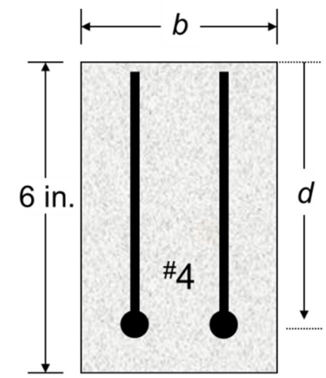
Example 2 - TopHat

Let's use the failure models to predict the ultimate strength-to-weight (SWR) of one of our reinforced concrete beams from the lab.



Consider a beam with the following characteristics:

- Concrete strength $f'_c = 6{,}000$ psi
- Steel strength $f_y = 60{,}000$ psi
- The tension reinforcement will be 2 #4 rebars
- The shear reinforcement will be #3 rebars installed vertically at 4 in. spacing
- Use a minimum cover of 1 in., a bar spacing of 0.75 in., and a width to accommodate the reinforcement

Consider the following mix for a yd^3 of concrete developed using the ACI mix design procedure.

Component	Amount (lb.)
water	315
cement	768
coarse aggregate	1,641
fine aggregate	1,251

Bar #	Diameter (in.)	As (in.^{2})
3	0.375	0.11
4	0.500	0.20
5	0.625	0.31
6	0.750	0.44

2.1 What is the minimum width *b* of the beam:

b =

2.2 What is the beam's minimum depth d (in.)?

d =

2.3 Compute As =

2.4 What is the value for a (depth of the Whitney compression block)?

$$a = \frac{A_s f_y}{0.85 f'_c\, b} =$$

2.5 What is the beam's moment capacity M (lb.-in.)?

$$M = A_s f_y \left(d - \frac{a}{2} \right) =$$

2.6 What is the predicted strength *P* (k) based on the tension model?

2.7 What is the predicted strength *P* (k) based on the shear model?

$$P_{shear} = 2\left(\frac{A_v f_y d}{s} + 2\sqrt{f'_c}bd\right) =$$

2.7a Which value for *P* controls the design? *S* =

2.8 What is the beta value for this design?

$$f'_c \le 4000\ psi \quad \Rightarrow \quad \beta_1 = 0.85$$

$$f'_c \ge 4000\ psi \quad \Rightarrow \quad \beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) \ge 0.65$$

2.9 What is reinforcement ratio ρ for tension control (c/d = 0.375)

$$\rho = 0.85\beta_1 \frac{c}{d}\frac{f'_c}{f_y} =$$

2.10 What is the reinforcement ratio ρ for the RC beam

$$\rho = \frac{A_s}{bd} =$$

2.11 What is the estimated weight of the beam (lb.)?

$$W = \frac{bhL}{1728\,\text{in.}^3/\text{ft.}^3}\left(\frac{145\,\text{lb.}}{\text{ft.}^3}\right) + \frac{A_s L}{1728\ \text{in.}^3/\text{ft.}^3}\left(\frac{490\,\text{lb.} - 145\,\text{lb.}}{\text{ft.}^3}\right)$$

W =

2.12 What is the estimated *SWR*?

$$SWR = \frac{Ultimate\,Load(lb.)}{Beam\,Weight(lb.)} =$$

2.13 What is the estimated cost of this beam?

$$Cost\ of\ steel = \frac{A_s L}{1{,}728\,\text{in.}^3/\text{ft.}^3}\left(490\,\frac{\text{lb.}}{\text{ft.}^3}\right)\left(\frac{\$530}{ton}\right)\left(\frac{ton}{2{,}000\ \text{lb.}}\right) =$$

$$Cost\ of\ cement = \frac{bhL}{1{,}728\,\text{in.}^3/\text{ft.}^3}\left(\frac{W_{cement}}{27\ ft.^3}\right)\left(\frac{\$130}{\text{ton}}\right)\left(\frac{\text{ton}}{2{,}000\ \text{lb.}}\right) =$$

$$Cost\ of\ coarse\ aggregate = \frac{bhL}{1{,}728\,\text{in.}^3/\text{ft.}^3}\left(\frac{W_{CA}}{27\ ft.^3}\right)\left(\frac{\$18}{\text{ton}}\right)\left(\frac{\text{ton}}{2{,}000\ \text{lb.}}\right) =$$

$$Cost\ of\ fine\ aggregate = \frac{bhL}{1{,}728\,\text{in.}^3/\text{ft.}^3}\left(\frac{W_{FA}}{27\ ft.^3}\right)\left(\frac{\$10}{\text{ton}}\right)\left(\frac{\text{ton}}{2{,}000\ \text{lb.}}\right) =$$

2.14 What is the cost-adjusted *SWR* of this beam?

If the cost > \$1.50, then: $Cost\,Factor = \frac{\$1.50}{Cost}$

$$SWR_{Adjusted} = SWR \times Cost\ Factor =$$