}\right)\left(\frac{ton}{2{,}000\,\text{lb.}}\right)$$

$$= \$0.08$$

The cost concrete is estimated as: $0.62

The cost of the reinforced concrete beam is estimated as: $1.07

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Reinforced Concrete Beam Example #1

The cost adjustment for the reinforced concrete beam is:

If cost < $1.50 then: *Cost Factor* = 1

If cost > $1.50 then:

$$Cost\ Factor = \frac{\$1.50}{Cost}$$

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Reinforced Concrete Beam Example #1

If the unadjusted ***SWR*** for a beam is 279 and the cost is $1.07, then the cost adjusted ***SWR*** is:

$$SWR_{Adjusted} = SWR \times Cost\ Factor$$

$$SWR_{Adjusted} = 279 \times 1 = 279$$

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Reinforced Concrete Beam Example #1

Questions?

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