Reinforced Concrete Beam Analysis

Let's use the failure models to predict the ultimate strength-to-weight (SWR) of one of our reinforced concrete beams from lab

Consider a beam with the following characteristics:

- Concrete strength f'_c = 5,000 psi
- Steel strength f_y = 60,000 psi
- The tension reinforcement will be 2 #3 rebars
- The shear reinforcement will be #3 rebars bent in a U-shape spaced at 4 inches.
- Concrete cover and bar spacing is ¾ inches.
- Use the minimum width to accommodate the reinforcement

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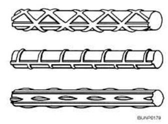
Reinforcing bars are denoted by the bar number. The diameter and area of standard rebars are shown below.



Bar #	Diameter (in.)	As (in.^{2})
3	0.375	0.11
4	0.500	0.20
5	0.625	0.31
6	0.750	0.44
7	0.875	0.60
8	1.000	0.79
9	1.128	1.00
10	1.270	1.27
11	1.410	1.56

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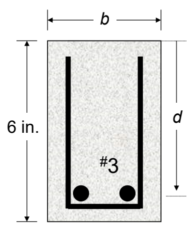
Based on the choice of reinforcement we can compute an estimate of *b* and *d*



Minimum cover | #3 tension rebars

$$b \geq 2(0.75\ in) + 2(0.375\ in)$$
$$+2(0.375\ in) + 0.75\ in = \boxed{3.75\,\text{in.}}$$

#3 shear rebar | Space between bars

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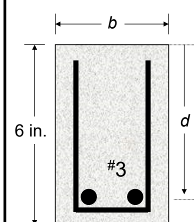
If we allow a minimum cover under the rebars were can estimate *d*



Minimum cover | #3 shear rebar | Half of #3 tension rebar

$$d = 6 - 0.75 - 0.375 - \frac{0.375}{2}$$

$$d = \boxed{4.69\text{ in.}}$$

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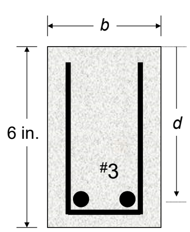
We now have values for *b*, *d*, and A_s



$$\boxed{M = A_s f_y \left(d - 0.59 \frac{A_s f_y}{f'_c\, b} \right)}$$

The A_s for two #3 rebars is:

$$A_s = 2(0.11\,\text{in.}^2) = 0.22\ \text{in.}^2$$

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Compute the moment capacity

$$M = A_s f_y \left(d - 0.59 \frac{A_s f_y}{f'_c\, b} \right)$$

$$= 0.22\text{in.}^2(60\,\text{ksi})\left(4.69\,\text{in.} - 0.59 \frac{0.22\text{in.}^2(60\,\text{ksi})}{5ksi(3.75\,\text{in.})} \right)$$

$$= 56.4\ \text{k}\cdot\text{in.} \quad \Rightarrow \quad \boxed{P = \frac{M}{4} = 14.1\,\text{kips}}$$

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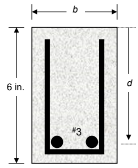
Reinforced Concrete Beam Analysis

Let's check the shear model



$$P_{shear} = 2\left(\frac{A_v f_y d}{s} + 2\sqrt{f'_c}\,bd\right)$$

Area of two #3 rebars

$$= 2\left(\frac{2(0.11\text{in.}^2)(60{,}000\text{psi})4.69\text{in.}}{4\text{in.}} + 2\sqrt{5{,}000\text{psi}}\,(3.75\text{in.})(4.69\text{in.})\right)$$

Shear reinforcement spacing

$$= 35{,}928\text{lb.} = \boxed{35.9\text{kips}}$$

$P_{tension}$ = 14.1 k < P_{shear} = 35.9 k; therefore $P_{tension}$ controls

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Let's check the reinforcement ratio

Reinforcement ratio definition

$$\rho = \frac{A_s}{bd}$$

ρ as function of *c/d*

$$\rho = 0.85\beta_1 \frac{c}{d}\frac{f'_c}{f_y}$$

To compute ρ, first we need to estimate β_1

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An β_1 estimate is given as:

$$f'_c \le 4000\ psi \quad \Rightarrow \quad \beta_1 = 0.85$$

$$f'_c \ge 4000\ psi$$

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) \ge 0.65$$

$$\beta_1 = 0.85 - 0.05\left(\frac{5{,}000 - 4{,}000}{1{,}000}\right) = \boxed{0.80}$$

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Check the reinforcement ratio for the maximum steel allowed for tension controlled behavior or c/d = 0.375

$$\rho = 0.85\beta_1 \frac{c}{d}\frac{f'_c}{f_y} = 0.85(0.80)0.375\frac{5\text{ksi}}{60\text{ksi}}$$

$$= 0.021$$

c/d = 0.375 for tension controlled behavior

$$\rho = \frac{A_s}{bd} = \frac{0.22\text{ in.}^2}{3.75\text{in.}(4.69\text{in.})} = 0.0125$$

The amount of steel in this beam is tension-controlled behavior.

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An estimate of the weight of the beam can be made as:

Size of concrete beam

Unit weight of concrete

$$W = \frac{b\,h\,L}{1728\text{in.}^3/\text{ft.}^3}\left(\frac{145\text{lb.}}{\text{ft.}^3}\right)$$

Additional weight of rebars

Unit weight of steel

$$+\frac{A_s L}{1728\text{ in.}^3/\text{ft.}^3}\left(\frac{490\text{lb.} - 145\text{lb.}}{\text{ft.}^3}\right)$$

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Reinforced Concrete Beam Analysis

An estimate of the weight of the beam can be made as:

Size of concrete beam

Unit weight of concrete

$$W = \frac{(3.75\text{in.})(6\text{in.})(30\text{in.})}{1728\text{in.}^3/\text{ft.}^3}\left(\frac{145\text{lb.}}{\text{ft.}^3}\right)$$

Additional weight of rebars

Unit weight of steel

$$+\frac{(0.22\text{in.}^2)(30\text{in.})}{1728\text{ in.}^3/\text{ft.}^3}\left(\frac{490\text{lb.} - 145\text{lb.}}{\text{ft.}^3}\right)$$

$$= 56.64\text{ lb.} + 1.32\text{ lb.} = \boxed{57.96\text{ lb.}}$$

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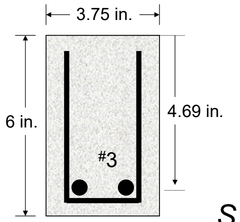
In summary, this reinforced concrete beam will fail in tension



$$\Rightarrow \begin{cases} S \Rightarrow P = 14.1\text{ kips} \\ W = 57.96\text{ lb.} \end{cases}$$

$$SWR = \frac{14{,}100\text{ lb.}}{57.96\text{ lb.}} = \boxed{243}$$

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Questions?

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