Reinforced Concrete Beam Example #1

Let's use the failure models to predict the ultimate strength-to-weight (SWR) of one of our reinforced concrete beams from the lab.

Consider a beam with the following characteristics:

- Concrete strength f'_c = 6,000 psi
- Steel strength f_y = 60,000 psi
- The tension reinforcement will be 1 #3 rebars
- The shear reinforcement will be #3 rebars, U-shaped, 3 in. spacing
- Use a minimum cover of 1 in. and width to accommodate the reinforcement

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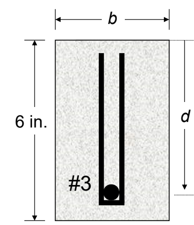
The bar number denotes reinforcing bars. The diameter and area of standard rebars are shown below.



Bar #	Diameter (in.)	As (in.^{2})
3	0.375	0.11
4	0.500	0.20
5	0.625	0.31
6	0.750	0.44
7	0.875	0.60
8	1.000	0.79
9	1.128	1.00
10	1.270	1.27
11	1.410	1.56

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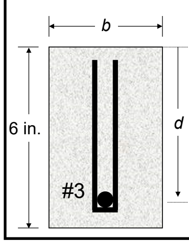
Based on the choice of reinforcement we can compute an estimate of *b* and *d*

Minimum cover | #3 rebar diameter | #3 rebar diameter

$$b \geq 2(1\,in) + 2(0.375\,in) + 1\,(0.375\,in)$$

$$= \boxed{3.13 \text{ in.}}$$



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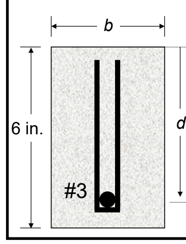
Reinforced Concrete Beam Example #1

If we allow a minimum cover under the rebars were can estimate *d*

Minimum cover | Half of #3 bar diameter | #3 rebar diameter

$$d = 6 - 1.0 - 0.375 - \frac{0.375}{2}$$

$$d = \boxed{4.44 \text{ in.}}$$



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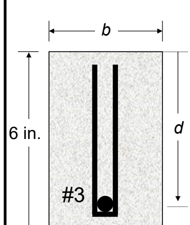
Reinforced Concrete Beam Example #1

We now have values for *b, d*, and A_s

$$\boxed{M = A_s f_y \left(d - 0.59 \frac{A_s f_y}{f'_c\, b} \right)}$$



The A_s for one #3 rebar is:

$$A_s = 0.11 \text{in.}^2$$

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Reinforced Concrete Beam Example #1

Compute the moment capacity.

$$M = A_s f_y \left(d - 0.59 \frac{A_s f_y}{f'_c\, b} \right)$$

$$= 0.11 \text{in.}^2 (60ksi) \left(4.44 \text{in.} - 0.59 \frac{0.11 \text{in.}^2 (60ksi)}{6ksi(3.13 \text{in.})} \right)$$

$$= 27.92\; k \cdot in \;\Rightarrow\; \boxed{P_{tension} = \frac{M}{4} = 6.98\; kips}$$

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Reinforced Concrete Beam Example #1

For proper anchorage, a minimum length of reinforcing, l_d is required:

$$l_d = \frac{f_y d_b}{24\sqrt{f_c'}\left(\frac{c}{d_b} - \frac{1}{2}\right)} = \frac{(60{,}000\,psi)(0.375\,in.)}{24\sqrt{6{,}000}\left(\frac{1\,in.}{0.375\,in.} - \frac{1}{2}\right)}$$

$= 5.59\,in.$ < 8 in. anchorage available at end of beam

For a hooked bar, the minimum L_{dh} is:

$$L_{dh} = \frac{1{,}200\,d_b}{\sqrt{f_c'}} = \frac{1{,}200(0.375\,in.)}{\sqrt{6{,}000}} = 5.81\,in.$$

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Reinforced Concrete Beam Example #1

Let's check the shear model

$$P_{shear} = 2\left(\frac{A_v f_y d}{s} + 2\sqrt{f'_c}\,bd\right)$$

Area of a #3 rebars

$$= 2\left(\frac{2(0.11\text{in.}^2)(60{,}000\,psi)4.44\text{in.}}{3\text{in.}} + 2\sqrt{6{,}000\,psi}\,(3.13\text{in.})(4.44\text{in.})\right)$$

Shear reinforcement spacing

$= 43.347\text{lb.} = \boxed{43.35\,kips}$

Since $P_{tension} < P_{shear}$ therefore $P_{tension}$ controls

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Reinforced Concrete Beam Example #1

Let's check the reinforcement ratio

$$\rho = \frac{A_s}{bd}$$

ρ as function of c/d

$$\rho = 0.85\beta_1 \frac{c}{d}\frac{f'_c}{f_y}$$

To compute ρ, first, we need to estimate β_1

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Reinforced Concrete Beam Example #1

The height of the stress box, ***a***, is defined as a percentage of the depth of the neural axis.

$$f'_c \leq 4000\ psi \quad \Rightarrow \quad \beta_1 = 0.85$$

$$f'_c \geq 4000\ psi$$

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) \geq 0.65$$

$$\beta_1 = 0.85 - 0.05\left(\frac{4{,}000 - 4{,}000}{1{,}000}\right) = \boxed{0.85}$$

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Reinforced Concrete Beam Example #1

Check the reinforcement ratio for the maximum steel allowed.

$$\rho = \frac{A_s}{bd} = \frac{0.11\text{in.}^2}{3.13\text{in.}(4.44\text{in.})} = 0.0079$$

$$\rho_{tension} = 0.85\beta_1 \frac{c}{d}\frac{f'_c}{f_y} = 0.85(0.85)0.375\frac{6\,ksi}{60\,ksi}$$

$$= 0.0271$$

$$\rho < \rho_{tension}$$

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Reinforced Concrete Beam Example #1

The minimum force controls

$$P_{tension} = 6.98\ kips$$

$$P_{shear} = 43.35\,kips$$

$$P = \boxed{6.98\,kips}$$

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Reinforced Concrete Beam Example #1

An estimate of the weight of the beam can be made as:

Size of concrete beam — Unit weight of concrete

$$W = \frac{(3.13\,in.)(6\,in.)(30\,in.)}{1728\,{}^{in.^3}/_{ft.^3}}\left(\frac{145\,lb.}{ft.^3}\right)$$

Additional weight of rebars — Unit weight of steel

$$+\frac{(0.11\,in.^2)(30\,in.)}{1728\,{}^{in.^3}/_{ft.^3}}\left(\frac{490\,lb. - 145\,lb.}{ft.^3}\right)$$

$$= 47.28\,lb. + 0.66\,lb. = \boxed{47.94\,lb.}$$

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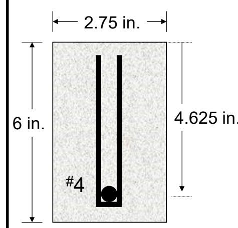
Reinforced Concrete Beam Example #1

In summary, this reinforced concrete beam will fail in tension



$$\Rightarrow P = 6.98\,kips$$

$$SWR = \frac{6{,}979\,lb}{47.94\,lb} = \boxed{146}$$

This beam should fail in tension

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Reinforced Concrete Beam Example #1

The cost of steel may be estimated as follows:

$$Cost\ of\ steel = \frac{A_s L}{1{,}728\,{}^{in.^3}/_{ft.^3}}\left(490\frac{lb.}{ft.^3}\right)\left(\frac{\$700}{ton}\right)\left(\frac{ton}{2{,}000\,lb.}\right)$$

where $\boldsymbol{A_s}$ is the cross-sectional area of steel rebars, ***L*** is the length of the steel rebars, and 490 lb./ft.^{3} is the unit weight of steel.

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Reinforced Concrete Beam Example #1

For example, if one #3 rebar is placed in the beam, the steel cost is estimated as follows:

Bar #	Diameter (in.)	As (in.^{2})
3	0.375	0.11
4	0.500	0.20
5	0.625	0.31
6	0.750	0.44
7	0.875	0.60
8	1.000	0.79
9	1.128	1.00
10	1.270	1.27
11	1.410	1.56

$$Cost\ of\ steel = $$

$$= \frac{(0.11\,in.^2)(30\,in.)}{1{,}728\,{}^{in.^3}/_{ft.^3}}\left(490\frac{lb.}{ft.^3}\right)\left(\frac{\$700}{ton}\right)\left(\frac{ton}{2{,}000\,lb.}\right)$$

$$= \$0.33$$

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Reinforced Concrete Beam Example #1

Consider the following mix for a yd.^{3} of concrete developed using the ACI mix design procedure.

Component	Amount (lb)
Water	315
Cement	553
Coarse aggregate	1,641
Fine aggregate	1,431

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Reinforced Concrete Beam Example #1

The cost of the concrete required for a 3.13 in. by 6 in. by 30 in. beam is estimated as:

$$Cost\ of\ cement = \frac{3.13\,in.(6\,in.)30\,in.}{1{,}728\,{}^{in.^3}/_{ft.^3}}\left(\frac{553\,lb.}{27\,ft.^3}\right)\left(\frac{\$150}{ton}\right)\left(\frac{ton}{2{,}000\,lb.}\right)$$

$$= \$0.50$$

$$Cost\ of\ coarse\ aggregate = \frac{3.13\,in^2(6\,in.)30\,in.}{1{,}728\,{}^{in.^3}/_{ft.^3}}\left(\frac{1{,}641\,lb.}{27\,ft.^3}\right)\left(\frac{\$18}{ton}\right)\left(\frac{ton}{2{,}000\,lb.}\right)$$

$$= \$0.18$$

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Reinforced Concrete Beam Example #1

The cost of the concrete required for a 3.13 in. by 6 in. by 30 in. beam is estimated as:

$$\textit{Cost of fine aggregate} = \frac{3.13\,\text{in.}(6\,\text{in.})30\,\text{in.}}{1{,}728\,{}^{\text{in.}^3}\!/_{\text{ft.}^3}}\left(\frac{1{,}431\,\text{lb.}}{27\,\text{ft.}^3}\right)\left(\frac{\$10}{ton}\right)\left(\frac{ton}{2{,}000\,\text{lb.}}\right)$$

$$= \$0.09$$

The cost of concrete is estimated at $0.77

The cost of the RC beam is estimated at $1.10

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Reinforced Concrete Beam Example #1

The cost adjustment for the reinforced concrete beam is:

If cost < $2 then: *Cost Factor* = 1

If cost > $2 then:

$$\textit{Cost Factor} = \frac{\$2}{\textit{Cost}}$$

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Reinforced Concrete Beam Example #1

If the unadjusted ***SWR*** for a beam is 146 and the cost is $1.10, then the cost-adjusted ***SWR*** is:

$$SWR_{Adjusted} = SWR \times \textit{Cost Factor}$$

$$SWR_{Adjusted} = 146 \times 1 = 146$$

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Reinforced Concrete Beam Example #1

Questions?

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