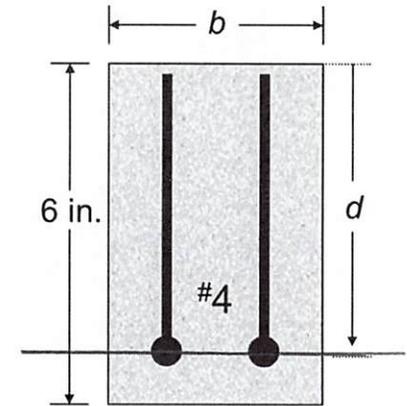


Example 2 - TopHat

Let's use the failure models to predict the ultimate strength-to-weight (SWR) of one of our reinforced concrete beams from the lab.

Consider a beam with the following characteristics:

- Concrete strength $f'_c = 6,000$ psi
- Steel strength $f_y = 60,000$ psi
- The tension reinforcement will be 2 #4 rebars
- The shear reinforcement will be #3 rebars installed vertically at 4 in. spacing
- Use a minimum cover of 1 in., a bar spacing of 0.75 in., and a width to accommodate the reinforcement



Consider the following mix for a yd^3 of concrete developed using the ACI mix design procedure.

Component	Amount (lb.)
water	315
cement	768
coarse aggregate	1,641
fine aggregate	1,251

Bar #	Diameter (in.)	As (in ²)
3	0.375	0.11
4	0.500	0.20
5	0.625	0.31
6	0.750	0.44

2.1 What is the minimum width b of the beam?

$$b = (2 \text{ COVER}) + \text{DIAMETER REBAR} + \text{BAR SPACING}$$

$$= 2(1\text{in}) + 2\left(\frac{1}{2}\text{in}\right) + 0.75\text{in} = \underline{3.75\text{in}}$$

2.2 What is the beam's minimum depth d (in.)?

$$d = h - \text{COVER} - \frac{1}{2} \text{REBAR DIAMETER}$$

$$= 6\text{in} - 1\text{in} - \frac{1}{2}\left(\frac{1}{2}\text{in}\right) = \underline{4.75\text{in}}^*$$

2.3 Compute As =

$$\longrightarrow 2 \text{ REBAR} = 2(0.20 \text{ in}^2) = \underline{0.40 \text{ in}^2}^*$$

2.4 What is the value for a (depth of the Whitney compression block)?

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{0.4 \text{ in}^2 (60,000 \text{ psi})}{0.85 (6,000 \text{ psi}) 3.75 \text{ in}} = 1.25 \text{ in}$$

$$M [\text{lb}\cdot\text{ft}] = 98,941 \text{ lb}\cdot\text{in} \left(\frac{\text{ft}}{12 \text{ in}} \right) = 8,245 \text{ lb}\cdot\text{ft}$$

2.5 What is the beam's moment capacity M (lb.-in.)?

$$M = A_s f_y \left(d - \frac{a}{2} \right) = 0.40 \text{ in}^2 (60,000 \text{ psi}) \left[4.75 \text{ in} - \frac{1.25 \text{ in}}{2} \right] = 98,941 \text{ lb}\cdot\text{in}$$

$$\beta_1 c = a^*$$

2.6 What is the predicted strength P (k) based on the tension model?

$$P_{\text{TENSION}} = \frac{M}{4 \text{ in}} = \frac{98,941 \text{ lb}\cdot\text{in}}{4 \text{ in}} = 24,735 \text{ lb}^*$$

2.7 What is the development length for the reinforcing steel?

$$l_d = \frac{f_y d_b}{24 \sqrt{f'_c} \left(\frac{c}{d_b} - \frac{1}{2} \right)} = \frac{60,000 \text{ psi} (0.5 \text{ in})}{24 \sqrt{6,000 \text{ psi}} \left[\frac{1.67 \text{ in}}{0.5 \text{ in}} - \frac{1}{2} \right]} = 5.7 \text{ in} < 8 \text{ in o.k.}$$

$$c = a / \beta_1 = \frac{1.25 \text{ in}}{0.75} = 1.67 \text{ in}$$

$$L_{dh} = \frac{1,200 d_b}{\sqrt{f'_c}}$$

2.8 What is the predicted strength P (k) based on the shear model?

$$P_{\text{shear}} = 2 \left(\frac{A_v f_y d}{s} + 2 \sqrt{f'_c} b d \right) = 2 \left[\frac{2 (0.11 \text{ in}^2) 60,000 \text{ psi} (4.75 \text{ in})}{4 \text{ in}} + 2 \sqrt{6,000 \text{ psi}} (3.75 \text{ in}) 4.75 \text{ in} \right] = 30,869 \text{ lb}$$

2.8a Which value for P controls the design? $S =$

2.9 What is the beta value for this design?

$$\beta_1 = 0.75$$

$$f'_c \leq 4000 \text{ psi} \Rightarrow \beta_1 = 0.85$$

$$f'_c \geq 4000 \text{ psi} \Rightarrow \beta_1 = 0.85 - 0.05 \left(\frac{f'_c - 4000}{1000} \right) \geq 0.65 = 0.75$$

6,000 psi

2.10 What is the reinforcement ratio ρ for tension control ($c/d = 0.375$)

$$\rho = 0.85 \beta_1 \frac{c}{d} \frac{f'_c}{f_y} =$$

2.11 What is the reinforcement ratio ρ for the RC beam

$$\rho = \frac{A_s}{bd} =$$

2.12 What is the estimated weight of the beam (lb.)?

$$W = \frac{bhL}{1728 \text{ in.}^3/\text{ft.}^3} \left(\frac{145 \text{ lb.}}{\text{ft.}^3} \right) + \frac{A_s L}{1728 \text{ in.}^3/\text{ft.}^3} \left(\frac{490 \text{ lb.} - 145 \text{ lb.}}{\text{ft.}^3} \right)$$

$$W = \frac{3.75 \text{ in} (6 \text{ in}) 30 \text{ in}}{1728 \text{ in.}^3/\text{ft.}^3} \left[\frac{145 \text{ lb}}{\text{ft.}^3} \right] = 56.64 \text{ lb}$$

$$= \underline{59.04 \text{ lb}}$$

$$\frac{2(0.2 \text{ in}^2)(30 \text{ in})}{1728 \text{ in.}^3/\text{ft.}^3} [490 - 145] \frac{\text{lb}}{\text{ft.}^3} = 2.40 \text{ lb}$$

2.13 What is the estimated SWR?

$$\text{SWR} = \frac{\text{Ultimate Load (lb.)}}{\text{Beam Weight (lb.)}} = \frac{24,735 \text{ lb}}{59.04 \text{ lb}} = \underline{\underline{419}}$$

2.14 What is the estimated cost of this beam?

$$\text{Cost of steel} = \frac{A_s L}{1,728 \frac{\text{in.}^3}{\text{ft.}^3}} \left(490 \frac{\text{lb.}}{\text{ft.}^3} \right) \left(\frac{\$1,000}{\text{ton}} \right) \left(\frac{\text{ton}}{2,000 \text{ lb.}} \right) = \$1.70$$

(0.4 in²) 30 in

$$\text{Cost of cement} = \frac{bhL}{1,728 \frac{\text{in.}^3}{\text{ft.}^3}} \left(\frac{W_{\text{cement}}}{27 \text{ ft.}^3} \right) \left(\frac{\$150}{\text{ton}} \right) \left(\frac{\text{ton}}{2,000 \text{ lb.}} \right) = \$0.83$$

768

$$3.75 \text{ in} (6 \text{ in}) (30 \text{ in}) = 675 \text{ in}^3$$

$$\text{Cost of coarse aggregate} = \frac{bhL}{1,728 \frac{\text{in.}^3}{\text{ft.}^3}} \left(\frac{W_{\text{CA}}}{27 \text{ ft.}^3} \right) \left(\frac{\$25}{\text{ton}} \right) \left(\frac{\text{ton}}{2,000 \text{ lb.}} \right) = \$0.30$$

1641

$$\text{Cost of fine aggregate} = \frac{bhL}{1,728 \frac{\text{in.}^3}{\text{ft.}^3}} \left(\frac{W_{\text{FA}}}{27 \text{ ft.}^3} \right) \left(\frac{\$15}{\text{ton}} \right) \left(\frac{\text{ton}}{2,000 \text{ lb.}} \right) = \$0.14$$

1251

$$\underline{\text{COST} = \$2.97}$$

2.15 What is the cost-adjusted SWR of this beam?

$$\text{If the cost} > \$2.25, \text{ then: } \text{Cost Factor} = \frac{\$2.25}{\text{Cost}}$$

$$\text{SWR}_{\text{Adjusted}} = \text{SWR} \times \text{Cost Factor} = 419 \times \frac{\$2.25}{\$2.97} = \underline{\underline{317}}$$

