











## **Reinforced Concrete Beams**

Mathematical model for failure in a reinforced concrete beam





















The shear between the applied load and the support is constant V = P/2.

The shear force V = P/2 is constant between the applied load and the support.





# Reinforced Concrete Beams

- Let's look at the internal moment at the section between the supports and applied load
- The bending moment is the internal reaction to forces that cause a beam to bend.
- Bending moment can also be referred to as torque



















### **Reinforced Concrete Beams**

Shear failure in a reinforced concrete beam













### Whitney Rectangular Stress Distribution

In the 1930s, Whitney proposed the use of a rectangular compressive stress distribution.





















### Whitney Rectangular Stress Distribution

The internal moment is the value of the tension or compression force multiplied by the distance between them.







### **Reinforced Concrete Beams**

We can approximate the shear failure in unreinforced concrete as follows:

$$V_c = 2\sqrt{f'_c}bd$$

If we include some reinforcing for shear, the total shear capacity of a reinforced concrete bean would be approximated as follows:

$$V_n = V_c + V_s$$





### **Reinforced Concrete Beams**

There is a "balanced" condition where the stress in the steel reinforcement and the stress in the concrete are both at their yield points.

The amount of steel required to reach the balanced strain condition is defined in terms of the reinforcement ratio:







# Reinforced Concrete BeamsLet's consider compression failure in over-reinforced concrete.First, let's define an equation that gives the stress in the<br/>tensile steel when the concrete reaches its ultimate strain. $f_{steel} = 87,000 \, psi \left( \frac{d-c}{c} \right)$ If $f_{steel} < f_y$ then or $\frac{c}{d} > 0.600$ $M_{compression} = A_s \left( \frac{d-c}{c} \right) \left( d - \frac{a}{2} \right) 87,000 \, psi$















### **Reinforced Concrete Beam Analysis**

Let's use the failure models to predict the ultimate strength-toweight (SWR) of one of our reinforced concrete beams from the lab

Consider a beam with the following characteristics:

Concrete strength  $f'_c$  = 5,000 psi

- Steel strength  $f_y = 60,000$  psi
- The tension reinforcement will be 2 #3 rebars
- The shear reinforcement will be #3 rebars bent in a U-shape spaced at 4 inches.

The concrete cover and bar spacing is ¾ inches.

Use the minimum width to accommodate the reinforcement

### **Reinforced Concrete Beam Analysis**

The bar number denotes reinforcing bars. The diameter and area of standard rebars are shown below.











Reinforced Concrete Beam Analysis For proper anchorage, a minimum length of reinforcing,  $I_d$  is required:  $I_d = \frac{f_y d_b}{24\sqrt{f_c} \left(\frac{c}{d_b} - \frac{1}{2}\right)} = \frac{(60 \text{ ksi})(0.375 \text{ in.})}{24\sqrt{5,000} \left(\frac{0.75 \text{ in.}}{0.375 \text{ in.}} - \frac{1}{2}\right)}$  = 8.8 in. > 8 in. anchorage available at end of beamBar should be hooked; minimum  $L_{dh}$  is:

$$L_{dh} = \frac{1,200\,d_{b}}{\sqrt{f_{c}}} = \frac{1,200(0.375\,in.)}{\sqrt{5,000}} = 6.4\,in.$$









# Reinforced Concrete Beam Analysis An $\beta_1$ estimate is given as: $f'_c \le 4000 \ psi \implies \beta_1 = 0.85$ $f'_c \ge 4000 \ psi$ $\beta_1 = 0.85 - 0.05 \left(\frac{f'_c - 4000}{1000}\right) \ge 0.65$ $\beta_1 = 0.85 - 0.05 \left(\frac{5,000 - 4,000}{1,000}\right) = 0.80$



The amount of steel in this beam is tension-controlled behavior.







