**Example 2 - TopHat**

Let's use the failure models to predict the ultimate strength-to-weight (SWR) of one of our reinforced concrete beams from the lab.

Consider a beam with the following characteristics:

- Concrete strength f'_c = 6,000 psi
- Steel strength f_y = 60,000 psi
- The tension reinforcement will be 2 #4 rebars
- The shear reinforcement will be #3 rebars installed vertically at 4 in. spacing
- Use a minimum cover of 1 in., a bar spacing of 0.75 in., and a width to accommodate the reinforcement
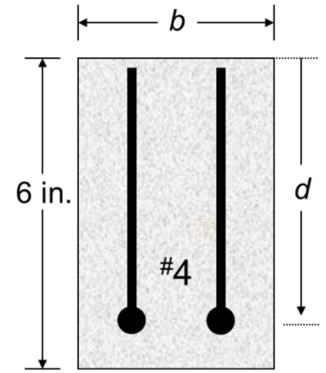
Consider the following mix for a yd³ of concrete developed using the ACI mix design procedure.

Component	Amount (lb.)
water	315
cement	768
coarse aggregate	1,641
fine aggregate	1,251

Bar #	Diameter (in.)	As (in.²)
3	0.375	0.11
4	0.500	0.20
5	0.625	0.31
6	0.750	0.44

2.1 What is the minimum width b of the beam?

b =

2.2 What is the beam's minimum depth d (in.)?

d =

2.3 Compute As =

2.4 What is the value for a (depth of the Whitney compression block)?

$$a = \frac{A_s f_y}{0.85 f'_c b} =$$

2.5 What is the beam's moment capacity M (lb.-in.)?

$$M = A_s f_y \left(d - \frac{a}{2} \right) =$$

2.6 What is the predicted strength P (k) based on the tension model?

2.7 What is the predicted strength P (k) based on the shear model?

$$P_{shear} = 2\left(\frac{A_v f_y d}{s} + 2\sqrt{f'_c}\, bd\right) =$$

2.7a Which value for P controls the design? $S =$

2.8 What is the beta value for this design?

$$f'_c \leq 4000\ psi \quad \Rightarrow \quad \beta_1 = 0.85$$

$$f'_c \geq 4000\ psi \quad \Rightarrow \quad \beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) \geq 0.65$$

2.9 What is the reinforcement ratio ρ for tension control ($c/d = 0.375$)

$$\rho = 0.85\beta_1 \frac{c}{d}\frac{f'_c}{f_y} =$$

2.10 What is the reinforcement ratio ρ for the RC beam

$$\rho = \frac{A_s}{bd} =$$

2.11 What is the estimated weight of the beam (lb.)?

$$W = \frac{bhL}{1728\,in.^3/ft.^3}\left(\frac{145\,lb.}{ft.^3}\right) + \frac{A_s L}{1728\ in.^3/ft.^3}\left(\frac{490\,lb. - 145\,lb.}{ft.^3}\right)$$

$W =$

2.12 What is the estimated SWR?

$$SWR = \frac{Ultimate\,Load(lb.)}{Beam\,Weight(lb.)} =$$

2.13 What is the estimated cost of this beam?

$$Cost\,of\,steel = \frac{A_s L}{1,728\,^{in.^3}\!/_{ft.^3}} \left(490\,\frac{lb.}{ft.^3}\right)\left(\frac{\$700}{ton}\right)\left(\frac{ton}{2,000\,lb.}\right) =$$

$$Cost\,of\,cement = \frac{bhL}{1,728\,^{in.^3}\!/_{ft.^3}} \left(\frac{W_{cement}}{27\,ft.^3}\right)\left(\frac{\$150}{ton}\right)\left(\frac{ton}{2,000\,lb.}\right) =$$

$$Cost\,of\,coarse\,aggregate = \frac{bhL}{1,728\,^{in.^3}\!/_{ft.^3}} \left(\frac{W_{CA}}{27\,ft.^3}\right)\left(\frac{\$18}{ton}\right)\left(\frac{ton}{2,000\,lb.}\right) =$$

$$Cost\,of\,fine\,aggregate = \frac{bhL}{1,728\,^{in.^3}\!/_{ft.^3}} \left(\frac{W_{FA}}{27\,ft.^3}\right)\left(\frac{\$10}{ton}\right)\left(\frac{ton}{2,000\,lb.}\right) =$$

2.14 What is the cost-adjusted *SWR* of this beam?

If the cost is > \$2, then: $Cost\,Factor = \dfrac{\$2}{Cost}$

$$SWR_{Adjusted} = SWR \times Cost\,Factor =$$