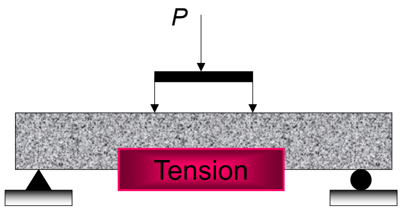
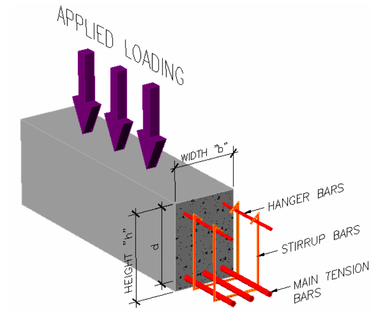
Reinforced Concrete Beams

Let’s focus on how to model the ultimate tensile load in a reinforced concrete beam.



Reinforced Concrete Beams

Typical rebar configuration to handle tension and shear loads.



Reinforced Concrete Beams

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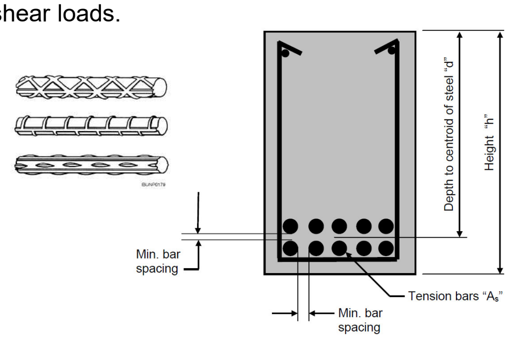

Reinforced Concrete Beams

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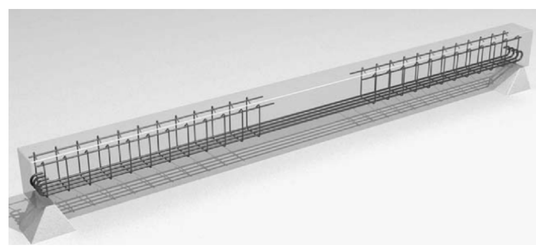
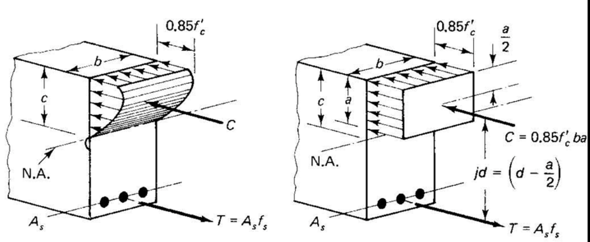


Whitney Rectangular Stress Distribution

In the 1930s, Whitney proposed the use of a rectangular compressive stress distribution.



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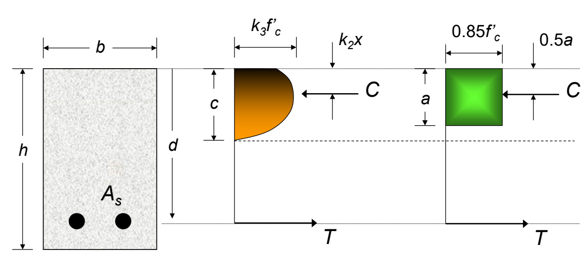
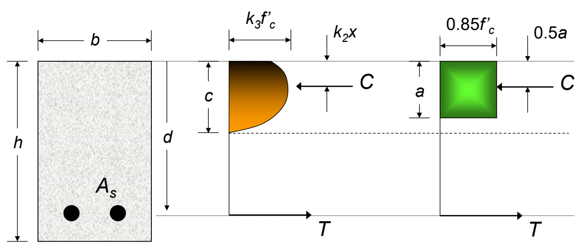


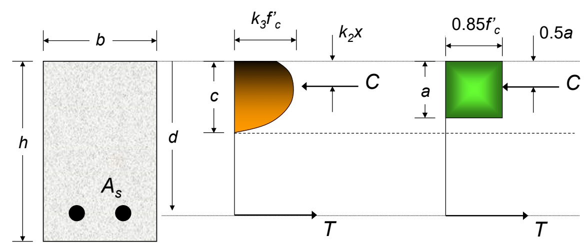
Whitney Rectangular Stress Distribution

Assume that the concrete contributes nothing to the tensile strength of the beam.



Whitney Rectangular Stress Distribution

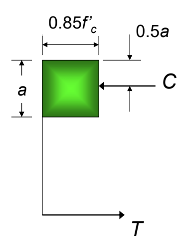
Assume that the complex distribution of compressive stress in the concrete can be approximated by a rectangle.



Whitney Rectangular Stress Distribution

The height of the stress box, *a*, is defined as a percentage of the depth to the neural axis.

$$a = \beta_1 c$$



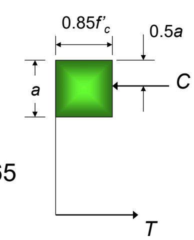
Whitney Rectangular Stress Distribution

The height of the stress box, *a*, is defined as a percentage of the depth to the neural axis.

$$f'_c \leq 4000\ psi \quad \Rightarrow \quad \beta_1 = 0.85$$

$$f'_c \geq 4000\ psi$$

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) \geq 0.65$$



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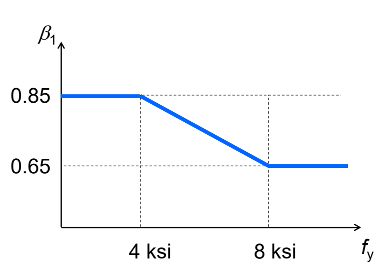
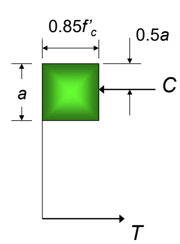




Whitney Rectangular Stress Distribution

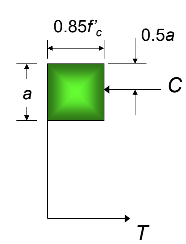
The values of the tension and compression forces are:

$$C = 0.85 f'_c\, ba$$

$$T = A_s f_y$$

$$\sum F = 0 = T - C$$

$$a = \frac{A_s f_y}{0.85 f'_c\, b}$$

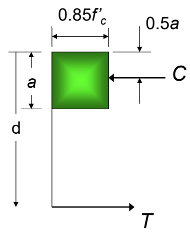


Whitney Rectangular Stress Distribution

If the tension force capacity of the steel is too high, then the value of ***a*** is large.

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

If ***a* > *d***, then you have too much steel

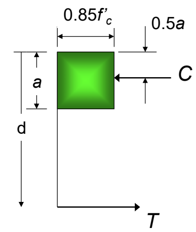


Whitney Rectangular Stress Distribution

If the tension force capacity of the steel is too high, then the value of ***a*** is large.

$$\sum M = T\left(d - \frac{a}{2}\right)$$

$$M = A_s f_y \left(d - \frac{a}{2}\right)$$



Whitney Rectangular Stress Distribution

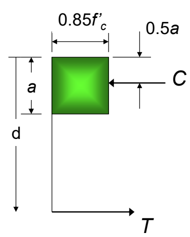
The internal moment is the value of the tension or compression force multiplied by the distance between them.

$$M = A_s f_y \left(d - \frac{a}{2}\right)$$

Substitute the value for ***a***

$$M = A_s f_y \left(d - 0.59 \frac{A_s f_y}{f'_c b}\right)$$

$M = 4P$ This is just for our 30-inch beams



Whitney Rectangular Stress Distribution

The internal moment is the value of the tension or compression force multiplied by the distance between them.

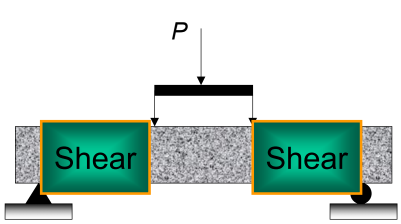
$$M = A_s f_y \left(d - 0.59 \frac{A_s f_y}{f'_c b}\right)$$

We know that the moment in our reinforced concrete beans is $M = 4P$

$$P_{tension} = \frac{A_s f_y}{4}\left(d - 0.59 \frac{A_s f_y}{f'_c b}\right)$$

Reinforced Concrete Beams

Let's focus on how to model the ultimate shear load in a reinforced concrete beam.



Reinforced Concrete Beams

We can approximate the shear failure in unreinforced concrete as follows:

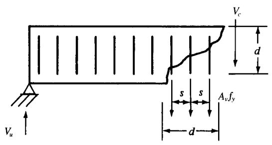
$$V_c = 2\sqrt{f'_c}\,bd$$

If we include some reinforcing for shear, the total shear capacity of a reinforced concrete bean would be approximated as follows:

$$V_n = V_c + V_s$$

Reinforced Concrete Beams

Let's consider shear failure in reinforced concrete.



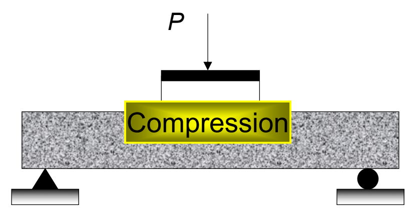
$$V_s = \frac{A_v f_y d}{s}$$

$$V_n = \frac{P}{2}$$

$$P_{shear} = 2\left(\frac{A_v f_y d}{s} + 2\sqrt{f'_c}\,bd\right)$$

Reinforced Concrete Beams

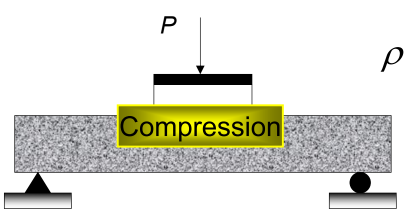
Let's focus on how to model the ultimate compression load in a reinforced concrete beam.



Reinforced Concrete Beams

There is a "balanced" condition where the stress in the steel reinforcement and the stress in the concrete are both at their yield points.

The amount of steel required to reach the balanced strain condition is defined in terms of the reinforcement ratio:



$$\rho = \frac{A_s}{bd}$$

Reinforced Concrete Beams

The limits of the reinforcement ratio are established as:

Reinforcement ratio definition

$$\rho = \frac{A_s}{bd}$$

ρ as function of c/d

$$\rho = 0.85\beta_1 \frac{c}{d}\frac{f'_c}{f_y}$$

Reinforced Concrete Beams

The limits of the reinforcement ratio are established as:

$\frac{c}{d} > 0.600$ Beam failure is controlled by ***compression***

$0.375 < \frac{c}{d} < 0.600$ ***Transition*** between tension and compression control

$\frac{c}{d} < 0.375$ Beam failure is controlled by ***tension***

Reinforced Concrete Beams

Let's consider compression failure in over-reinforced concrete.

First, let's define an equation that gives the stress in the tensile steel when the concrete reaches its ultimate strain.

$$f_{steel} = 87{,}000\,psi\left(\frac{d-c}{c}\right)$$

If $f_{steel} < f_y$ then or $\frac{c}{d} > 0.600$

$$M_{compression} = A_s\left(\frac{d-c}{c}\right)\left(d - \frac{a}{2}\right)87{,}000\,psi$$

Reinforced Concrete Beams

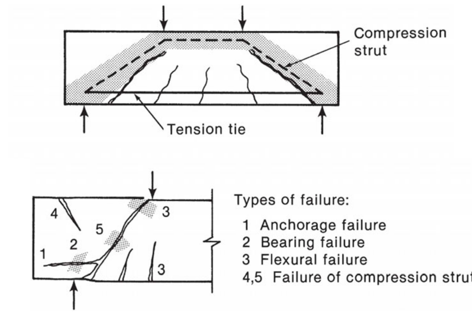
Let's consider compression failure in over-reinforced concrete. First, let's define an equation that gives the stress in the tensile steel when the concrete reaches its ultimate strain.

$$M = 4P \qquad \text{only if } f_s < f_y$$

$$P_{compression} = \frac{A_s}{4}\left(\frac{d-c}{c}\right)\left(d-\frac{a}{2}\right)87{,}000\,psi$$

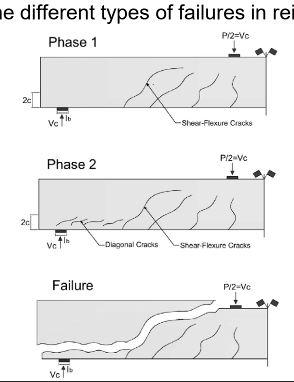
Reinforced Concrete Beams

Consider the different types of failures in reinforced concrete:



Reinforced Concrete Beams

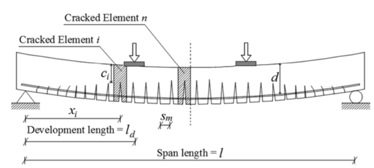
Consider the different types of failures in reinforced concrete:



Reinforced Concrete Beams

Because the actual bond stress varies along the length of the reinforcing bar anchored in the tension zone, the ACI code uses the ***development length*** concept.

The development length l_d is the shortest length of the bar in which the bar stresses can increase from zero to the yield stress f_y.

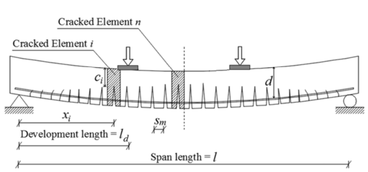


Reinforced Concrete Beams

For proper anchorage, a minimum length of reinforcing, l_d is required:

$$l_d = \frac{f_y d_b}{24\sqrt{f'_c}\left(\frac{c}{d_b}-\frac{1}{2}\right)}$$

where d_b is the diameter of the reinforcing bars and c is the bottom cover of the bars.

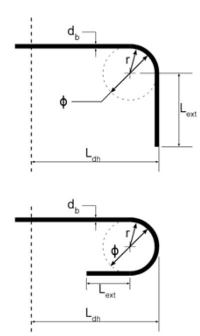


Reinforced Concrete Beams

For hooked bars, a minimum length of reinforcing, L_{dh} is required:

$$L_{dh} = \frac{1{,}200\, d_b}{\sqrt{f'_c}}$$

ϕ_{inner} = 2.250 in.
r_{inner} = 1.125 in.
ϕ_{outer} = 3.000 in.
r_{outer} = 1.500 in.
L_{ext} = 2.500 in.



Reinforced Concrete Beams

Questions?