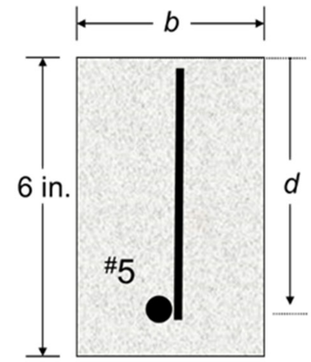
**Example 4 - TopHat**

Let's use the failure models to predict the ultimate strength-to-weight (SWR) of one of our reinforced concrete beams from the lab.

Consider a beam with the following characteristics:

- Concrete strength $f'_c$ = 6,000 psi
- Steel strength $f_y$ = 60,000 psi
- The tension reinforcement will be one #5 rebar
- The shear reinforcement will be one #3 rebar installed vertically at 3 in. spacing
- Use a minimum cover of 1 in. and width to accommodate the reinforcement




Consider the following mix for a yd$^3$ of concrete developed using the ACI mix design procedure.

| Component | Amount (lb.) |
|---|---|
| water | 315 |
| cement | 768 |
| coarse aggregate | 1,658 |
| fine aggregate | 1,242 |

| Bar # | Diameter (in.) | As (in.$^2$) |
|---|---|---|
| 3 | 0.375 | 0.11 |
| 4 | 0.500 | 0.20 |
| 5 | 0.625 | 0.31 |
| 6 | 0.750 | 0.44 |

4.1 What is the minimum width *b* of the beam?

*b* =

4.2 What is the beam's minimum depth d (in.)?

*d* =

4.3 Compute $A_s$ =

4.4 What is the value for *a* (depth (in.) of the Whitney compression block)?

$$a = \frac{A_s f_y}{0.85 f'_c b} =$$

4.5 What is the beam's moment capacity M (lb.-in.)?

$$M = A_s f_y \left( d - \frac{a}{2} \right) =$$

4.6 What is the predicted strength *P* (k) based on the tension model?

4.7 What is the predicted strength $P$ (k) based on the shear model?

$$P_{shear} = 2\left(\frac{A_v f_y d}{s} + 2\sqrt{f'_c}bd\right) =$$

4.8 What is the beta value for this design?

$$f'_c \leq 4000\ psi \quad \Rightarrow \quad \beta_1 = 0.85$$

$$f'_c \geq 4000\ psi \quad \Rightarrow \quad \beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) \geq 0.65$$

4.9 What is the reinforcement ratio $\rho$ for tension control ($c/d$ = 0.375)

$$\rho = 0.85\beta_1 \frac{c}{d}\frac{f'_c}{f_y} =$$

4.10 What is the reinforcement ratio $\rho$ for the RC beam

$$\rho = \frac{A_s}{bd} =$$

4.12 Which value for $P$ controls the design? $S$ =

4.13 What is the estimated weight of the beam (lb.)?

$$W = \frac{bhL}{1728\,\text{in.}^3/\text{ft.}^3}\left(\frac{145\,\text{lb.}}{\text{ft.}^3}\right) + \frac{A_s L}{1728\ \text{in.}^3/\text{ft.}^3}\left(\frac{490\,\text{lb.} - 145\,\text{lb.}}{\text{ft.}^3}\right)$$

$W$ =

4.14 What is the estimated $SWR = \dfrac{Ultimate\,Load(lb.)}{Beam\,Weight(lb.)} =$

4.15 16

$$Cost\ of\ steel = \frac{A_s L}{1{,}728\,\text{in.}^3/\text{ft.}^3}\left(490\,\frac{\text{lb.}}{\text{ft.}^3}\right)\left(\frac{\$700}{ton}\right)\left(\frac{ton}{2{,}000\ \text{lb.}}\right) =$$

$$Cost\ of\ cement = \frac{bhL}{1{,}728\,\text{in.}^3/\text{ft.}^3}\left(\frac{W_{cement}}{27\ ft.^3}\right)\left(\frac{\$150}{\text{ton}}\right)\left(\frac{\text{ton}}{2{,}000\ \text{lb.}}\right) =$$

$$Cost\ of\ coarse\ aggregate = \frac{bhL}{1{,}728\,\text{in.}^3/\text{ft.}^3}\left(\frac{W_{CA}}{27\ ft.^3}\right)\left(\frac{\$18}{\text{ton}}\right)\left(\frac{\text{ton}}{2{,}000\ \text{lb.}}\right) =$$

$$Cost\ of\ fine\ aggregate = \frac{bhL}{1{,}728\,\text{in.}^3/\text{ft.}^3}\left(\frac{W_{FA}}{27\ ft.^3}\right)\left(\frac{\$10}{\text{ton}}\right)\left(\frac{\text{ton}}{2{,}000\ \text{lb.}}\right) =$$

4.16 What is the cost-adjusted *SWR* of this beam?

If the cost is > \$2, then: $Cost\ Factor = \frac{\$2}{Cost}$

$$SWR_{Adjusted} = SWR \times Cost\ Factor =$$