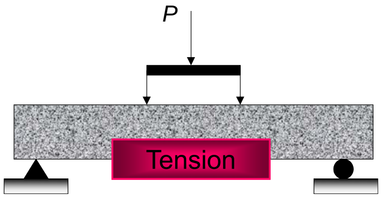
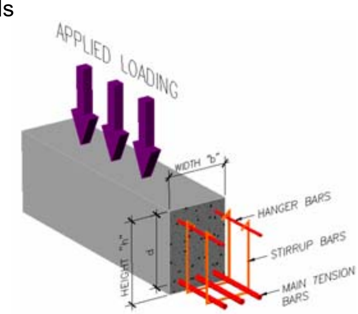
Reinforced Concrete Beams

Let's focus on how to model the ultimate tensile load in a reinforced concrete beam



Reinforced Concrete Beams

Typical rebar configuration to handle tension and shear loads



Reinforced Concrete Beams

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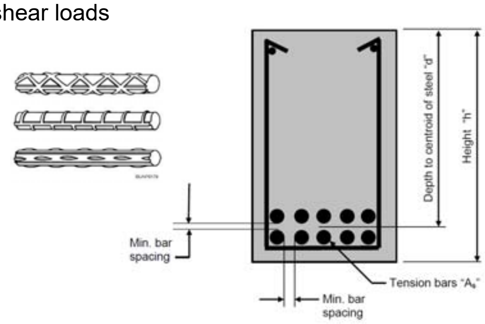
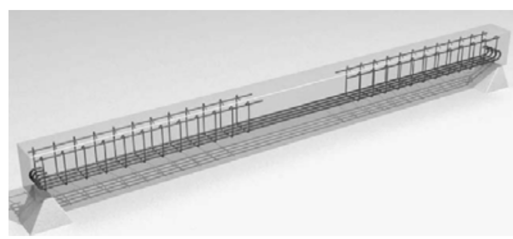


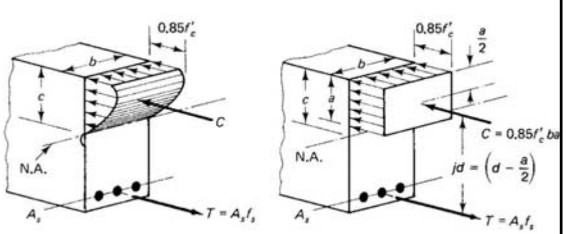
Reinforced Concrete Beams

Typical rebar configuration to handle tension and shear loads



Whitney Rectangular Stress Distribution

In the 1930s, Whitney proposed the use of a rectangular compressive stress distribution



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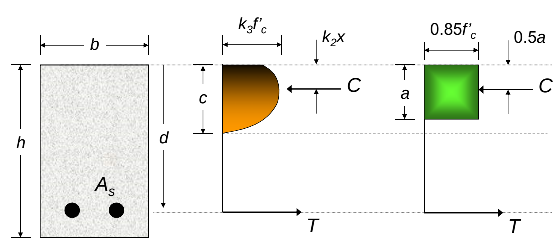
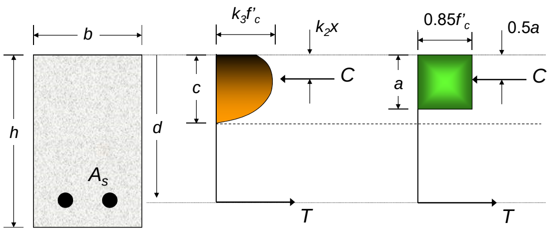


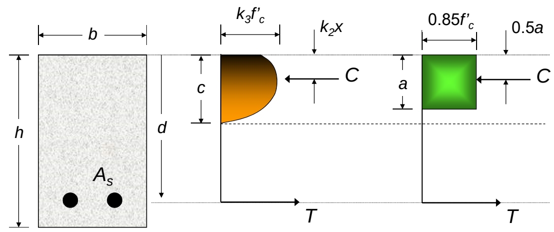
Whitney Rectangular Stress Distribution

Assume that the concrete contributes nothing to the tensile strength of the beam



Whitney Rectangular Stress Distribution

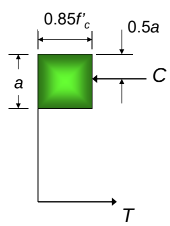
Assume that the complex distribution of compressive stress in the concrete can be approximated by a rectangle



Whitney Rectangular Stress Distribution

The height of the stress box, *a*, is defined as a percentage of the depth to the neural axis

$$a = \beta_1 c$$



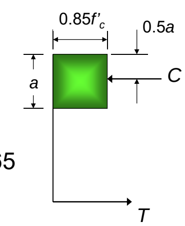
Whitney Rectangular Stress Distribution

The height of the stress box, *a*, is defined as a percentage of the depth to the neural axis

$$f'_c \leq 4000\ psi \quad \Rightarrow \quad \beta_1 = 0.85$$

$$f'_c \geq 4000\ psi$$

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) \geq 0.65$$



Whitney Rectangular Stress Distribution

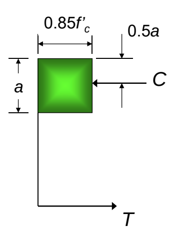
The values of the tension and compression forces are:

$$C = 0.85 f'_c\, ba$$

$$T = A_s f_y$$

$$\sum F = 0 = T - C$$

$$a = \frac{A_s f_y}{0.85 f'_c\, b}$$

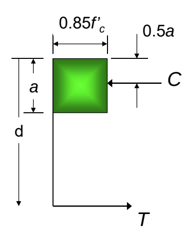


Whitney Rectangular Stress Distribution

If the tension force capacity of the steel is too high, than the value of ***a*** is large

$$a = \frac{A_s f_y}{0.85 f'_c\, b}$$

If $a > d$, then you have too much steel

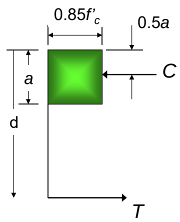


Whitney Rectangular Stress Distribution

If the tension force capacity of the steel is too high, than the value of ***a*** is large

$$\sum M = T\left(d - \frac{a}{2}\right)$$

$$M = A_s f_y\left(d - \frac{a}{2}\right)$$



Whitney Rectangular Stress Distribution

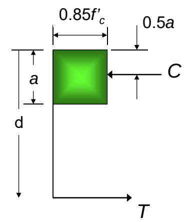
The internal moment is the value of either the tension or compression force multiplied the distance between them.

$$M = A_s f_y\left(d - \frac{a}{2}\right)$$

Substitute the value for ***a***

$$M = A_s f_y\left(d - 0.59\frac{A_s f_y}{f'_c\, b}\right)$$

$$M = 4P$$



Whitney Rectangular Stress Distribution

The internal moment is the value of either the tension or compression force multiplied the distance between them

$$M = A_s f_y\left(d - 0.59\frac{A_s f_y}{f'_c\, b}\right)$$

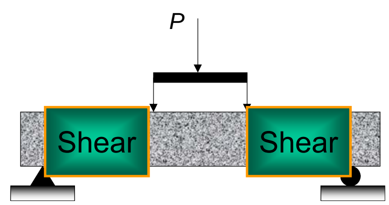
We know that the moment in our reinforced concrete beans is

$$M = 4P$$

$$P_{tension} = \frac{A_s f_y}{4}\left(d \text{ - } 0.59\frac{A_s f_y}{f'_c b}\right)$$

Reinforced Concrete Beams

Let's focus on how to model the ultimate shear load in a reinforced concrete beam



Reinforced Concrete Beams

We can approximate the shear failure in unreinforced concrete as:

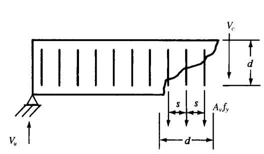
$$V_c = 2\sqrt{f'_c}\,bd$$

If we include some reinforcing for shear the total shear capacity of a reinforce concrete bean would be approximated as:

$$V_n = V_c + V_s$$

Reinforced Concrete Beams

Lets consider shear failure in reinforced concrete



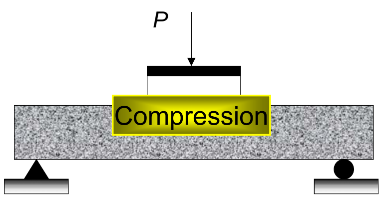
$$V_s = \frac{A_v f_y d}{s}$$

$$V_n = \frac{P}{2}$$

$$P_{shear} = 2\left(\frac{A_v f_y d}{s} + 2\sqrt{f'_c}\,bd\right)$$

Reinforced Concrete Beams

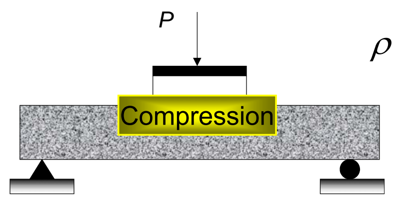
Let’s focus on how to model the ultimate compression load in a reinforced concrete beam



Reinforced Concrete Beams

There is a “balanced” condition where the stress in the steel reinforcement and the stress in the concrete are both at their yield points

The amount of steel required to reach the balanced strain condition is defined in terms of the reinforcement ratio:



$$\rho = \frac{A_s}{bd}$$

Reinforced Concrete Beams

The limits of the reinforcement ratio are established as:

Reinforcement ratio definition

$$\rho = \frac{A_s}{bd}$$

ρ as function of c/d

$$\rho = 0.85\beta_1 \frac{c}{d}\frac{f'_c}{f_y}$$

Reinforced Concrete Beams

The limits of the reinforcement ratio are established as:

$\frac{c}{d} > 0.600$ Beam failure is controlled by ***compression***

$0.375 < \frac{c}{d} < 0.600$ Transition between tension and compression control

$\frac{c}{d} < 0.375$ Beam failure is controlled by ***tension***

Reinforced Concrete Beams

Lets consider compression failure in over reinforced concrete. First, let define an equation that given the stress in the tensile steel when concrete reaches its ultimate strain.

$$f_{steel} = 87{,}000\,psi\left(\frac{d-c}{c}\right)$$

If $f_{steel} < f_y$ then or $\frac{c}{d} > 0.600$

$$M_{compression} = A_s\left(\frac{d-c}{c}\right)\left(d-\frac{a}{2}\right)87{,}000\,psi$$

Reinforced Concrete Beams

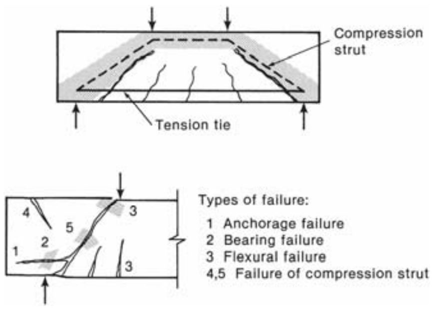
Lets consider compression failure in over reinforced concrete First, let define an equation that given the stress in the tensile steel when concrete reaches its ultimate strain

$M = 4P$ only if $f_s < f_y$

$$P_{compression} = \frac{A_s}{4}\left(\frac{d-c}{c}\right)\left(d-\frac{a}{2}\right)87{,}000\,psi$$

Reinforced Concrete Beams

Consider the different types of failures in reinforced concrete:



Reinforced Concrete Beams

Questions?