A model is a pattern, plan, representation, or description designed to show the main object or workings of an object, system, or concept.

Models may also refer to abstractions, concepts, and theories.

What is a heuristic?

A heuristic is a plausible or reasonable approach that often proves useful.

We cannot guarantee the efficacy of heuristics.

Modeling is an art rather than a science.
Why Models are Important?

- The model is as important as your answer
- You cannot evaluate your answer unless you know the assumptions made in the model
- It is often more important to identify your model than to compare answers

Modeling

- Group Modeling Problem
  - Form groups of about three or four people
  - Assign one person to be the note taker

Time for Ping-Pong?

- Take 60 seconds to answer the following question:
  - How many ping-pong balls could you fit into this room?

What is your Answer?

- How did you get your answer?
- Did you guess?
- Did you build a model?
- Can you describe your model?

Review of 60 Second Model

- Did you answer "lots" or "hundreds" or "millions"?
- What have you accomplished with your answer?
- Did you develop a predictive model?
- Did you try a volumetric calculation?

Review of 60 Second Model

- This is not a completely useless exercise; there is a difference between "some" and "lots"
- If you used a volumetric model, how did you model the room?
- What simplifications or assumptions did you make?
Review of 60 Second Model

- Did you ignore the furniture?
- Did you account for the irregular shape of the room?
- Did you assume the ping-pong balls could deform?

60 Second Model

Time for Ping-Pong?

- Take 2 minutes to answer the following question:
  - How many ping-pong balls could you fit into this room?

What is your Answer?

- How did you solve the problem this time?
  - Did you refine your 60 second model?
  - Did you change your model?
  - Did you modify your assumptions?
  - Given more resources, did you build a more sophisticated model?

Three Minute Model

- Assume the following:
  - $L$ is the length of the room (in.)
  - $W$ is the width of the room (in.)
  - $H$ is the height of the room (in.)
  - $D$ is the diameter of a ping-pong ball (in.)

$$V_{room} = LWH$$
Three Minute Model

- The volume of a ping-pong ball is:
  \[ V_{\text{ball}} = D^3 \]

- Therefore the number of ping-pong ball, \( n \), can be estimated by:
  \[ n = \frac{V_{\text{room}}}{V_{\text{ball}}} \]

Visual Comparison of the Models

60-second model

2-minute model

What Did We Learn?

- Some things to consider about this example:
  - A "rough" answer is better than no answer
  - A model is a partial not a complete representation
  - The design of a model depends on the constraints
  - A symbolic representation is "clean" and powerful
  - An explicit model is an indispensable tool for solving problems

The Real World and the Model World

- The room is the real world
- The model world is likely more like this:
The Real World and the Model World

A model is like a caricature

- Certain features are emphasized at the expense of other features
- Identify aspects of the real world that are relevant

Occam’s Razor

William of Occam was a 14th century English philosopher who propounded the heuristic:

"entia non sunt multiplicanda praeter necessitatem"


Occam’s Razor

- You should eliminate all unnecessary information relating to a problem
- Occam was reputed to have a sharp, cutting mind - thus this heuristic is called: Occam’s Razor

Occam’s Razor

Think of the model world connected to the real world by a passage guarded by a mythical customs officer with Occam’s razor

- Leaf Area
- Tree Mass
- Oxygen Production

Occam’s Razor

A "bad" model is either:

- Using the razor too little (letting irrelevant details in the model)
- Using the razor too much (cutting essential features of the real world)
Occam's Razor

- How do we reach a balance?
  - Should the furniture be in the model?
  - Is the cube a good model of the ball?

Occam's Razor

- There are no hard-and-fast rules
- We do have some guidelines:
  - The objective of the model
  - Constraints on resources
- How do we apply these guidelines to the ping-pong ball problem?

Time for Ping-Pong?

What is the "best" answer to the question:

How many ping-pong balls could you fit into this room?

Considerations for “Best” Model?

- Define the problem
- What are your objectives?
- Should we measure the room more accurately?
- Fill the room up with ping-pong balls and count them?
- Thinking of the "best" answer is equivalent to making a wish list of things you would like to have in your model world
  - “Musts” and “Wants”

Upper and Lower Bounds of the Model

- Suppose you model a ping-pong ball as a sphere instead of a cube
  \[ V_{ball} = \frac{\pi D^3}{6} \]
- Suppose the balls are packed so that there are no air gaps (upper bound)
  \[ n = \frac{6LWH}{\pi D^3} \]

Upper and Lower Bounds of the Model

- If you compare the upper bound to the lower bound:
  \[ \frac{Upper}{Lower} = \frac{6}{\pi} \]
- This is a ratio of nearly 2
Comparing Assumptions

- The room is shaped like a box
- The ping-pong ball is assumed to be a cube
- Furniture in the room is ignored
- Windows and door spaces are ignored

Which assumptions should we relax?

Packing Spheres

- A more "realistic" representation of the ping-pong balls

H/D = sin 60°  \[ H = 0.866D \]

The distance between the center of two balls is reduced by nearly 14%

What Did We Learn?

- You should risk making "back of the envelope" calculations and recognize when they are appropriate
- Match model resolution with available resources
- Awareness of assumptions
- Power of symbolic representation
- "Looking Back" is a good heuristic for problem solving, but it is vital for learning

Questions?